

The shear Alfvén continuum of optimized stellarators

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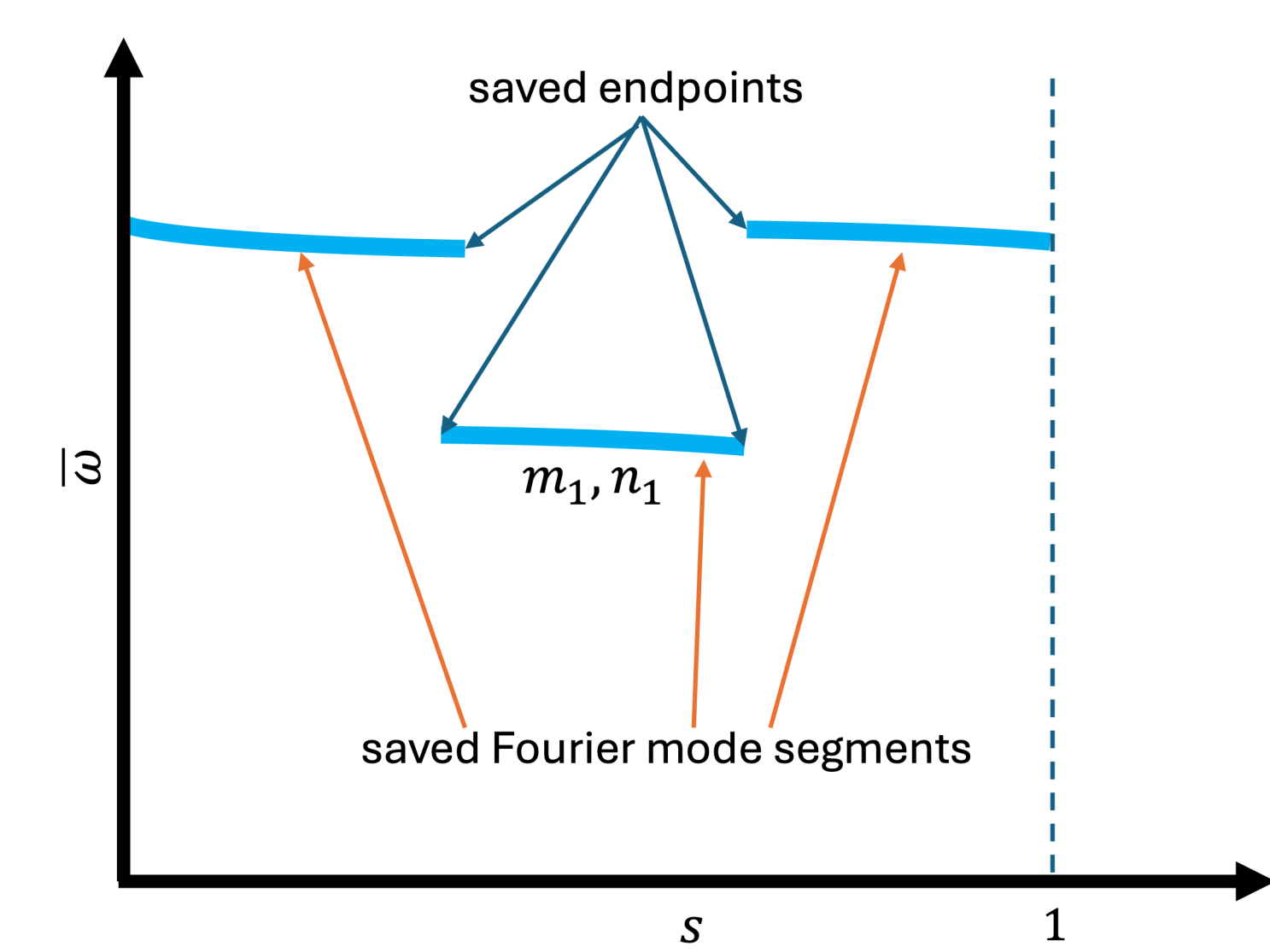
Background

- Waves satisfying $\omega = k_{||,m,n} v_A$, where $k_{||,m,n} = (lm - n)/R$, form a continuous frequency spectrum known as the shear Alfvén continuum^[1]
- Using normalized frequency $\bar{\omega} = \omega/\omega_A = \omega R/v_A$, modes have the same frequency at $\bar{\omega}_x^2 = \left(\frac{\iota(s)\Delta m - \Delta n}{2}\right)^2$
- When coupling is present, counter-propagating Alfvén waves ($k_{||,m,n} = -k_{||,m+\delta m, n+\delta n}$) of similar ω destructively interfere creating **avoided crossings in the continuum** labeled by $\delta m, \delta n$
 - A collection of avoided crossings forms a gap ($\Delta\omega$)
 - TAE: $|\delta m| = 1, \delta n = 0$, EAE: $|\delta m| = 2, \delta n = 0$, NAE: $|\delta m| \geq 3, \delta n = 0$, MAE: $\delta m = 0, |\delta n| \geq 1$, HAE: $|\delta m| \geq 1, |\delta n| \geq 1$
- Shear Alfvén continuum equation: $B \nabla_{||} \left(\frac{|\nabla\psi|^2}{B} \nabla_{||} \Phi \right) + \frac{\omega^2 |\nabla\psi|^2}{v_A^2} \Phi = 0$ (Φ : electric potential, ψ : flux)^[2]
- The continuum is labeled by the dominant mode (m, n) of $\Phi = \sum_{m,n} \Phi_{m,n} \cos(m\theta - n\zeta)$

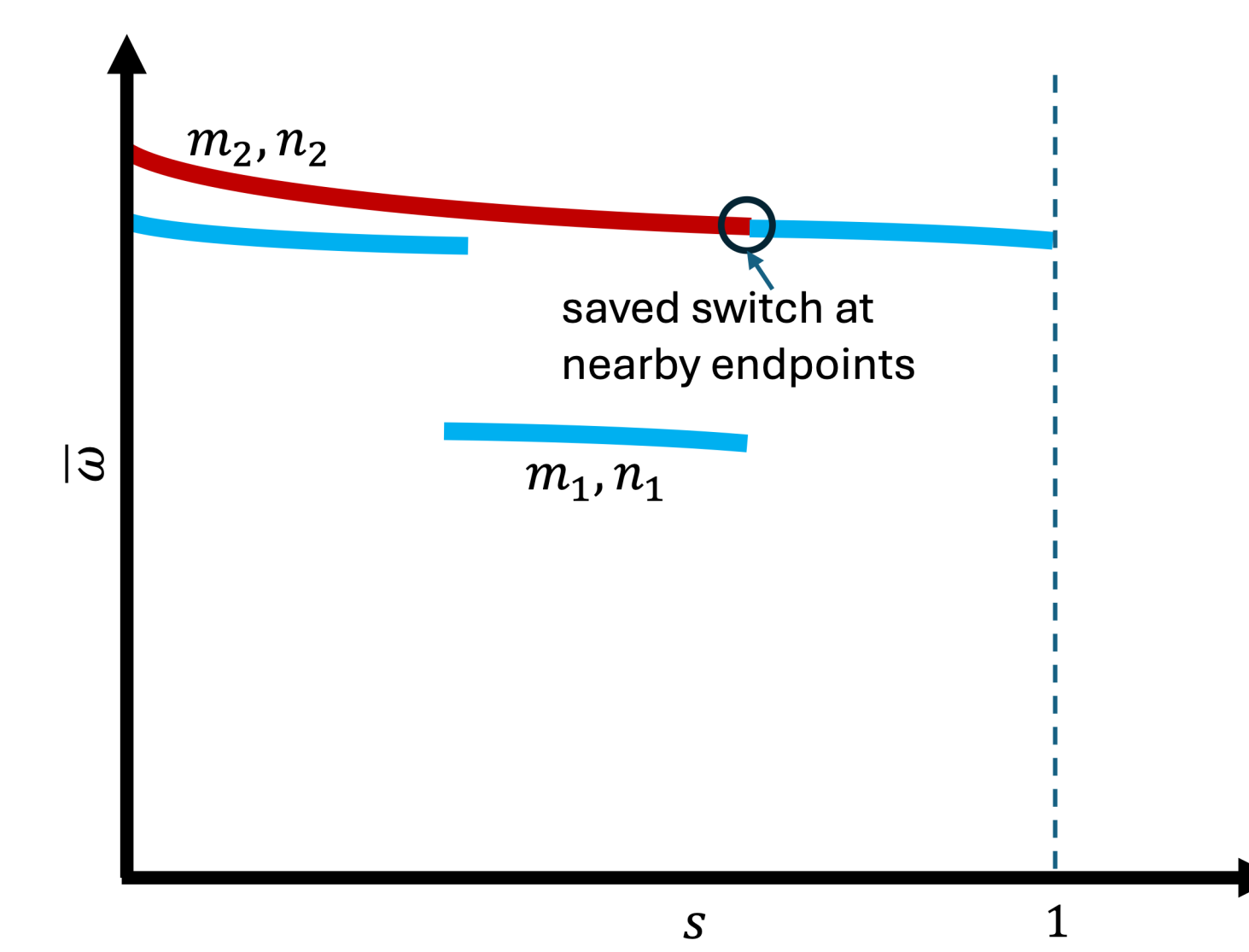
- Alfvén modes intersecting the continuum are damped
- Discrete Alfvén eigenmodes in the gaps are easily excited by energetic particles (EPs) via inverse Landau damping leading to particle transport → **We favor small gaps at EP resonant frequencies**
- To reduce Alfvénic instabilities, stellarators can be further optimized to **favor low-frequency gaps**
- Why avoid high-frequency gaps?
 - Resonance at high-frequency gaps: $\omega_A/\omega_\zeta \approx 1/4$ for reactor-scale stellarators
- Which Alfvén eigenmodes dominate different stellarator equilibria and are in the frequency region for resonance with EPs?

Computing the continuum: automatic gap detection method

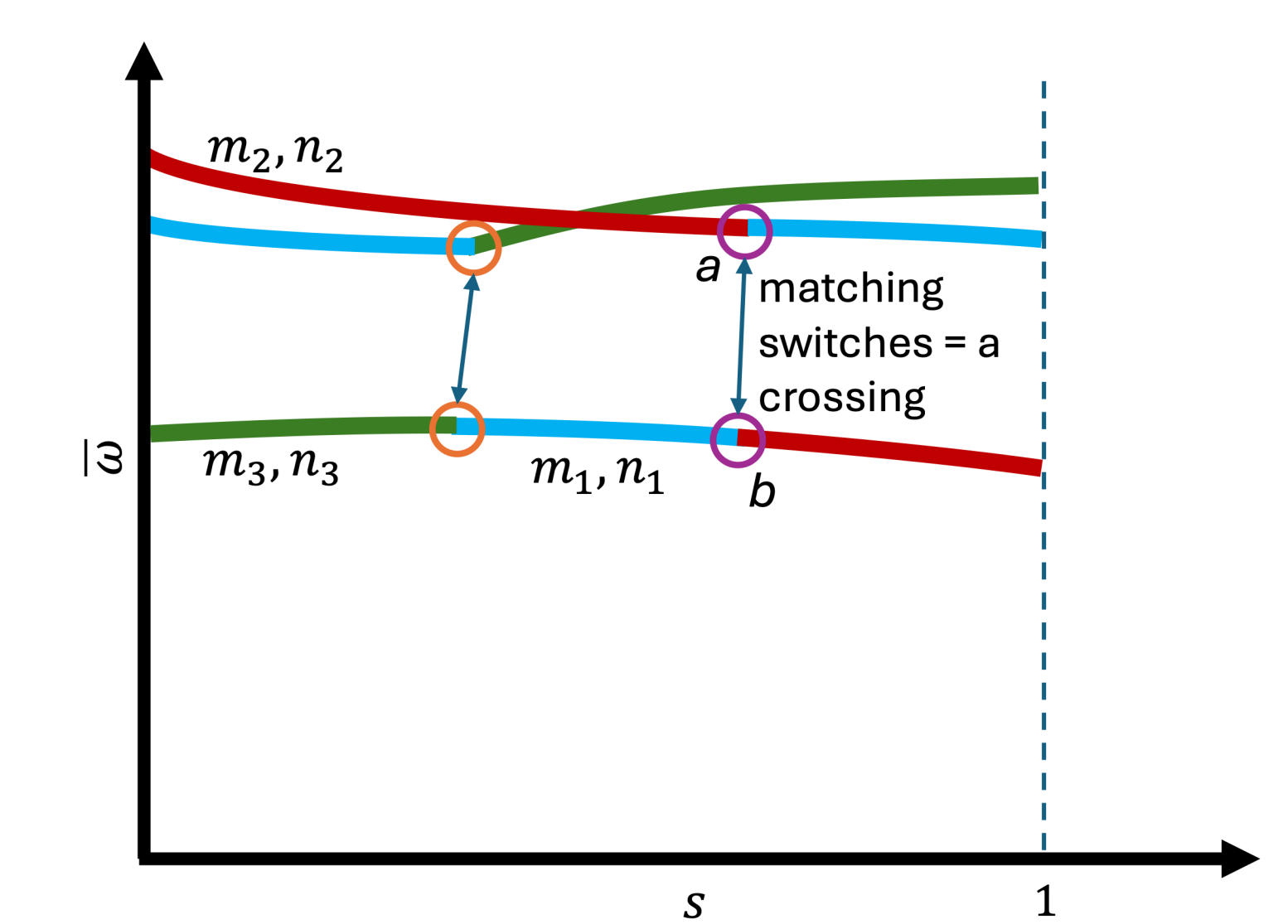
Step a) Forming Fourier mode segments



Step b) Finding switches



Step c) Find avoided crossings



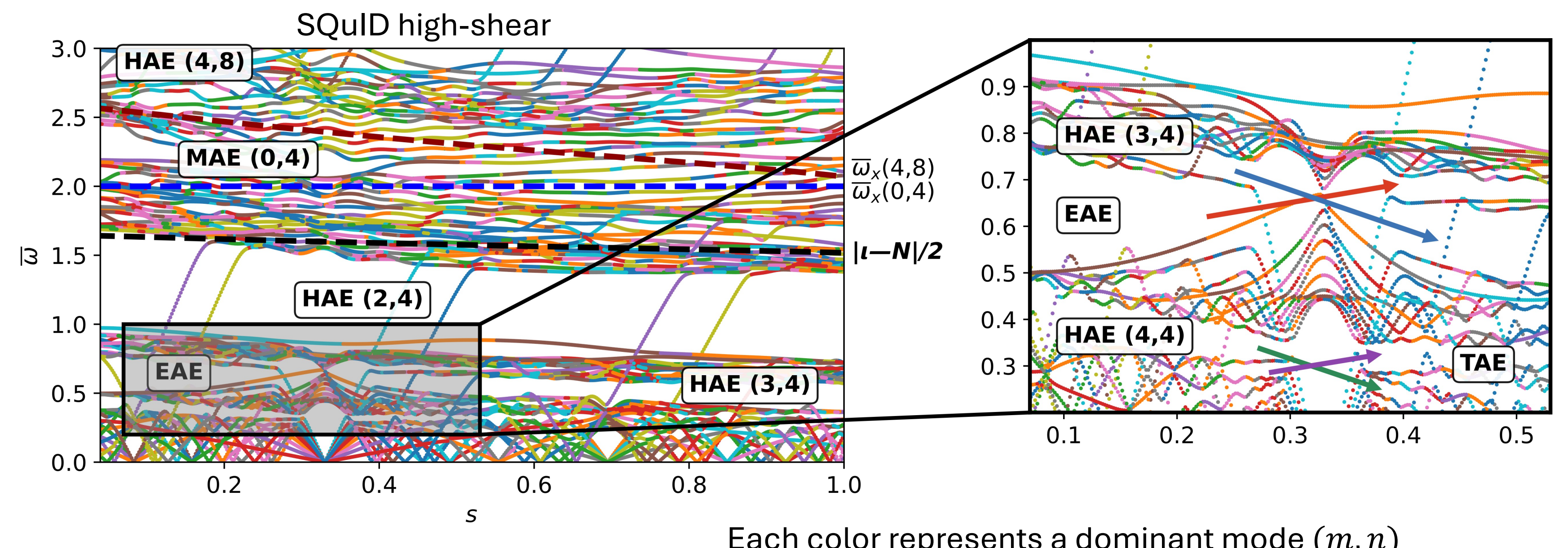
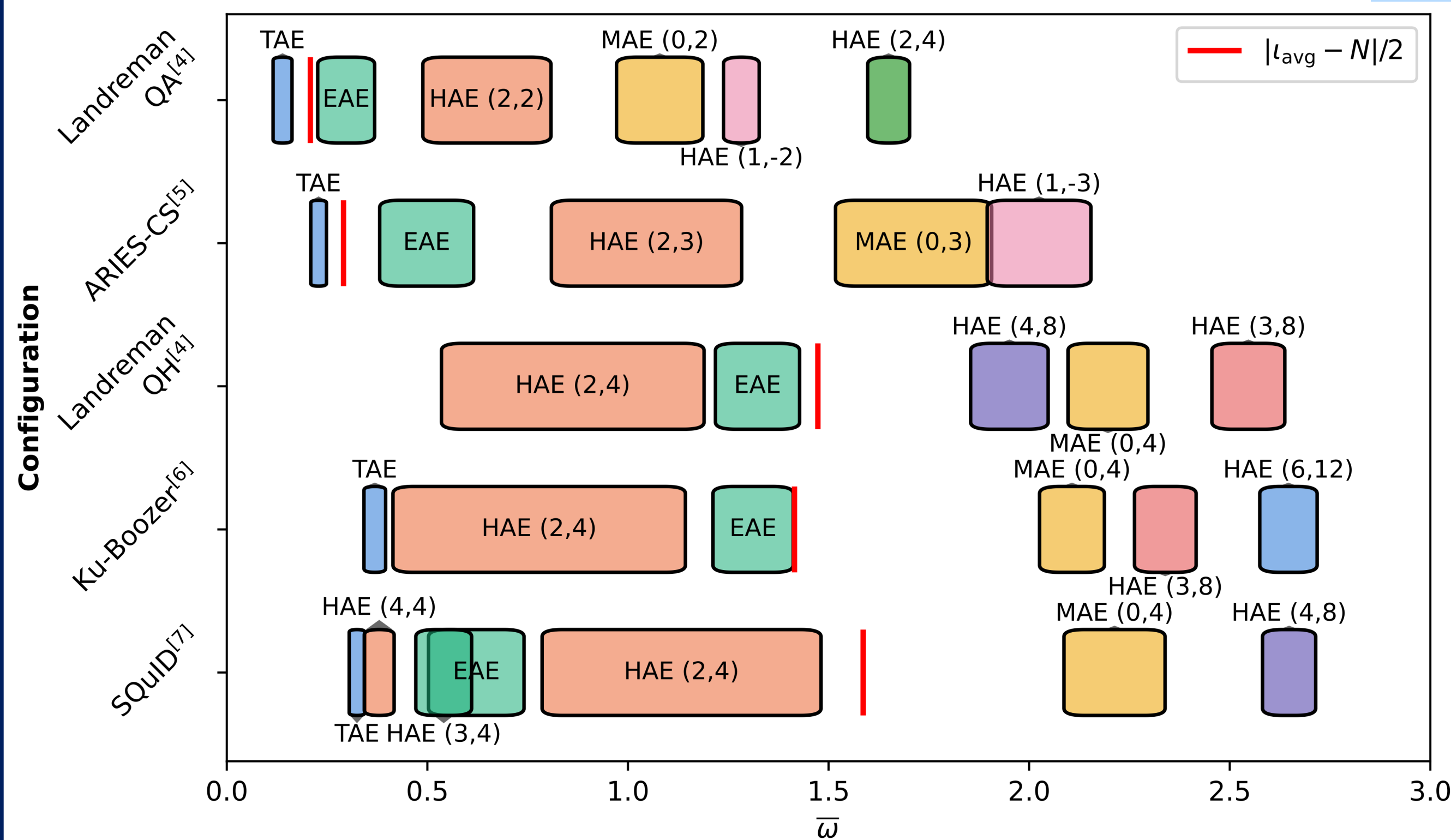
- A novel gap detection method was implemented to automatically identify gaps from Stellgap^[3] frequency and dominant mode data
- The frequency must be normalized to negate the effect of mass density shear

The method follows four steps:

- Divide the frequencies of each dominant mode (m, n) into segments wherever adjacent frequencies differ by more than $\bar{\omega}_{\text{threshold}}$

- Match segment endpoints to find where a continuum curve switches from one mode to another
- Match switches to find avoided crossings
- Group avoided crossings to identify gaps and remove avoided crossings at distant frequencies from the group

Results



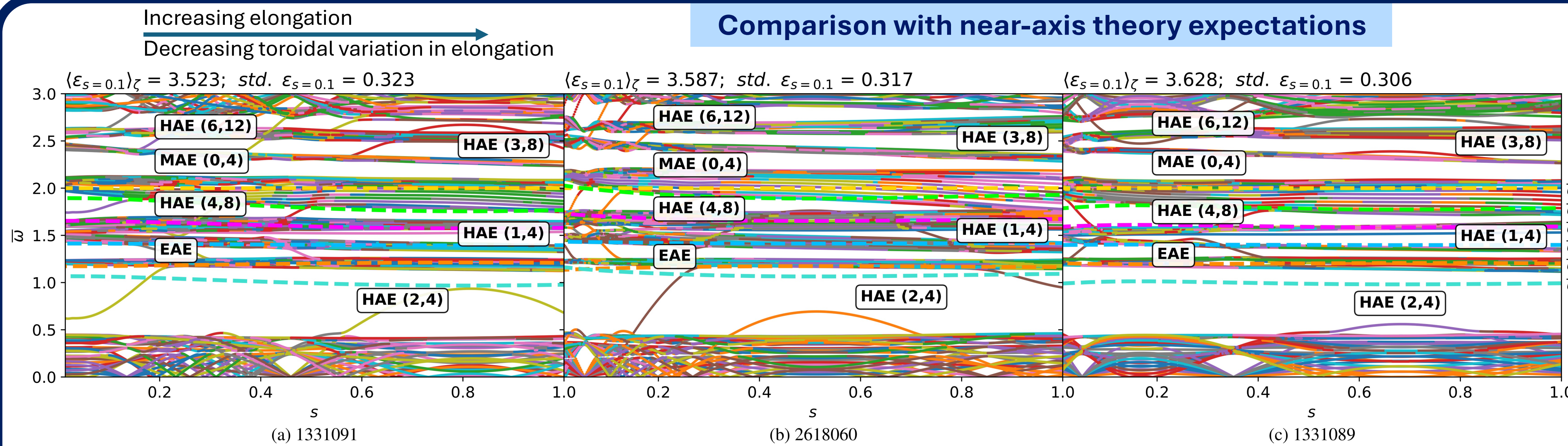
- Frequencies and dominant modes are from Stellgap^[3]
- 2 QA^{[4][5]}, 2 QH^{[4][6]}, and 1 SQUID^[7] configurations are illustrated
- A single mode family is shown: $(n = n_0 + cN_{fp})$, where n_0 is the lowest non-negative toroidal mode number and c is an integer
- Driven by rotating ellipticity, HAE($2, N_{fp}$) gaps have the largest width for all configurations
- Fusion-born alpha resonance when $\bar{\omega} > |\iota - N|/2$, where N is the helicity^[8]
 - QAs have more gaps in the resonant region, including the large HAE($2, N_{fp}$) and EAE gaps

- Unlike many tokamaks, the TAE is small and always below relevant fusion-born alpha frequencies

SQUID:

- The SQUID continuum shows two gap crossings: EAE with HAE(3,4), HAE(4,4) with TAE
- The gaps narrow and annihilate at the gap crossings^[9]
- Analytical $\bar{\omega}_x$ suggests that the oscillatory behavior of the continua around the MAE(0,4) is due to interactions between the MAE(0,4) and HAE(4,8) gaps

Comparison with near-axis theory expectations



- Three QUASR^[10] configurations with similar QS error, aspect ratio, mean ι , and minor radius
- $std. \epsilon_{s=0.1}$ is the relative standard deviation of elongation (ϵ) at the $s = 0.1$ surface and $\langle \epsilon \rangle_\zeta$ is the toroidal average

Expectation from near-axis theory for QA and QH^[8]:

- Elongation ϵ drives the EAE
- Toroidal variation in elongation drives the MAE

Result:

- $\Delta\bar{\omega}_{MAE}: (c) > (a) > (b) \mid std. \epsilon_{s=0.1}: (a) > (b) > (c)$
- $\Delta\bar{\omega}_{EAE}: (b) > (c) > (a) \mid \langle \epsilon_{s=0.1} \rangle_\zeta: (c) > (b) > (a)$
- Interaction is strong between HAE(7,12) and MAE / HAE(5,8) and EAE for (a) and (b) but not (c)
- The theory may not represent the interaction between gaps well

Conclusions

- A gap detection method for identifying the gaps of a continuum and calculating its gap widths was developed and implemented
- A common structure of large HAE($2, N_{fp}$), MAE, and EAE gaps was found across all equilibria
- The interaction and crossing between gaps play a strong role in continuum shaping and can produce regions of continuum damping
 - Near-axis theory may not represent the interaction between gaps well
- When optimizing against high-frequency gaps, QA configurations may prove challenging because of the HAE($2, N_{fp}$) gap falling in the resonance region of fusion-born alphas
- A. S. Hyder, E. J. Paul, A. R. Knyazev; The shear Alfvén continuum of optimized stellarators. *In prep.*