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Frequency-guessing method for reduced modeling of energetic particle modes (EPM)

... utilizing resonant auto-optimization remnants of imperfect time-scale separation

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Presenter: Andreas Bierwage QST/Japan

Contact: bierwage.andreas@qst.go.jp



Vinícius N. Duarte, Pablo Oyola,
Roscoe B. White **PPPL / U.S.A.**



Collaborators

Kouji Shinohara
Univ. of Tokyo / Japan



1. Subject of interest: Bursts of chirping EPMs

Challenges:

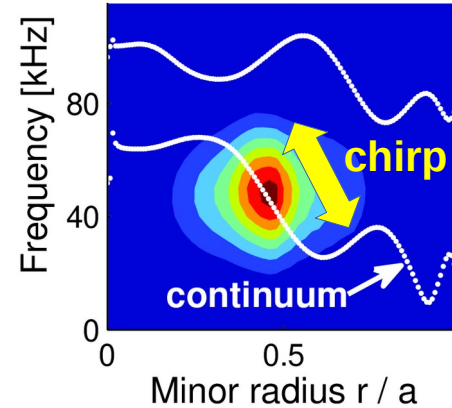
- ▶ MHD-PIC hybrid sims (with sources & collisions) can take **days** for single burst to **months** for computation of quasi-steady fast ion distribution.
- ▶ Existing **wave function-based perturbative models** require **prior knowledge of mode structure & freq.**
 - Easy for discrete AEs. More difficult for modes inside dense / continuous spectra, like EPMs.

Long-t sim: Bierwage *et al*,
Nucl. Fusion **57** (2017) 016036;
Nat. Comms. **9** (2018) 3282.

Starting point:

- **Long-wavelength $n=1$ EPM** as in N-NB-driven JT-60U has spatial structure that varies little during up+down chirp (as evidenced by beating).
 - **Need to guess only initial (pre-chirp) seed freq. ω_0 .**

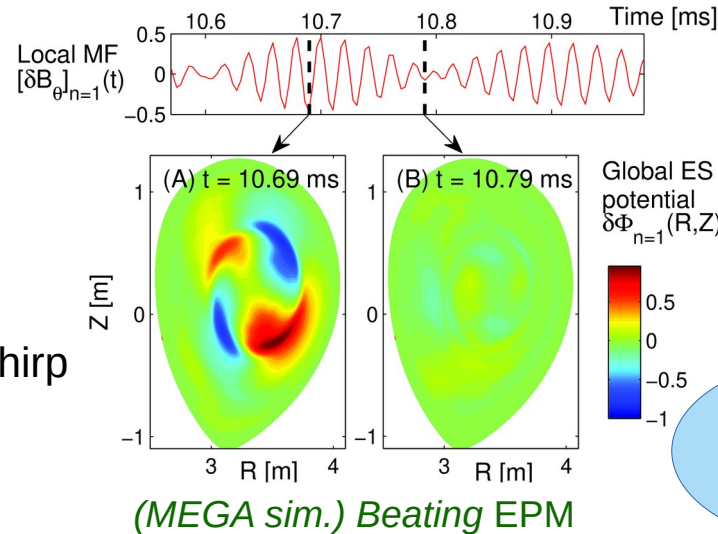
(MEGA simulation)
EPM spectrogram



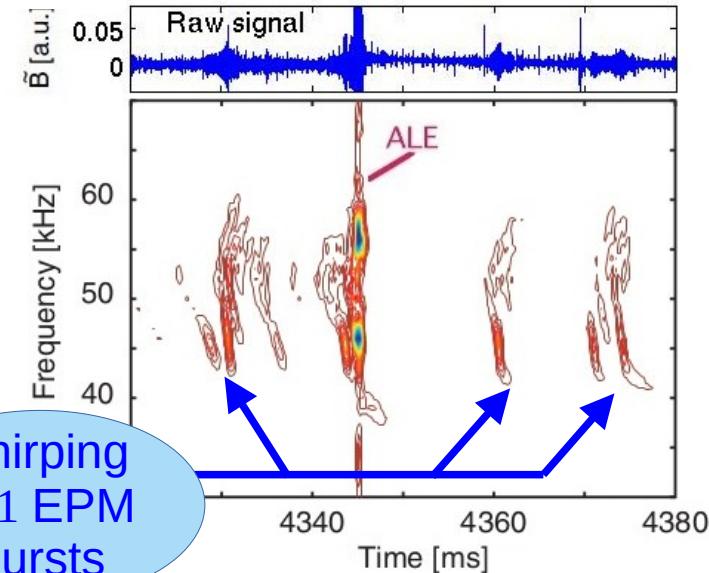
Theory: L. Chen *et al*,
Phys. Rev. Lett. **52** (1984) 1122;
Phys. Plasmas. **1** (1994) 1519.

JT-60U exp: Shinohara *et al*,
Nucl. Fusion **41** (2001) 603;
Nucl. Fusion **42** (2002) 942.

JT-60U sim: Todo *et al*,
JPFRR **79** (2003) 1107;
Phys. Plasmas **12** (2005) 012503.



(MEGA sim.) Beating EPM



2. Focus: Frequency $\omega(t)$ of large-scale EPM with robust structure

Effective signal seen by simulation particles:

$$\delta\Phi = \underbrace{A_0 \delta\tilde{\Phi}(r) e^{in\zeta - im\vartheta - i\omega_0 t}}_{\text{prescribed seed wave}} \underbrace{\hat{A}(t) e^{-i\phi(t)}}_{\text{dynamic part}} - \text{c.c.}$$

- Amplitude $A(t)$
- Phase $\phi(t)$
- Chirp $\delta\omega = \dot{\phi}$

Assumptions:

- (I) The radial mode structure $\delta\tilde{\Phi}(r)$ for each set of poloidal and toroidal Fourier mode numbers (m, n) as well as the mode number spectrum can be prescribed arbitrarily and does not evolve in time.
- (II) The evolution of the frequency $\omega(t) = \omega_0 + \dot{\phi}(t)$ is not subject to any constraints from the bulk plasma response function. The choice of ω_0 is arbitrary and $\phi(t)$ is determined by interactions with fast ion.
- (III) The mode is subject to ω - and t -independent, spatially uniform damping $0 \leq \gamma_d = \text{const.}$
- (IV) The amplitude modulations $A(t)$ and dynamic phase shifts $\phi(t)$ are assumed to vary slowly in the sense that $d \ln |A| / dt \ll \omega_0$ and $d\phi / dt \ll \omega_0$

• Lagrangian formulation of perturbative model:

Pinches *et al*, *Comp. Phys. Comm.* **111** (1998) 133

HAGIS code

$$\mathcal{L}_{\text{sys}} = \underbrace{\mathcal{L}_{\text{fp}} + \mathcal{L}_{\text{int}}}_{\substack{\text{Particle Lagrangian given} \\ \text{in Eq. (6)}}} + \underbrace{\mathcal{L}_{\text{bulk}} + \mathcal{L}_{\text{em}}}_{\substack{\text{Wave Lagrangian,} \\ \mathcal{L}_w \\ \text{Wave equations}}}$$

perturbative changes in the frequency eigenvalue

$$\mathcal{L}_w = \sum_{k=1}^{N_w} \frac{E_k}{\omega_k} [A_k^2 \dot{\phi}_k], \quad \boxed{A_k(t) e^{-i\phi_k(t)} = \mathcal{X}_k(t) - i\mathcal{Y}_k(t)}$$

(k : mode index)

$$\boxed{\delta\omega_k / \omega_k \ll 1}$$

$$\text{where } E_k = \frac{1}{2\mu_0} \int_V \frac{|\nabla_{\perp} \tilde{\Phi}_k|^2}{v_A^2} d^3x$$

and v_A is the Alfvén velocity.

See also

Y. Chen & White, *PoP* **4** (1997) 3591

Y. Chen *et al.*, *PoP* **6** (1999) 226.

R.B. White *et al.*, *PoP* **27** (2020) 052108.

ORBIT code
→ used here

P. Oyola, 2026 *TM-EP* invited (Monday 9/28)

ASCOT code

3. “Flaw” & opportunity: Time-scale separation is not enforced

Model in textbook & early ORBIT papers:

- Avg. $\langle \dots \rangle$ over wave period τ_0 acts as low-pass filter

$$\frac{dA}{dt} = -\frac{C_A^2}{A\omega_0^2} \left\langle \int d\mathbf{x} \mathbf{J}_f \cdot \delta \mathbf{E}_\perp \right\rangle_{\tau_0} - \gamma_d A, \quad (4a)$$

$$A \frac{d\phi}{dt} = -\frac{C_A^2}{A\omega_0^3} \left\langle \int d\mathbf{x} \mathbf{J}_f \cdot \partial_t \delta \mathbf{E}_\perp \right\rangle_{\tau_0}, \quad (4b)$$

filtering

→ Currently testing this in ASCOT.

Model actually implemented: (ORBIT, HAGIS)

- Truncation of fast terms ($d_t^2 A$, $d_t^2 \phi$, $d_t A d_t \phi$, ...)
after time-scale-separation-based ordering. Result:

$$\frac{dA}{dt} = -\frac{v_A^2}{\omega_0} \sum_k [(\varrho_{\parallel} B^2 \alpha - \delta \Phi) \cos \Omega_{mn}]_k - \gamma_d A, \quad (5a)$$

$$\frac{d\phi}{dt} = -\frac{v_A^2}{A\omega_0} \sum_k [(\varrho_{\parallel} B^2 \alpha - \delta \Phi) \sin \Omega_{mn}]_k, \quad (5b)$$

where we have discretized the phase space integrals and represented them as sums over simulation particles k .

- No filtering, so remnants of fast dynamics ($\tau < \tau_0$) still exist in num. solution of initial value problem!
- This entails potential for uncertainty & utility ...

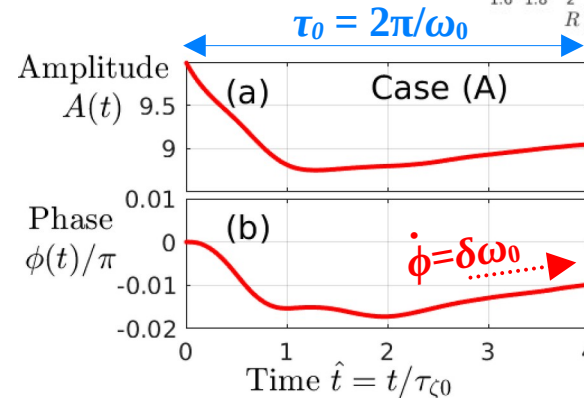
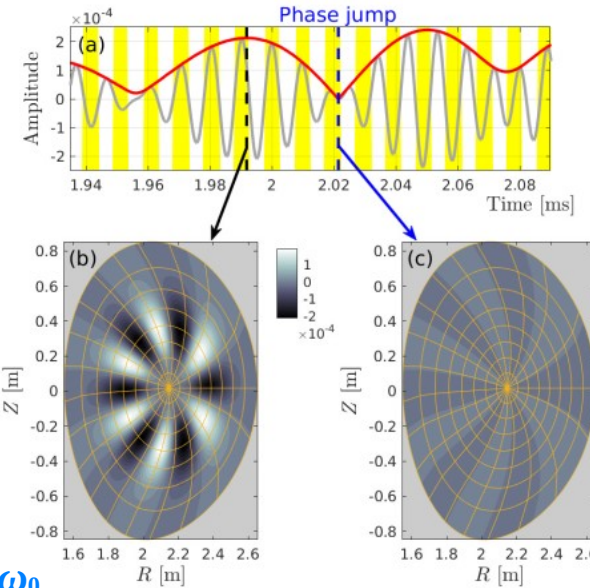
Consequence 1:

Model can capture linear beating with

$$[d/dt]^{-1} \ll \tau_0$$

(discontinuous growth & abrupt phase jumps in A and ϕ).

Bierwage, White & Duarte,
Plasma Fus. Res. **16**
(2021) 1403087



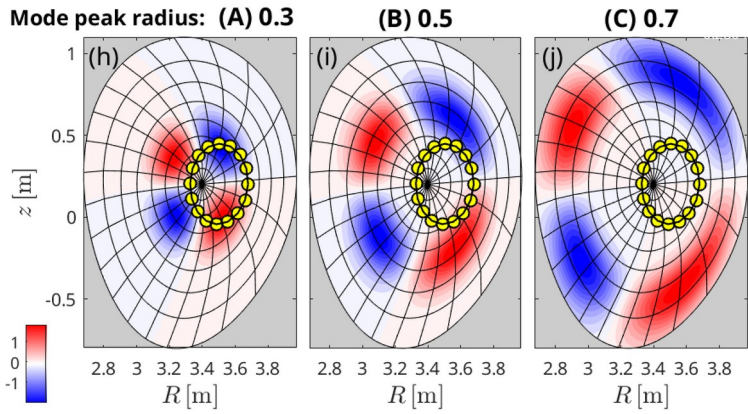
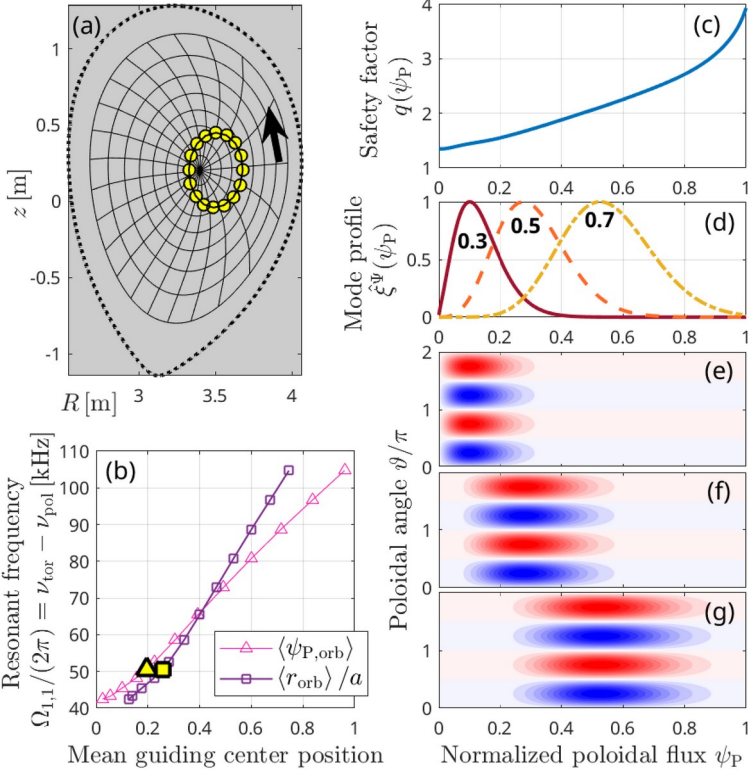
Consequence 2:

Prompt frequency shift $\omega_0 \rightarrow \omega_0 + \delta\omega_0$ occurs when initial guess ω_0 doesn't yield maximal resonant pwr transfer.

- Resonance auto-optimization!
- ... turns out to be incomplete ... but still helpful!

4. Characterization of prompt frequency shift

- ▶ 0 Setup of numerical experiments (with simplification & exaggeration)
- ▶ I Scan of mode peak location (fixed seed frequency & damping)
 - II Role of gradients and damping [→ See paper]
- ▶ III Scan of seed frequency (fixed mode structure & damping)
 - Test “resonant auto-optimization” hypothesis
- IV Discussion [→ See paper]
 - Potential caveats & systematic biases
 - Partial auto-optimization



- JT-60U-like geom. • Test mode structures
- Marker loading procedure (Hamiltonian guiding centers)

$$\mathcal{E}' = \mathcal{E} - \omega_0 P_{\zeta} / n \quad (\text{"rotating frame energy"})$$

$$P_{\zeta}(t) = -Ze\Psi_{\text{P,gc}}(t) + Mu(t)g(\Psi_{\text{P,gc}}(t)),$$

$$\mathcal{E}(t) = Mu^2(t)/2 + \mu B_{\text{gc}}(t) + Ze\delta\Phi(\mathbf{X}_{\text{gc}}(t), t),$$

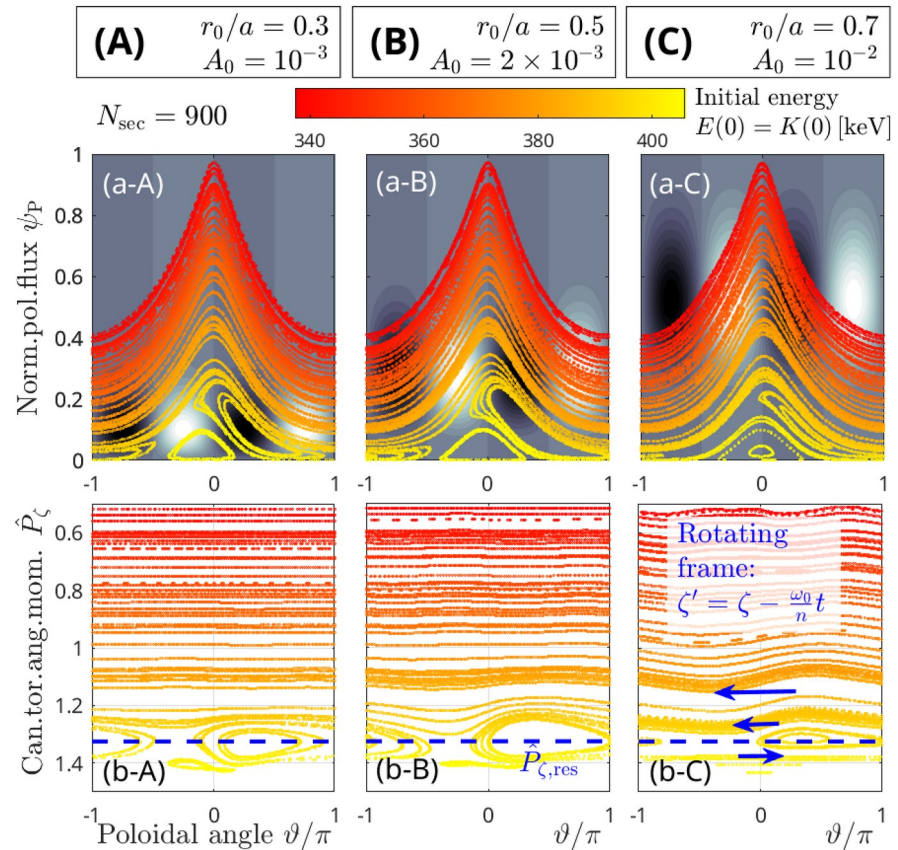
$$\mathcal{E}'(0) = K(0) - \omega_0 P_{\zeta}(0)/n = \text{const.}$$

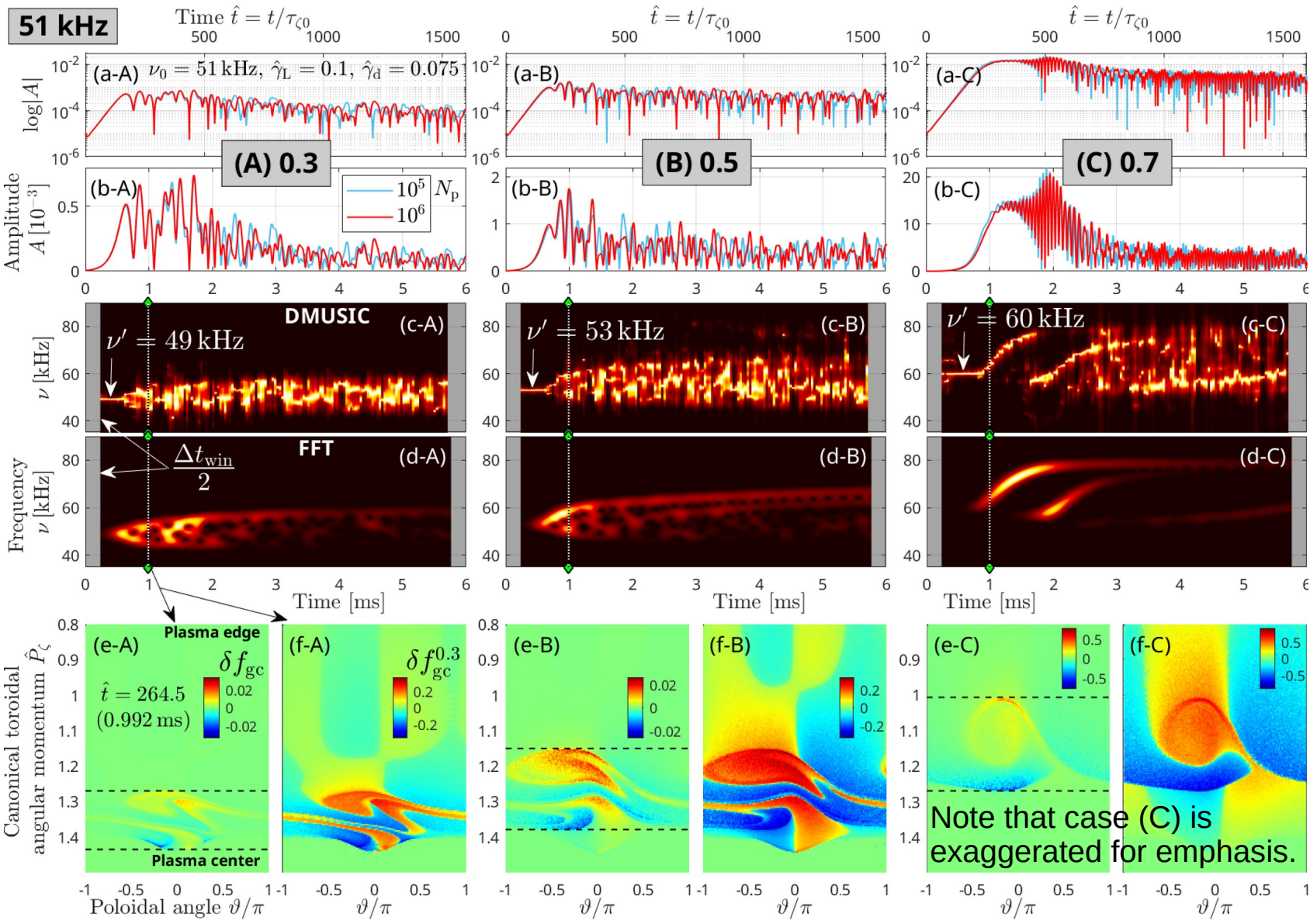
Assume constant mode damping and EP density gradient

→ Uniform distrib. of free energy.

Meanwhile, mode structure & magnetic drifts give nonuniform energy conversion.

Note that case (C) is exaggerated for emphasis.





Scan of mode peak location r_0

Frequency shifts

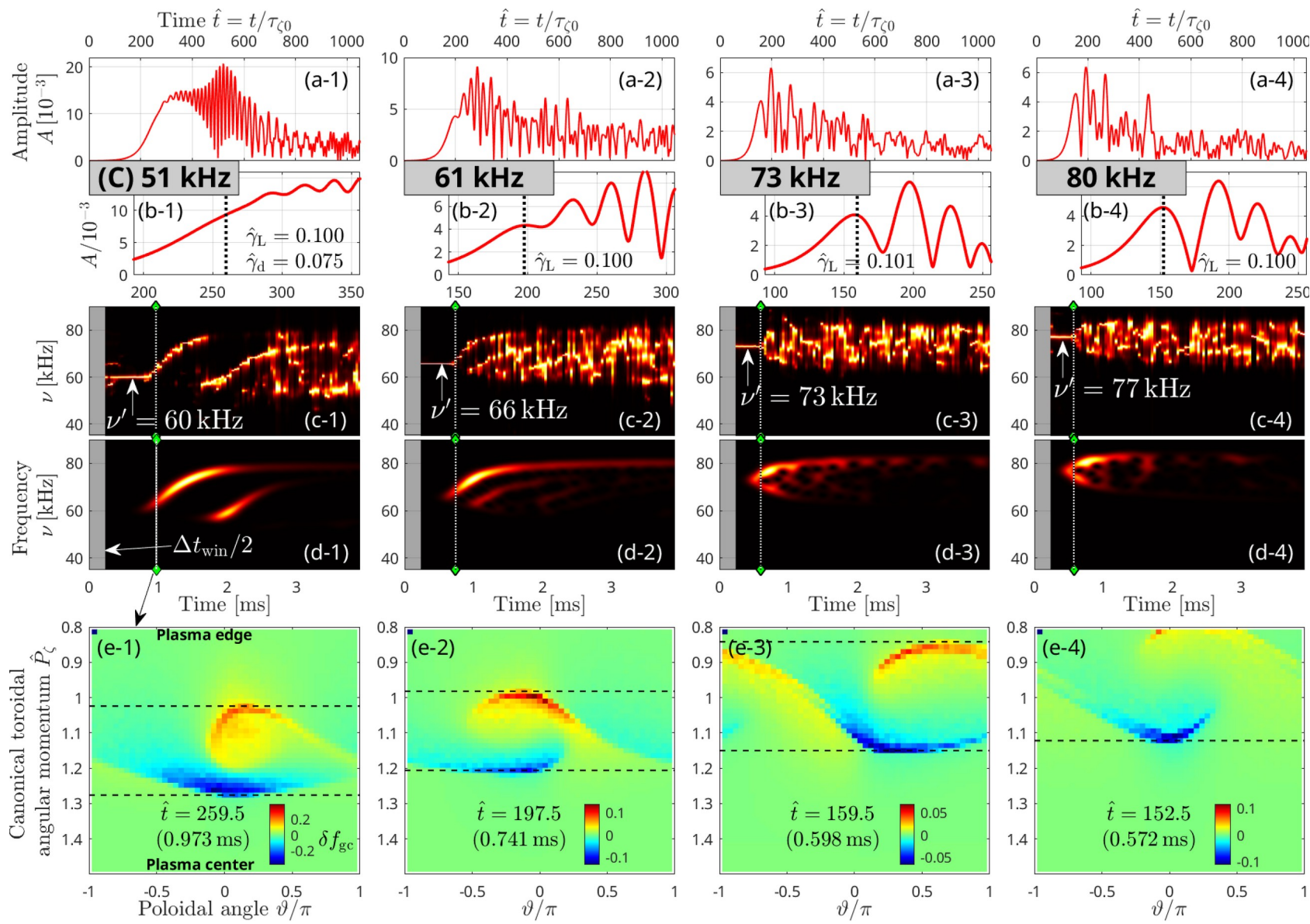
$$\nu_0 = 50.6, \text{ kHz}$$

$$\rightarrow \nu' = \nu_0 + \delta\nu_0$$

$$\approx \{49, 53, 60\} \text{ kHz.}$$

seem to be directed towards mode's peak, where energy conversion rate is maximized (for uniform gradient).

Lasting effect, even during long-term NL dynamics.



Scan of seed frequency for $r_0/a=0.7$

Test hypothesis that prompt frequency shift is a manifestation of “resonant auto-optimization”.

→ Iterate: New seed frequency is old shifted freq.

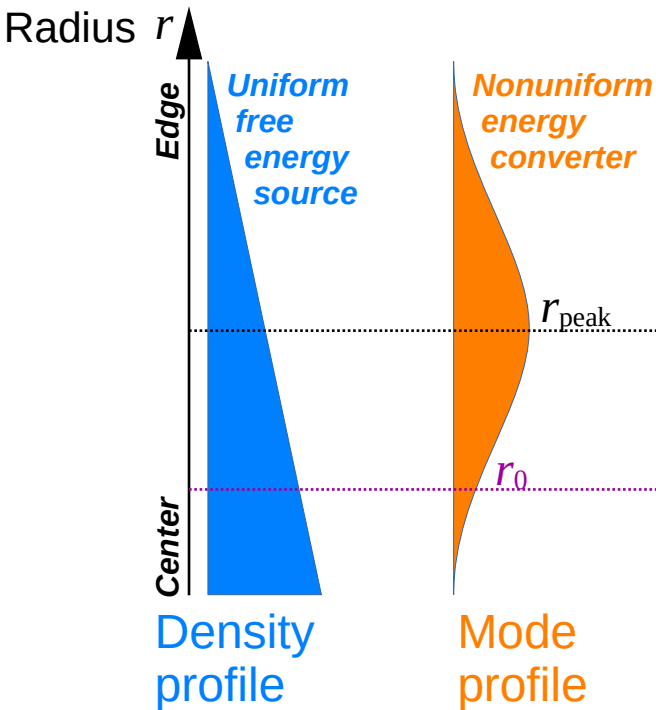
$$\nu_0^{(k)} = \nu_0^{(k-1)} + \delta\nu_0^{(k-1)}$$

Persistent prompt frequency shift implies incomplete optimization.

5. Summary & proposal: Use remnant auto-optimization as a guide

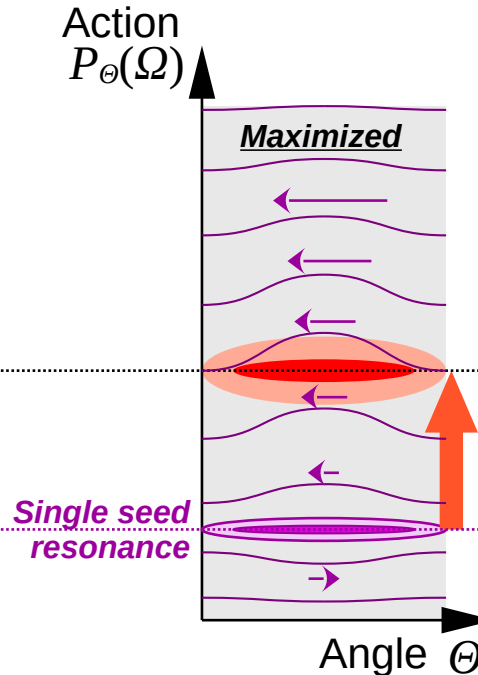
Simplified setup:

Used for characterization & proof of principle



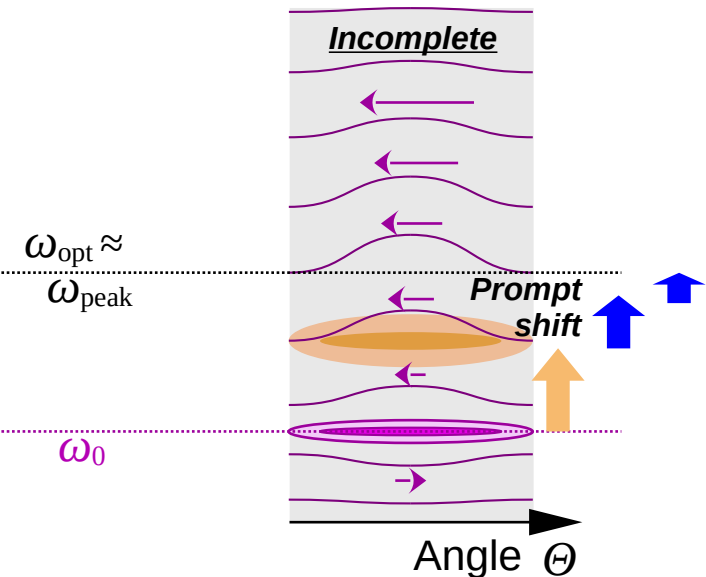
Non-perturbative model:

Complete auto-optimization expected



Perturbative model (without filter):

Incomplete auto-optimization found



Phase space topology

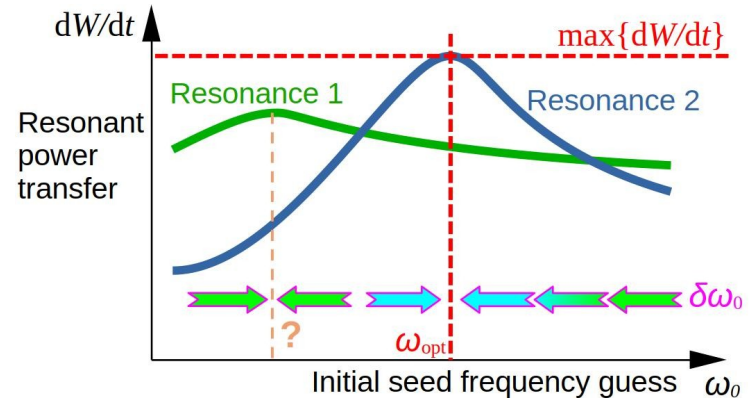
Can find ω_{opt} by iteration!

6. Conclusion & outlook:

The prompt frequency shift is interpreted as a remnant of imperfect time scale separation. Thus, its utility could only be investigated empirically. Results so far are promising that $\delta\omega_0$ can be useful for finding the EPM seed frequency ω_{opt} , which is largely determined by resonances.

In our simplified setup, the optimal seed frequency ω_{opt} was quite obvious (mode peak). This may not be so under in more realistic condition with nonuniform gradients and competing resonances.

When using complete fast ion distributions that may drive multiple resonances simultaneously, we recommend to begin with a scan of ω_0 over the expected range of possible EPM frequencies, because the optimal frequency ω_{opt} may be obscured by other resonances.



EPMs should also be guided by continuous spectra.

Thus, it is necessary to look for ways to include that information about the preferred plasma response in the reduced model. One idea is to incorporate information about continuous spectra in a radially nonuniform and frequency-dependent damping rate $\gamma(\omega, r)$. Similarly, changes in the mode's spatial structure $\delta\Phi(x)$ could also be modeled based on information about the continuous spectra ($\rightarrow$ peak location) and magnetic drifts ($\rightarrow$ radial width). The reduced simulations could be initialized with a set of test wavefunctions distributed along the continua.