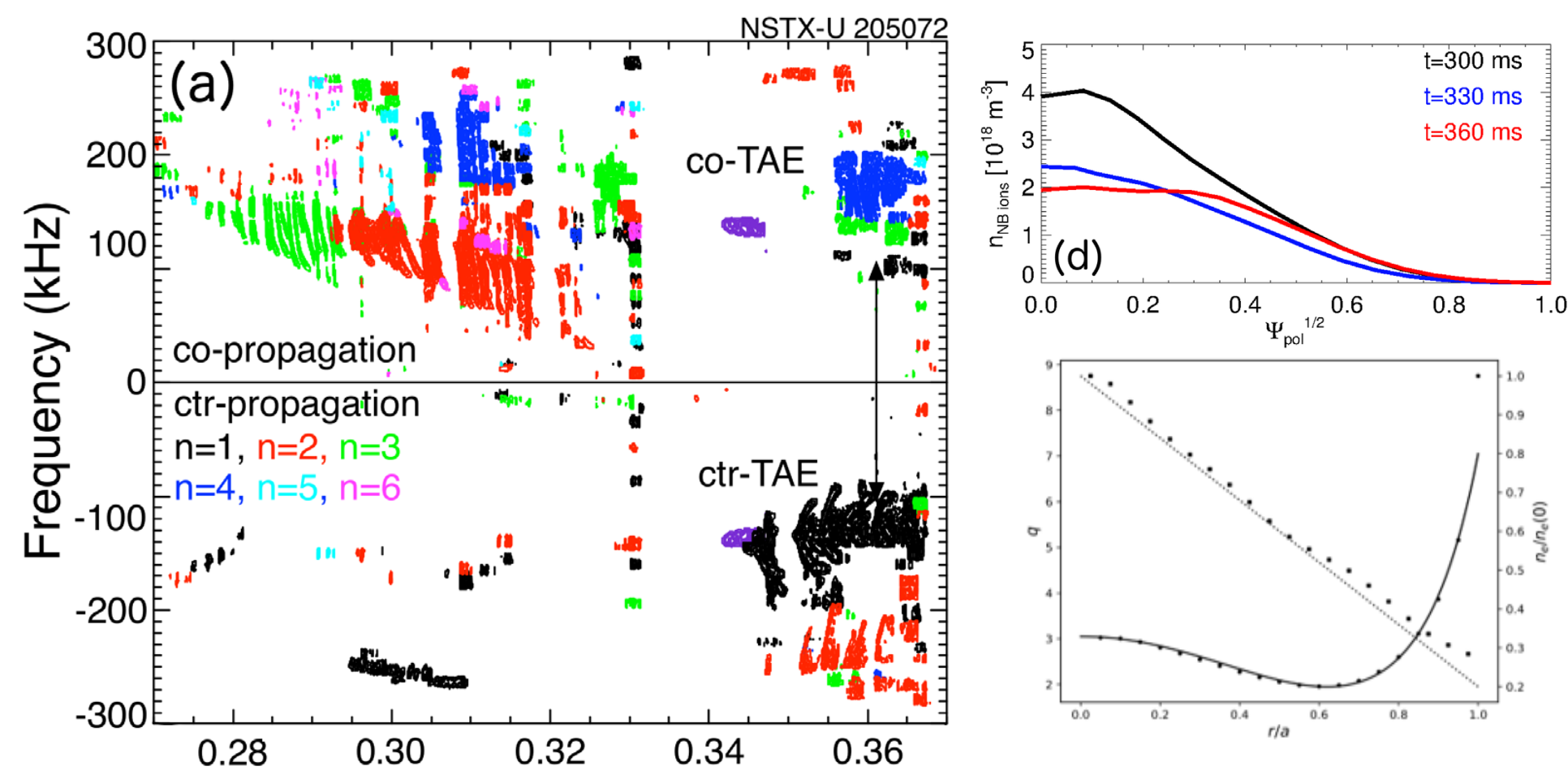


1. Summary

- Mechanisms of destabilization of toroidal Alfvén eigenmodes (TAE) in tokamaks are analyzed with the aim to reveal those leading to modes propagating in the direction opposite to plasma current, i.e. **counter-propagating modes (ctr-TAE)**. Plasmas with fast-ions sources [such as neutral beam injection (NBI), ion cyclotron resonance heating (ICRH), fusion reactions] and without them (Ohmic discharges) are considered.
- A particular NSTX-U experiment with NBI, where co- and counter-propagating TAEs were observed simultaneously [1], is considered.
- Destabilization of $m/n = 2/1$, $m/n = 3/1$ ctr-TAE modes is sensitive to the position of plateau edge of the radial distribution of beam ions. The modes can be destabilized as both co- and ctr-TAEs with only minor changes in the radial distribution.
- Finite width of pitch-angle distribution of beam ions generally reduces the mode growth rate, and can even completely suppress the instability. However, enhancement of instability can also take place.
- This work is based on the recent publication [2] extended to take into account finite width of the pitch distribution of the beam particles.

2. Motivation

In NSTX-U shot #205072 with unbalanced NBI, TAE modes propagating along current and against current were observed simultaneously while beam ion radial profile was flat.



Left, top: Fig. 3(a,d) from [M. Podesta et al. 2018 Nucl. Fusion 58 082023]. Bottom right: q and n_r for NSTX-U shot #205072 at $t = 360$ ms (TRANSP data).

3. Instabilities driven by passing particles

For $\omega < \omega_B$, the resonance condition of wave-particle interaction $\omega - m \langle \dot{\varphi} \rangle + n \langle \dot{\varphi} \rangle = s\omega_B$ for well passing particles with narrow orbits simplifies to

$$\omega = \left(k_{\parallel} \pm \frac{1}{qR} \right) v_{\parallel} - n\omega_{pr}^{pas}.$$

When precession can be neglected, for TAE this condition gives four resonance velocities: two along the plasma current and two opposite to the current. In the simplest case (near the intersection point of cylindrical continuum branches + narrow gap or modes do not lie near the continuum)

$$v_{\parallel*}^{res} = \frac{v_A}{\sigma_k \pm 2},$$

where $\sigma_k = \text{sgn} k_{\parallel}$, $k_{\parallel,m} + k_{\parallel,m+1} = 0$, v_A is the Alfvén velocity, and the subscript $*$ labels values at r_* .

I. Drive by fast ions with flat profile

When the radial profile of fast ions is sufficiently flat in the region of mode location, the contribution of velocity anisotropy dominates. Assuming $n > 0$, $k_{\parallel,m} > 0$ and $k_{\parallel,m+1} < 0$, which corresponds to ctr-TAE, then for co-TAE we will have $n < 0$, $k_{\parallel,m} < 0$ and $k_{\parallel,m+1} > 0$ for $n < 0$, i.e. changing the sign of n just switches which mode component is in resonance. Unbalanced injection is required for excitation of TAE in only one direction (beam ions have one sign of v and only two resonances remain).

II. TAEs driven by temperature gradient

TAE can be destabilized in the absence of fast ions, drawing energy from suprathermal ions in the tail of Maxwellian distribution, thanks to 'kinetic' enhancement of resonance interaction [3]:

$$\frac{d \ln F_{\parallel j}}{dr} = \frac{n'_j}{n_j} + \left(x_{rj}^2 - 0.5 + z_j \frac{\hat{I}_{lj}}{\hat{I}_{lj}} \right) \frac{T'_j}{T_j}$$

where $x_r = v_{\parallel}^{res}/v_T \sim \beta$. The 'kinetic' enhancement factor grows with ion velocity, but the fraction of such ions exponentially decreases, so only moderately suprathermal ions ($x_r \sim 2-3$) are important. Suprathermal ions can destabilize co-TAE for moderate values of β_{i*} . Ctr-TAE could be destabilized by suprathermal electrons, but this requires impractically small values of $\beta_{e*} \sim (M_e/M_i)\beta_{i*}$.

References

- [1] M. Podesta et al. 2018 Nucl. Fusion 58 082023
 [2] Ya.I. Kolesnichenko, E. Fredrickson, V.V. Lutsenko, A.V. Tykhyy 2026 Nucl. Fusion 66 046002
 [3] Ya.I. Kolesnichenko, V.V. Lutsenko, A.V. Tykhyy Phys. Plasmas 31 (2024) 042107

3. Instabilities driven by passing particles (cont.)

III. Drive by plasma current

TAE instabilities in Ohmic discharges can be driven by plasma current due to anisotropy in the velocity distribution of resonance electrons in a shifted Maxwell distribution $F_{\parallel} \propto \exp\{-(v_{\parallel} - u)^2/v_{Te}^2\}$:

$$\frac{d \ln F_{\parallel}}{dv_{\parallel}} = \frac{2(u - v_{\parallel})}{v_{Te}^2} > 0 \quad \text{for } u > v_{\parallel}$$

This mechanism requires a flat radial profile of temperature and concentration in the mode region. co- and ctr-TAE will be destabilized simultaneously, because changing the sign of n simply swaps resonances.

IV. Drive in plasmas with runaway electrons

TAE can be destabilized by barely circulating/trapped runaway electrons, because their resonance velocity is much higher than that of strongly passing particles and may exceed Alfvén velocity. Instability is caused by pressure gradient of runaway electrons, so TAE direction depends on its sign: ctr-TAE when $dp_e/dr < 0$, co-TAE when $dp_e/dr > 0$.

4. Instabilities driven by trapped particles

Trapped fast ions are created during ICRH and perpendicular NBI. Trapped ions have $\langle \dot{\varphi} \rangle \approx 0$, so their resonance condition takes the form

$$\omega = s\omega_B - n\omega_{pr}^{tr}$$

where

$$\omega_{pr}^{tr} \equiv \langle \dot{\varphi} \rangle = -\frac{qv_d}{\epsilon} \left\langle \frac{\partial \ln B}{\partial r} \right\rangle$$

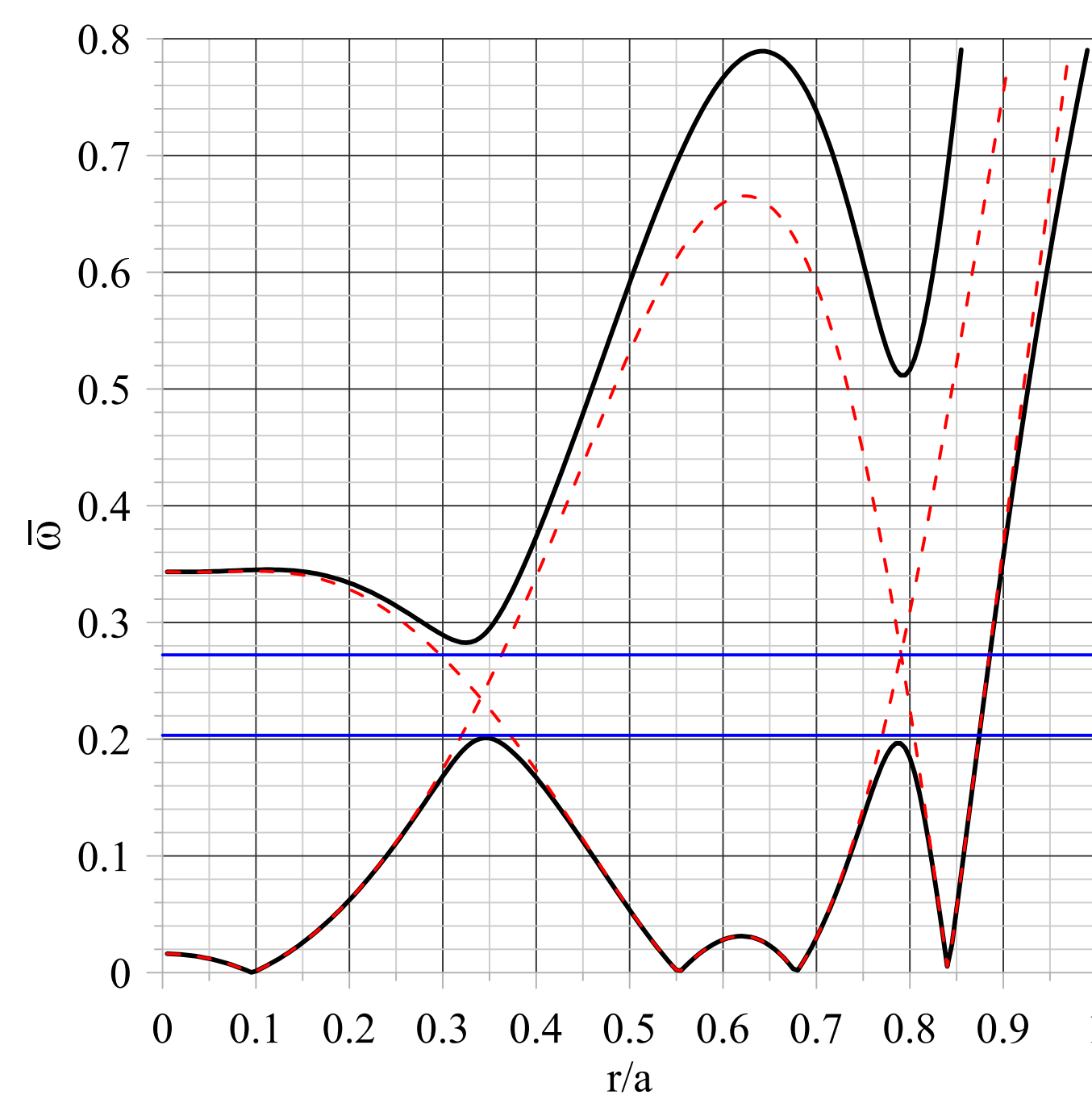
This resonance does not depend on m and exists for both signs of n . The direction of TAE propagation depends on the relation $\nu \equiv |n\omega_{pr}|/\omega_B$:

- $\nu \ll 1$: analogous to case (I) of passing particles (co- and ctr-TAE together)
- $\nu \gg 1$: ctr-TAE can be destabilized when $\omega_{pr} < 0$. In both cases the influence of velocity anisotropy competes with plasma inhomogeneity.
- $\nu \sim 1$: for deeply trapped particles resonance interaction is enhanced when $n\omega = s^2\omega_B^2/(4\omega_{pr})$. This makes modes easier to destabilize. For TAEs, this condition reduces to

$$\frac{2nq^2\rho_A}{s^2\epsilon r} = 1.$$

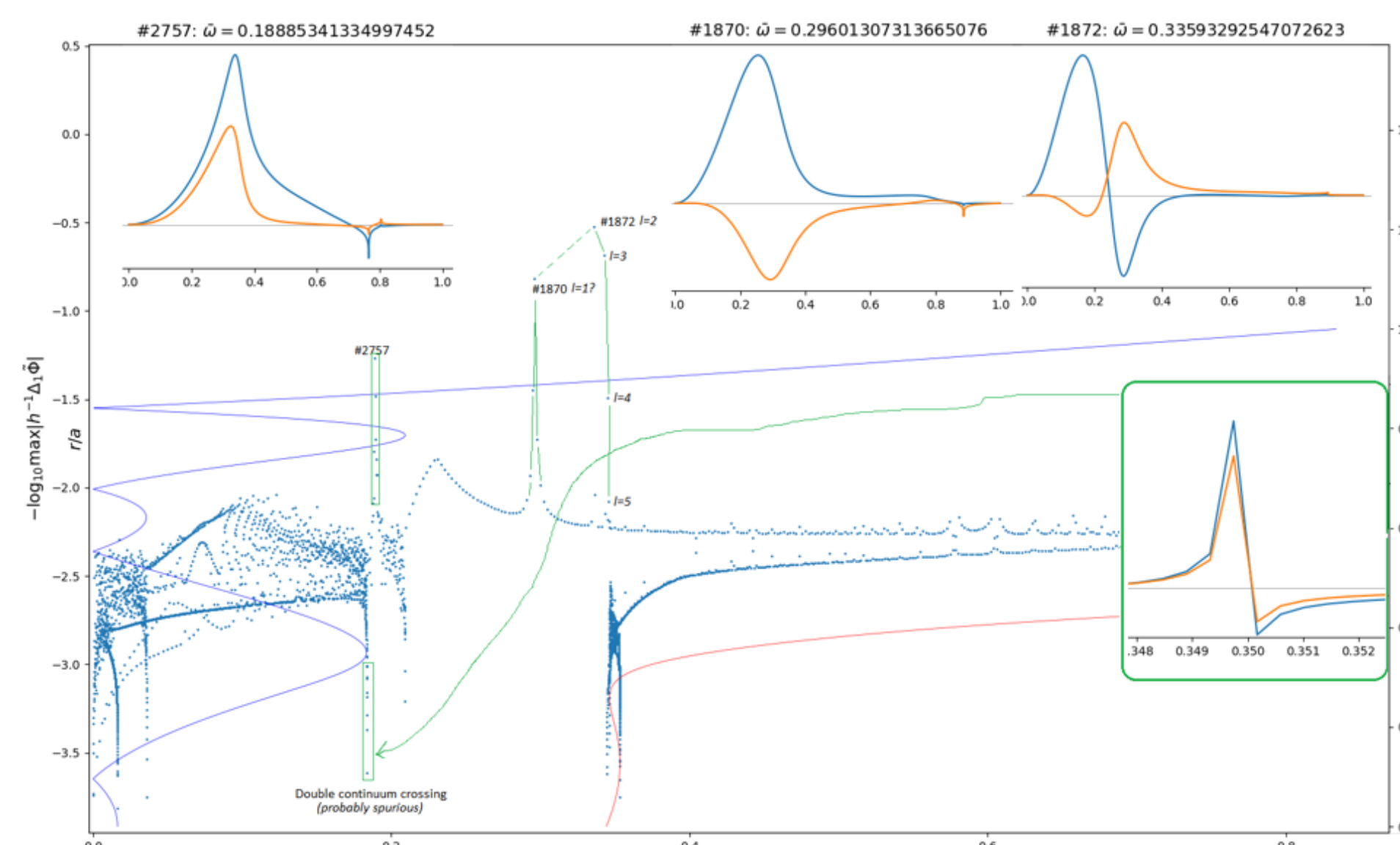
5. Modeling the shot: TAE continuum

The initial mode search was performed with a new code, and confirmed with the code BOA newly adapted for tokamaks.



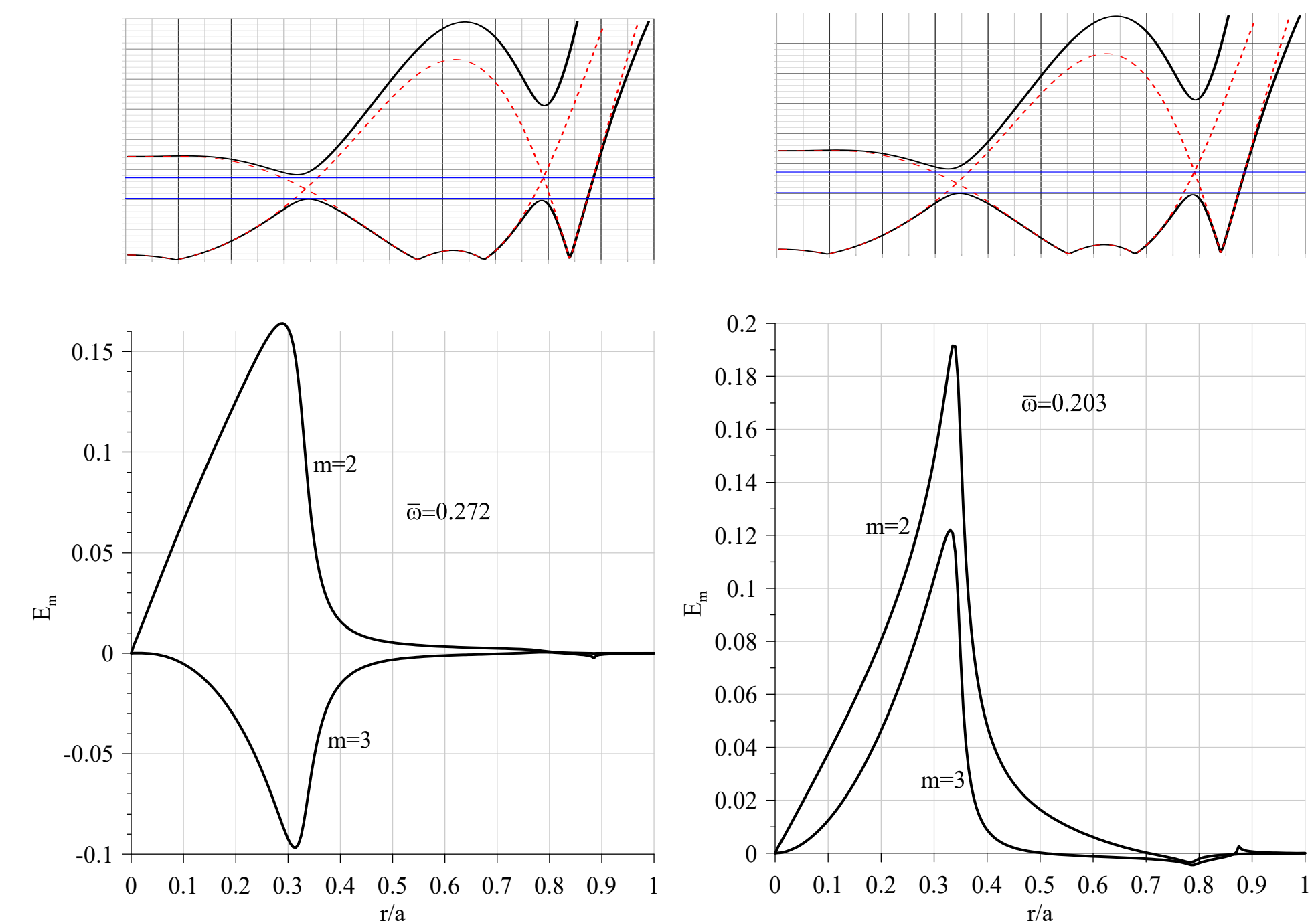
Alfvén continuum for $|n| = 1$ and $|m| = 2/3$

Continuum, numerical spectrum and some of the modes found by the new code with automatic filtering of continuum modes based on compressed Fourier transform of radial structure:

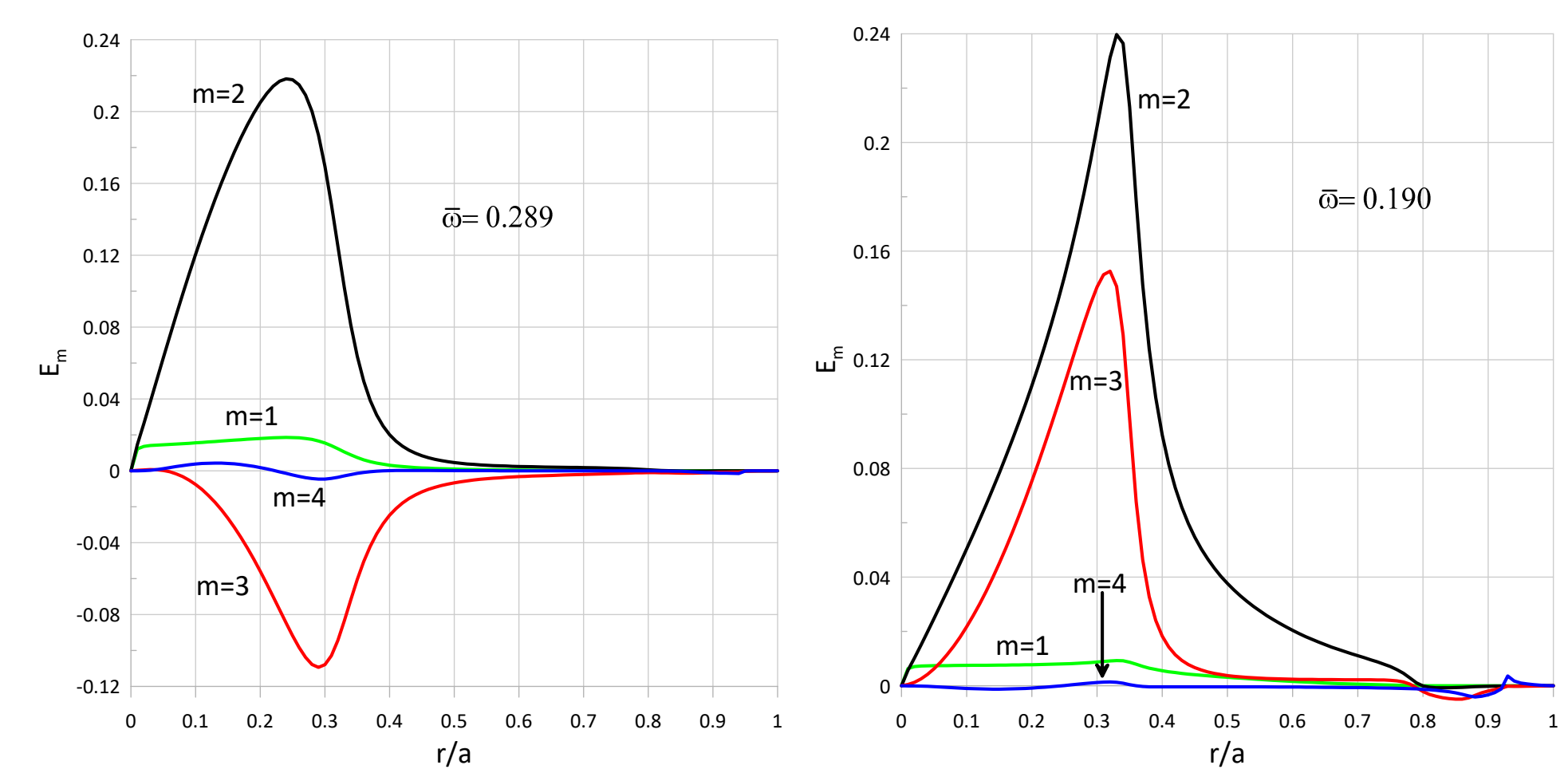


6. Modeling the shot: modes

TAE modes localized around $r/a = 1/3$ with $|n| = 1$, $|m| = 2/3$ were found.

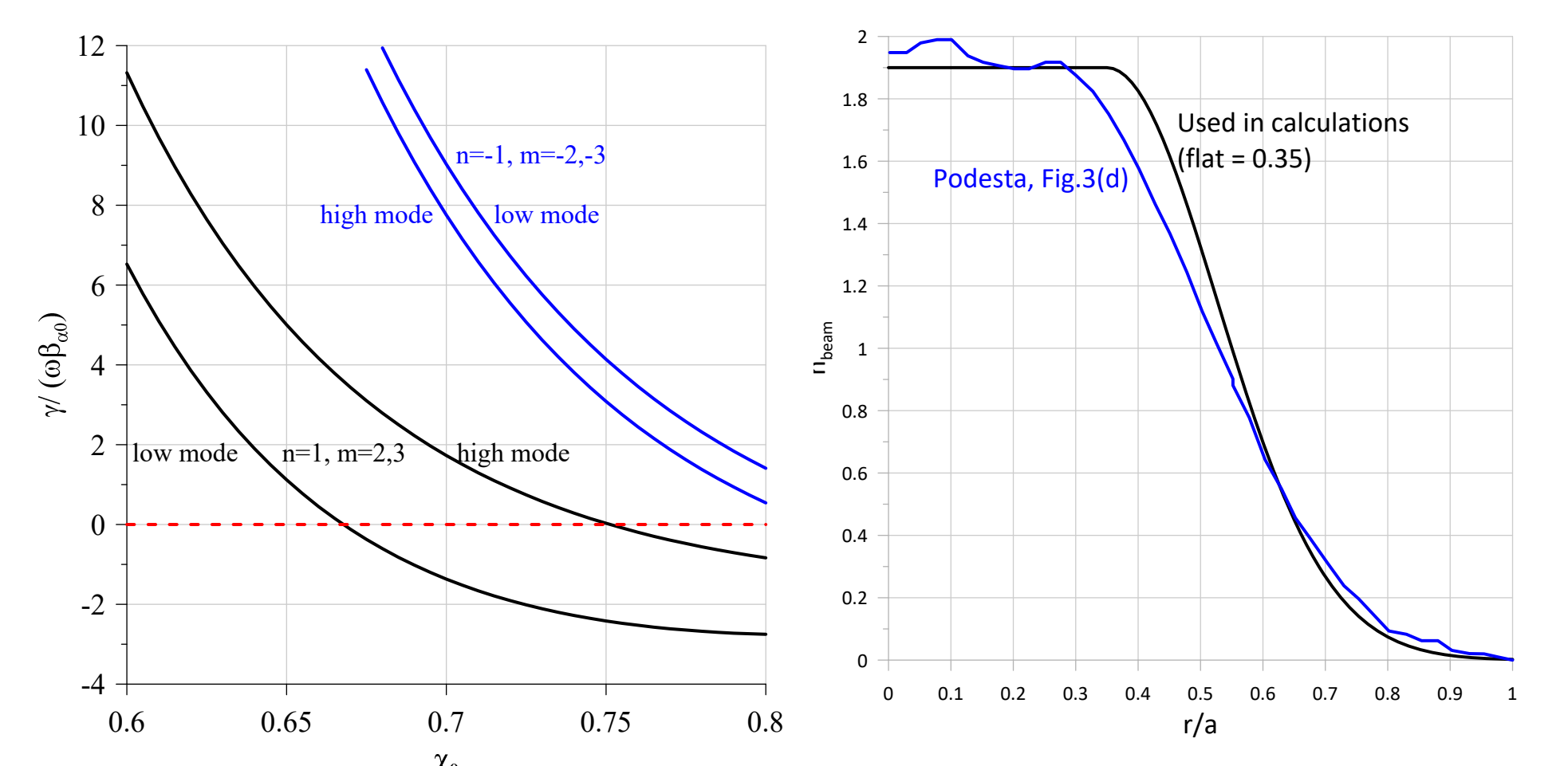


Two-component equations suffice to describe the modes:



7. Modeling the shot: mode drive

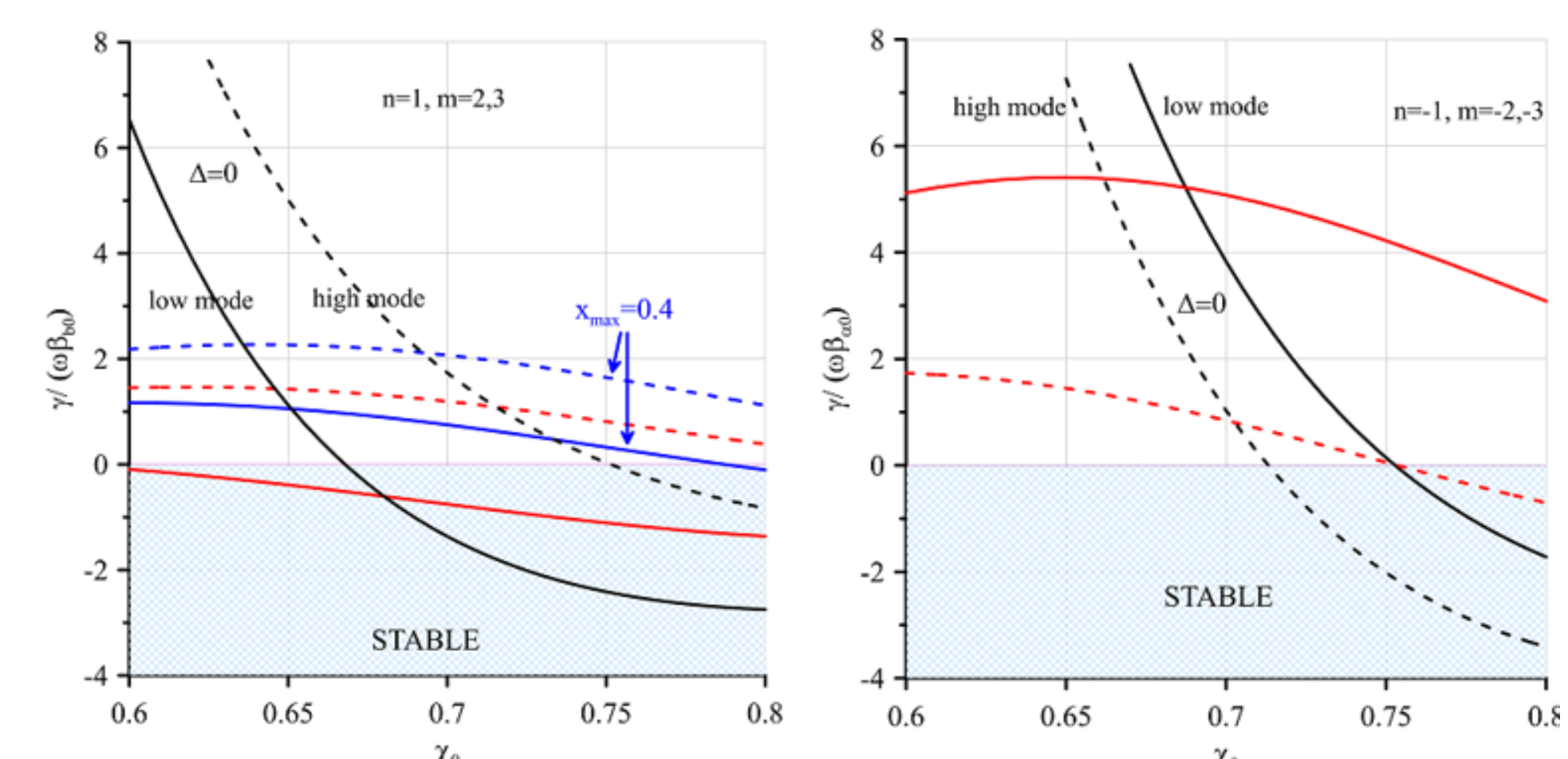
Destabilization of these modes as ctr-TAEs is sensitive to the position of plateau edge of the radial distribution of beam ions. The modes can be destabilized as both co- and ctr-TAEs with only minor changes in the radial distribution. The radial distribution of beam ions was not measured experimentally in this discharge, but calculated with TRANSP.



8. Finite-width pitch angle distribution

Above, we took $F_{\alpha} \sim \delta(\chi - \chi_0)$ because the pitch angle distribution of beam ions was not known. But it is possible that a finite width distribution may suppress the instability. To test this hypothesis, the numerical code was extended to account for $\Delta\chi$, with

$$F_{\alpha} = n_{\alpha}(r/a)(v^3 + v_e^3)^{-1} \exp[-(\chi - \chi_0)^2/(\Delta\chi)^2].$$



Finite width of pitch-angle distribution of beam ions reduces the mode increment, and can completely suppress the instability (left, lower frequency mode). However the opposite may be the case: when the region of $\partial F_{\alpha}/\partial\chi > 0$ shifts to lower χ due to $\Delta\chi > 0$ (right, higher frequency mode).

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