



**Politecnico
di Torino**

Department
of Applied Science
and Technology



Vertical Displacement Oscillatory Modes (VDOM) in Tokamak Plasmas

What they are and why they matter

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OUTLINE

Why $n=0$ matter: experimental observations of $n=0$ modes

Fast ion drive for $n=0$ modes

On the difference between VDOM and GAE

Back to basics: cylindrical plasma with circular cross section

Conclusions

First observation of high-frequency $n=0$ modes in tokamak plasmas go back to TFTR, CHANG, Z., et al., Nucl. Fusion 35 (1995) 1469.

More recently, they have been observed in JET and TCV.

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Axisymmetric global Alfvén eigenmodes in the TCV tokamak

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Axisymmetric Eigenmodes Excited by Alpha Particle Energy Gradients in JET D-T Plasmas

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Experimental results can be explained by either $n=0$ GAE or VDOM driven unstable by fast ions

- For GAE destabilized by fast ions, see J. Oliver, S. Sharapov, B. Breizman, L.-J. Zheng et al, PoP 2017.
- For VDOM destabilized by fast ions, see T. Barberis, F. Porcelli, A. Yolbarsop, Nuclear Fusion 2022.

Both modes oscillate with a frequency in the Alfvén frequency range; resonant interaction with fast ions at their transit/bounce frequency.

Fast ion drive

Hybrid kinetic MHD model and dispersion relation

We consider the fast ions effect perturbatively in the framework of the Hybrid Kinetic MHD, where the dispersion relation can be obtained using quadratic forms:

$$-\gamma^2 \delta I = \delta W_{MHD} + \delta W_{fast} \quad (1)$$

After standard manipulations discussed in detail in [2], the fast ion term for toroidal mode number $n=0$, can be expressed as:

$$\delta W_h = \frac{-2\pi^2 c}{Zem^2} \sum_{\sigma} \int dP_{\phi} d\mathcal{E} d\mu_{\perp} \tau_{\Omega} \omega \left[\left. \frac{\partial F}{\partial \mathcal{E}} \right|_{\Lambda} - \frac{\Lambda}{E} \left. \frac{\partial F}{\partial \Lambda} \right|_E \right] \sum_{p=-\infty}^{+\infty} \frac{|\Upsilon_p|^2}{\omega + p\omega_{\Omega}} \quad (2)$$

Ideal MHD analysis $\rightarrow \gamma^3 + \gamma^2 \frac{1}{\tau_{\eta}} \frac{1}{1-\hat{e}_0 D} + \gamma \omega_0^2 + \omega_0^2 \frac{1}{\tau_{\eta}} \frac{1}{1-D} = 0$, $\omega_0 \approx e_0^{1/2} \tau_A^{-1} \sqrt{D-1}$
With fast ions, defining $\lambda_h \propto \mathcal{I}m(\delta W_h)$

$$\omega = \omega_0 + i\gamma_{tot}, \quad \gamma_{tot} = \omega_0 \lambda_h / 2 - \gamma_d \quad (3)$$

Here we will focus on the term coming from the resonant interaction between the mode and the fast ions, neglecting other sources of damping γ_d .

$\partial f_h / \partial E > 0$
or pitch
angle
anisotropy
required for
VDOM
instability

- Experimental evidence:
 - M.A. Van Zeeland et al., NF (2021)
Beam Modulation and Bump-on-tail Effects On Alfvén Eigenmode Stability in DIII-D
 - V.G. Kiptily et al., NF (2021)
Self modulation of alpha particles production due to sawtooth oscillations in JET D-He³ experiments.
“Bump-on-tail” distribution for energetic particles when the sawtooth period is relatively short, i.e., $\tau_{\text{saw}}/\tau_s < 1$.
 - Interpretation:
 - T. Barberis and F. Porcelli, Pl. Phys. Contr. Fus. (2024) *Velocity space distribution function of fast ions in a sawtooth plasma*
 - L.—G. Eriksson and F. Porcelli, Nucl. Fusion (2025) *On the drive of n=0 modes by ICRF accelerated ions in a tokamak*
- See also L-G Eriksson and F. Porcelli, *On the drive of axisymmetric modes by fusion alpha particles in tokamaks*, this conference, poster this afternoon (29 September).

Difference between VDOM and GAE

VDOM

- The restoring force is the force of attraction between the plasma current and the current induced on the wall.
- External mode, with a frequency dependence on the plasma-wall distance.
- Broad spatial structure, nearly rigid shift (especially for the shearless case).
- Could interact with Alfvén continuum, in which case continuum damped.

GAE

- The restoring force is related to field line bending.
- Mostly internal mode, with a frequency insensitive to the plasma-wall distance.
- Mode structure varies rapidly near the minimum of $k_{\parallel} v_A$.
- Always immune of continuum damping.

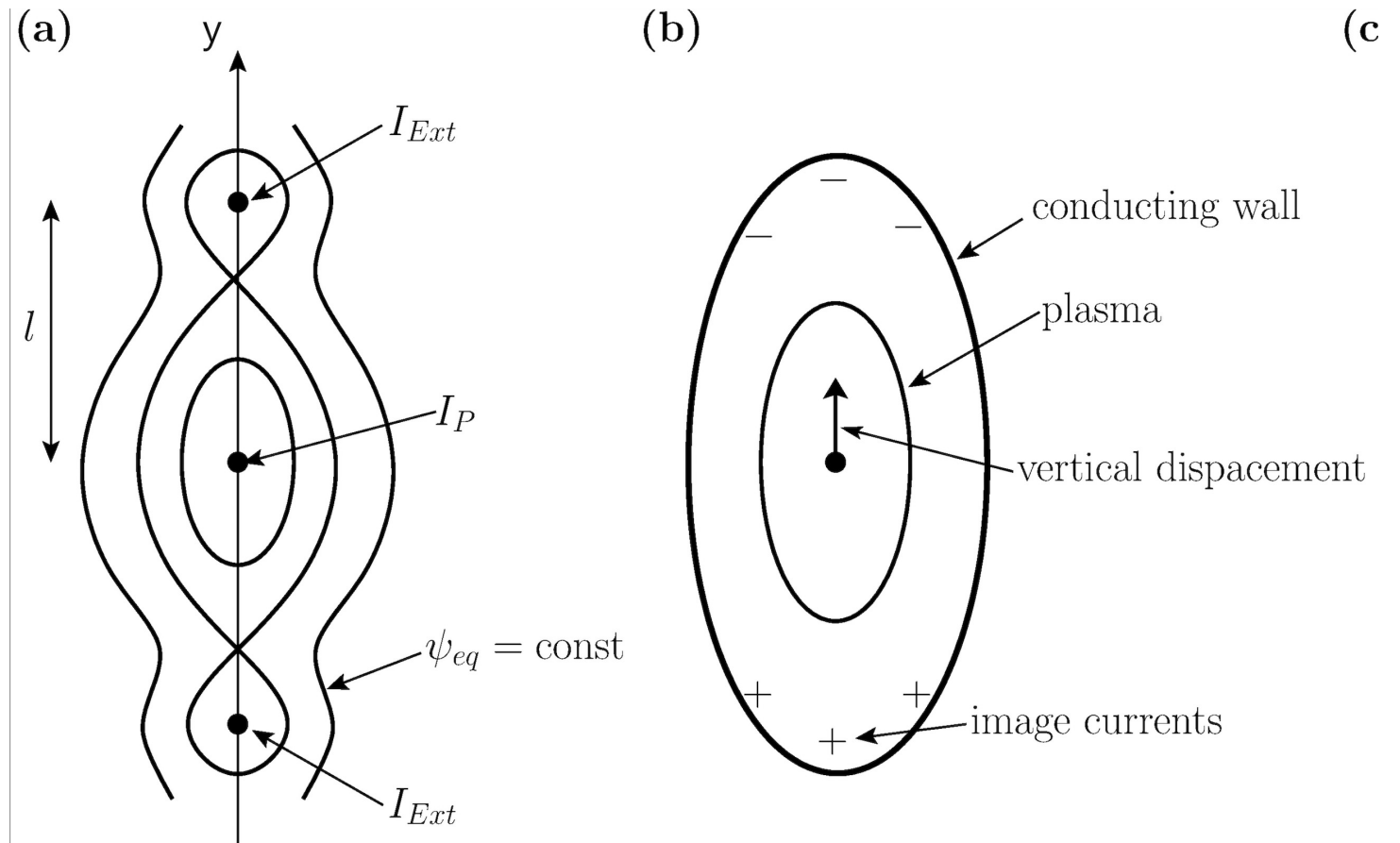
... and for the elliptical cross section

- | | |
|---|---|
| <ul style="list-style-type: none">• Odd parity wrt poloidal angle -> up-down vertical motion.• Frequency depends importantly on ellipticity, could become ideal-MHD unstable above a critical elongation. | <ul style="list-style-type: none">• Even parity wrt poloidal angle -> ballooning structure favored.• Weak dependence on elongation and always stable. |
|---|---|

Both VDOM and GAE can be driven unstable by the resonant interactions with energetic particles.

What are $n=0$ Vertical Displacement Oscillatory Modes (VDOM)?

$n=0$ VDOM are natural modes of oscillation in a tokamak plasma.



Back to basics

Straight tokamak with circular cross section

Acknowledgement: work done in collaboration with PhD student
Simone Cavallero (Politecnico di Torino)

The model

- Reduced Ideal MHD (Strauss, PF 1976)
- Linear theory

$$\frac{\partial \tilde{\psi}}{\partial t} + [\tilde{\varphi}, \psi_{eq}] = 0$$

$$\frac{\partial}{\partial t} \nabla \cdot (\rho_{eq} \nabla \tilde{\varphi}) = [\tilde{\psi}, J_{eq}] + [\psi_{eq}, \tilde{J}]$$

$$\mathbf{B} = \mathbf{e}_z \times \nabla \psi + B_z \mathbf{e}_z; \quad \mathbf{v} = \mathbf{e}_z \times \nabla \varphi + v_z \mathbf{e}_z; \quad J = \nabla^2 \psi$$

$$\psi(r, \vartheta, t) = \psi_{eq}(r) + \tilde{\psi}(r, \vartheta, t); \quad \tilde{\psi}(r, \vartheta, t) = \hat{\psi}(r) e^{-i\omega t + im\vartheta}$$

- Plasma boundary at $r=a$, wall at $r=r_w$, vacuum in between

Dispersion equation

$$\omega^2 \left\{ \frac{d}{dr} \left[r \varrho_{eq} \frac{d}{dr} \left(\frac{r}{\psi'_{eq}} \psi \right) \right] - \frac{\varrho_{eq}}{\psi'_{eq}} \psi \right\} = \psi'_{eq} J - J'_{eq} \psi; \quad J(r) = \frac{1}{r} \frac{d}{dr} \left(r \frac{d\psi}{dr} \right) - \frac{\psi}{r^2}$$

Boundary condition: $\hat{\psi}(r_w) = 0$ (ideal wall) (hats are dropped)

The solutions of the d.e. include “VDOM” and GAE*. Our aim is to understand the difference between the two. Only stable oscillations are found (in the circular cross section limit).

*Villard and Vaclavik, *Alfvén frequency modes and Global Alfvén Eigenmodes*, NF 1997; Appert and Vaclavik, JPP 1983.

At first, let us focus on “VDOM” with $m=1$

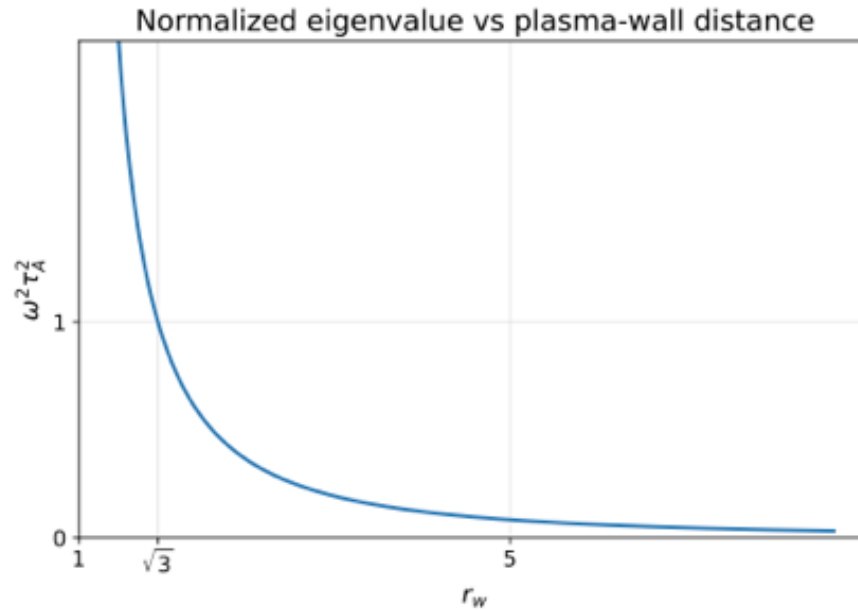
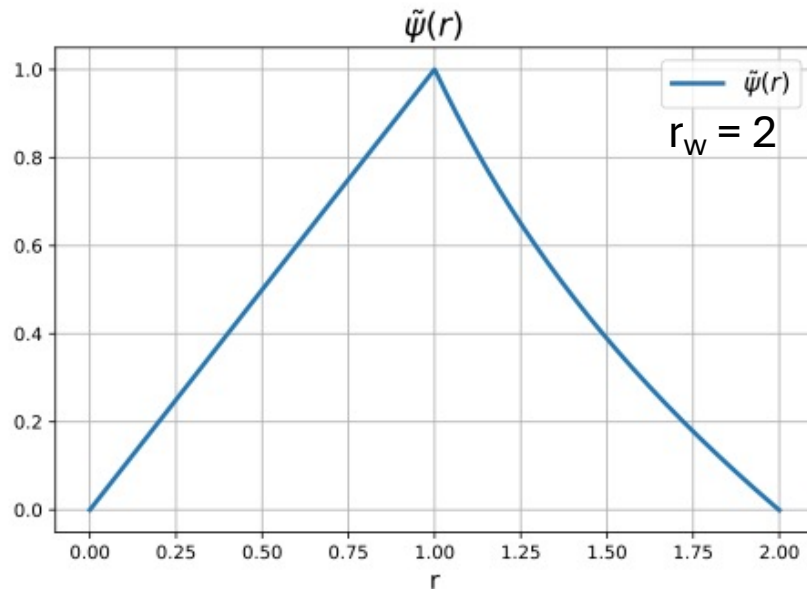
Idealized equilibrium:

$$\varrho_{eq}(r) = H(1 - r) \quad \text{and} \quad J_{eq}(r) = 2H(1 - r) \quad (r=1: \text{plasma boundary})$$

→ shearless

Solution:

$$\omega^2 = \frac{2}{r_w^2 - 1}$$



The “VDOM” solution

- The solution in this idealized case is a rigid-shift displacement of the plasma column;
- The restoring force is the force of attraction between the equilibrium plasma current and the current induced on the perfectly conducting wall;
- Intrinsically an “external” mode;
- Something must happen when $(\omega\tau_A)^2 \geq 1 \rightarrow$ continuum damping;
- “Vertical” sounds like a misnomer for the circular case, but the degeneracy is broken as soon as an elliptical deformation of the plasma cross section is introduced; also, for the equivalent toroidal case, a $n=0$ rigid shift can only occur along the “vertical” direction.

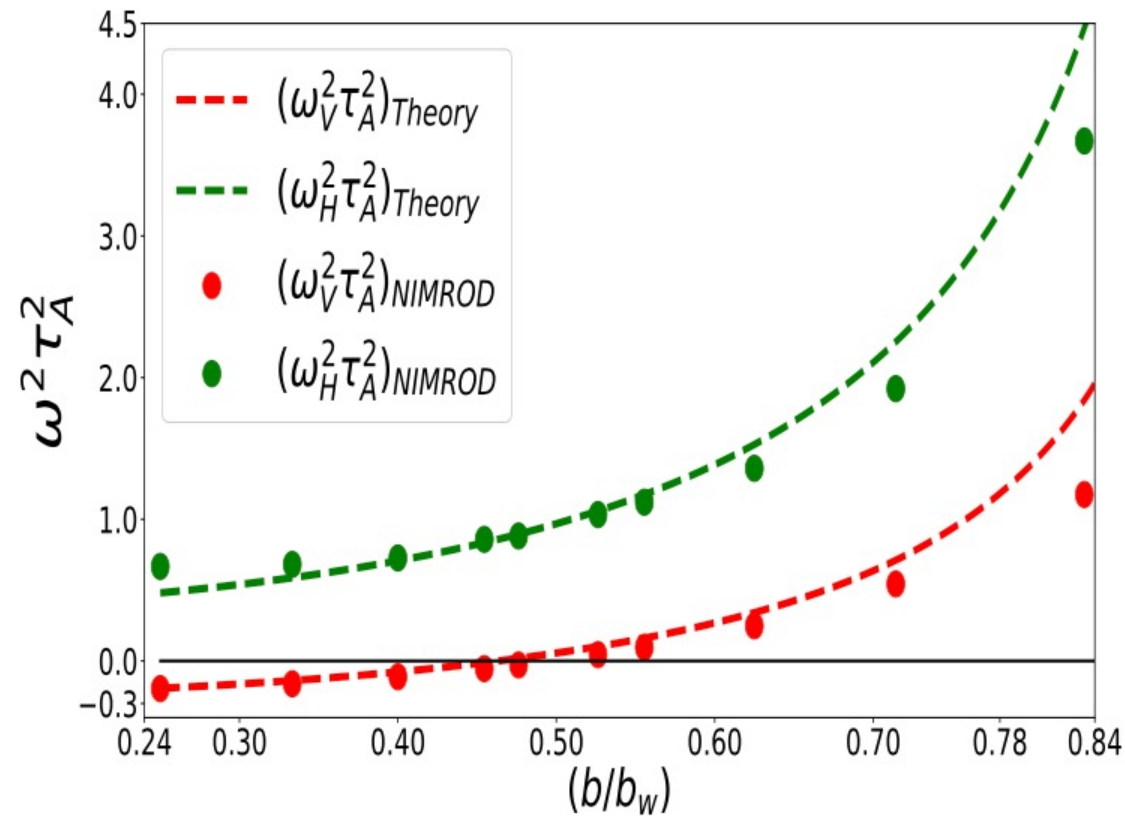
Elliptical plasma with minor semi-axis a and major semi-axis b

Elongation $\kappa = b/a$; Plasma-wall distance measured by b/b_w

Confocal wall, Ideal wall limit.

Shearless equilibrium (Gajevskii, PF 1972; Porcelli and Yolbarsop, PoP 2019)

“Horizontal” vs “Vertical” modes

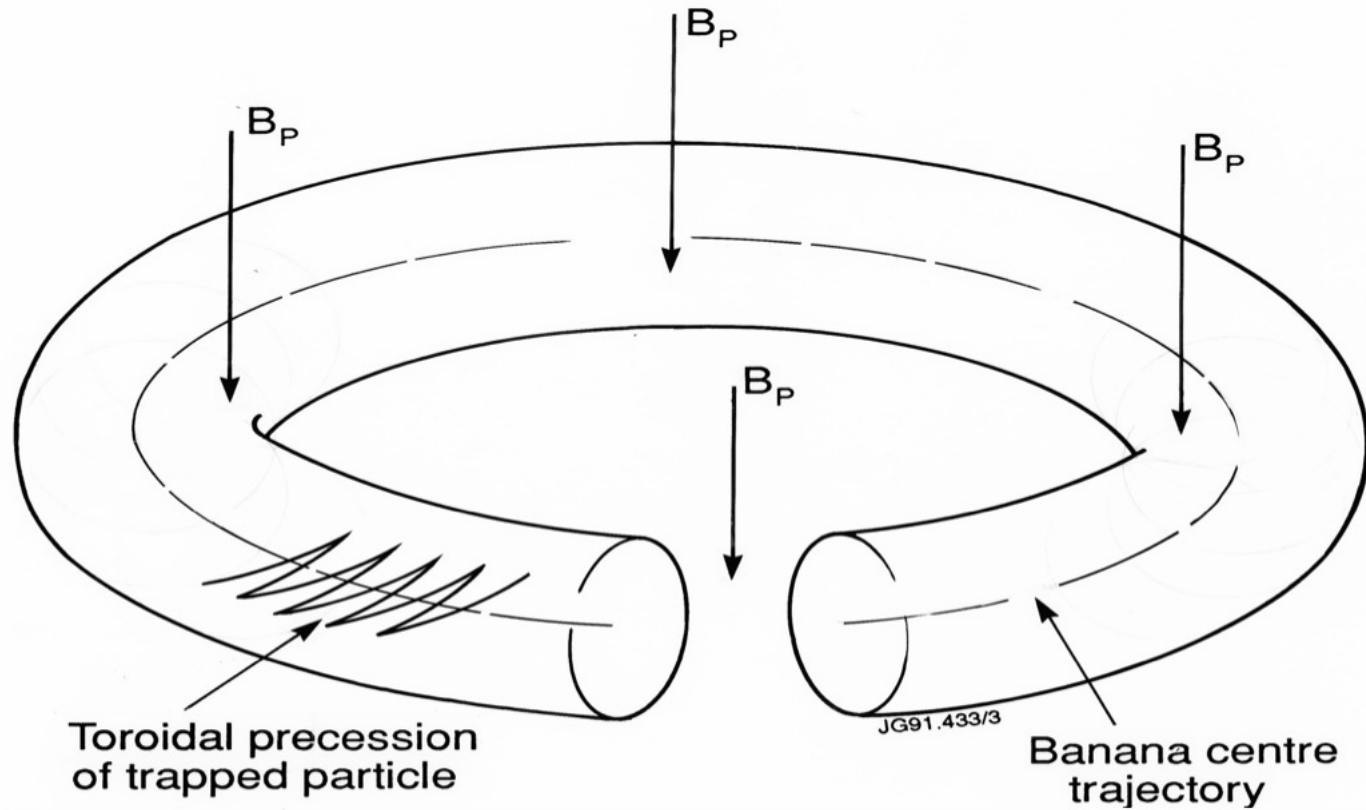


(a) elongation $\kappa = 1.4$

NIMROD simulation

A. Yolbarsop et al,
Phys. Lett. A 2023.

In the equivalent torus, a horizontal rigid shift would actually be a $n=1$ mode!



A different equilibrium with magnetic shear: VDOM vs GAE

Dispersion equation

$$\omega^2 \left\{ \frac{d}{dr} \left[r \varrho_{eq} \frac{d}{dr} \left(\frac{r}{\psi'_{eq}} \psi \right) \right] - \frac{\varrho_{eq}}{\psi'_{eq}} \psi \right\} = \psi'_{eq} J - J'_{eq} \psi; \quad J(r) = \frac{1}{r} \frac{d}{dr} \left(r \frac{d\psi}{dr} \right) - \frac{\psi}{r^2}$$

Boundary condition: $\hat{\psi}(r_w) = 0$ (ideal wall) (hats are dropped)

The solutions of the d.e. include “VDM” and GAE*. Our aim is to understand the difference between the two. Only stable oscillations are found (in the circular cross section limit).

However, in order to find GAE, we must consider an equilibrium with magnetic shear and a minimum in the $k_{\parallel} v_A$ spectrum.

Note: the d.e. is singular at $\omega^2 = (k_{\parallel} v_A)^2$.

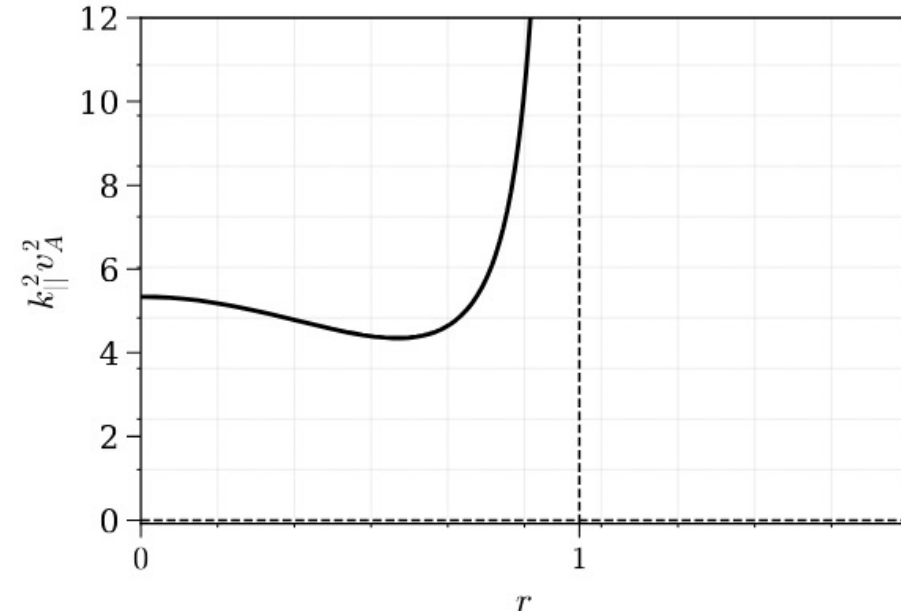
GAEs are solutions with ω^2 just below the minimum of $(k_{\parallel} v_A)^2$.

We will follow closely the analysis by*Villard and Vaclavik, *Alfvén frequency modes and Global Alfvén Eigenmodes*, NF 1997; Appert and Vaclavik, JPP 1983.

Consider the following equilibrium:

$$J_{eq}(r) = C_j(1 - r^2)^3;$$

$$\rho_{eq} = C_\rho(1 - r^2)^2$$



Use WKB to find an analytic solution assuming $|(k_{\parallel} v_A)_{min} - \omega|^2 \ll \omega^2$.

WKB analysis for GAEs

We assume solutions of the type

$$\psi(r) = \frac{1}{\sqrt{k_r}} e^{i \int_0^r k_r dr}, \text{ where } k_r a \gg 1.$$

Substituting in the d.e. and keeping the leading order terms in k_r , we find

$$k_r^2 = \frac{1}{r^2 \varrho_{eq}} \frac{\psi'_{eq} J'_{eq}}{\omega^2 - (k_{\parallel} v_A)^2}$$

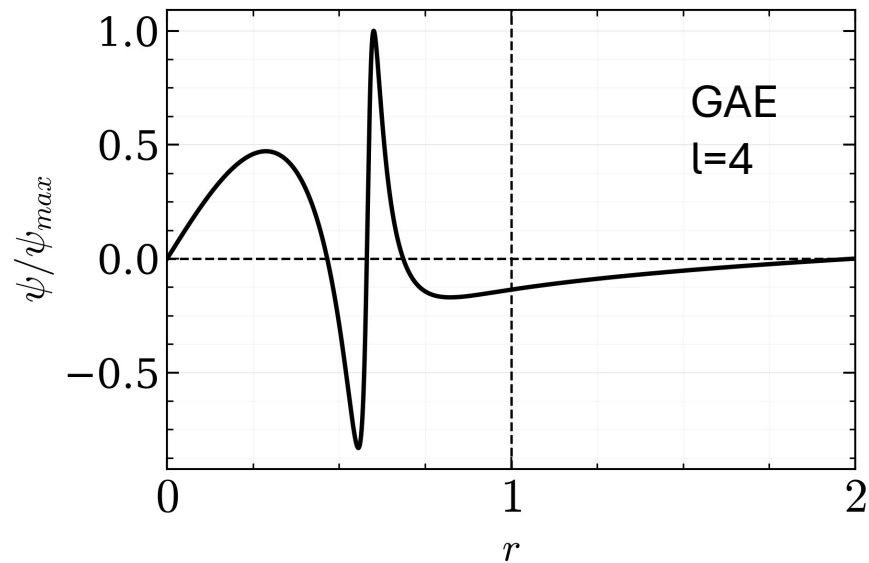
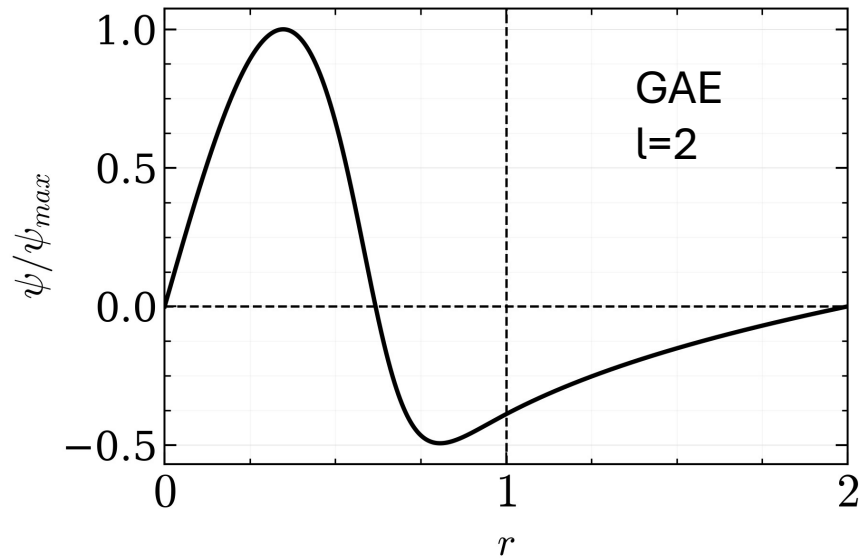
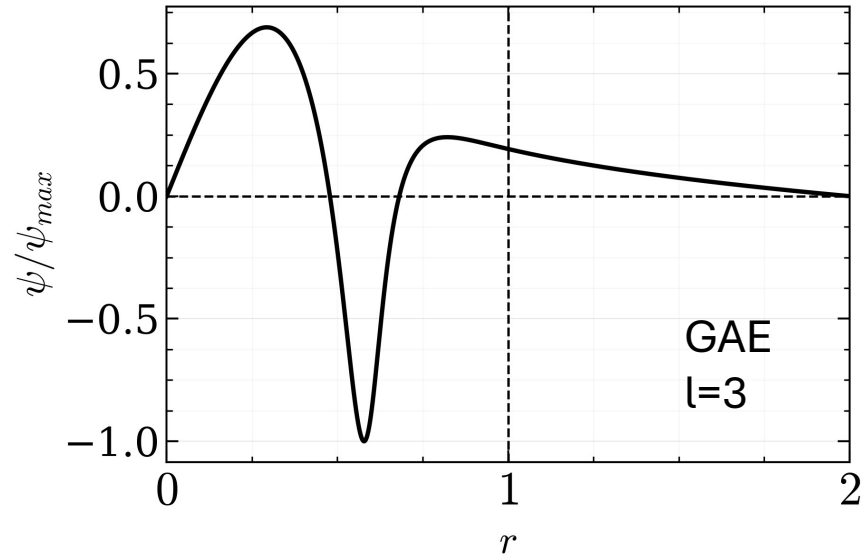
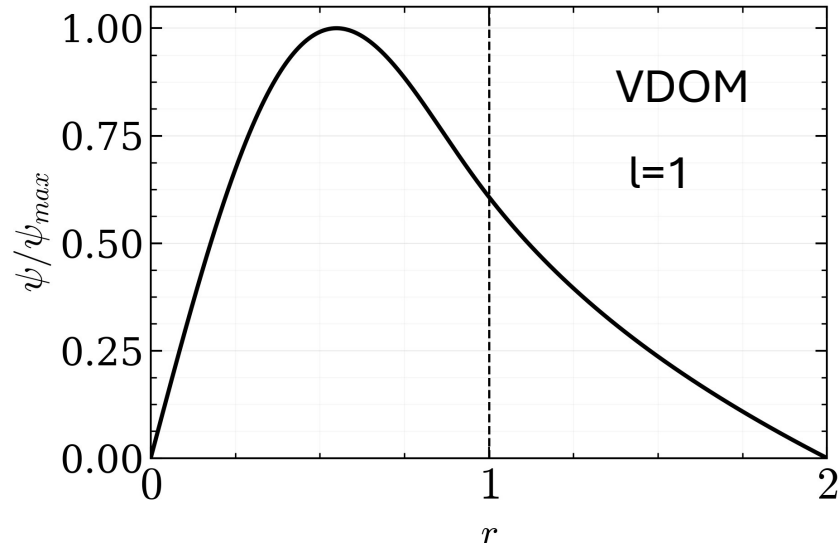
In order to obtain ω^2 , we use the quantization condition (assuming GAE are internal modes)

$$\int_0^a k_r dr = l\pi,$$

with l an integer number (“radial” mode number).

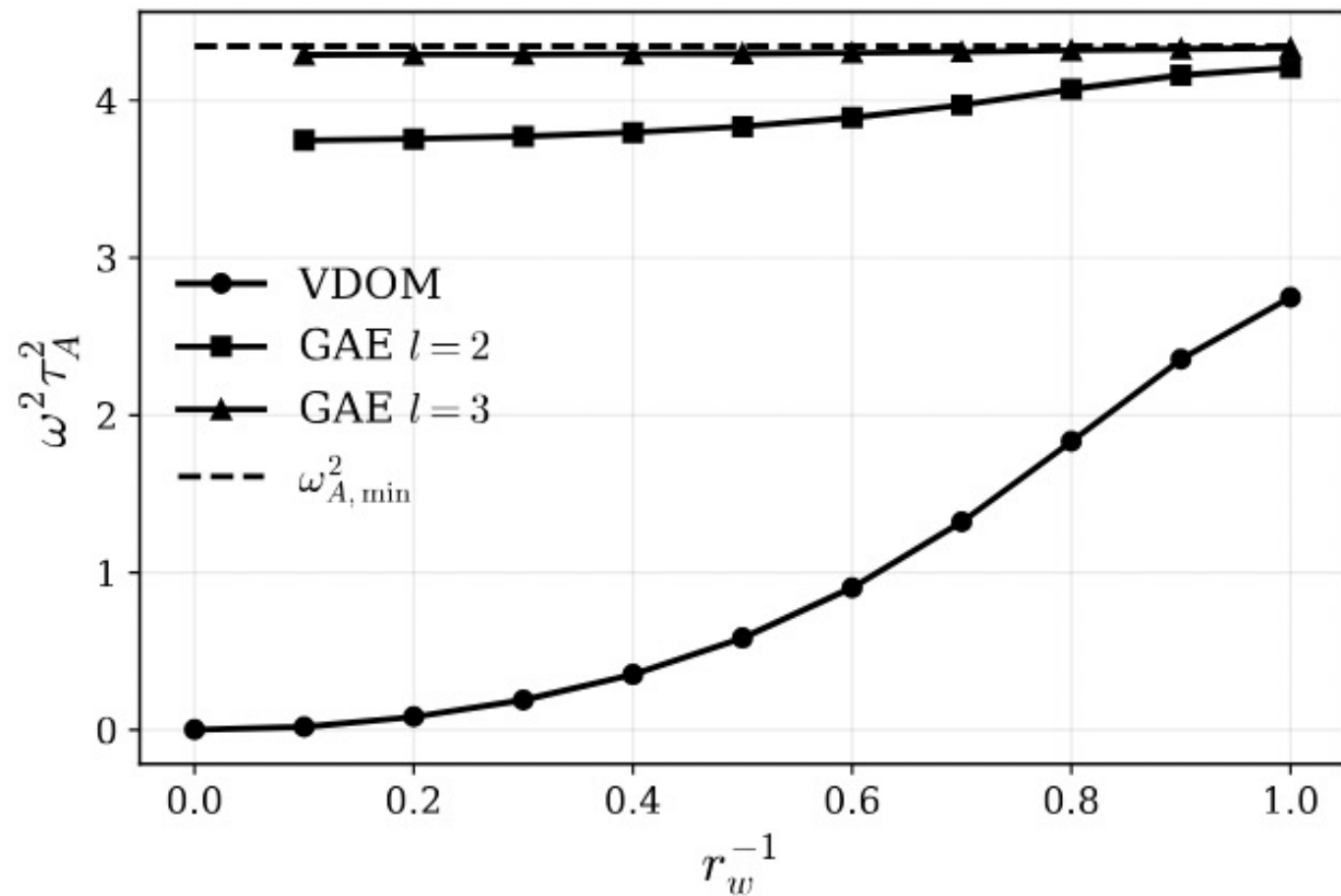
For the chosen equilibrium, we find that WKB works well for $l \geq 2$.

The dispersion equation is also solved numerically using a shooting code (Simone Cavallero).
We find that “external” VDOM solution and “internal” GAE solutions can coexist.

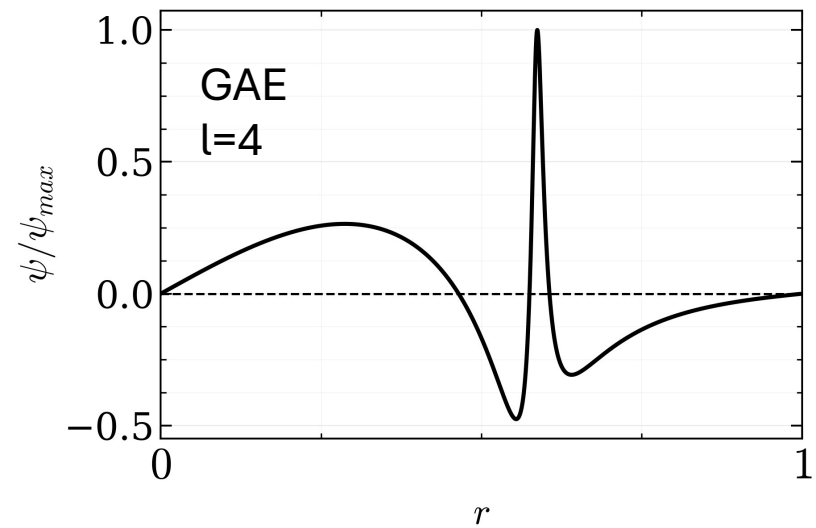
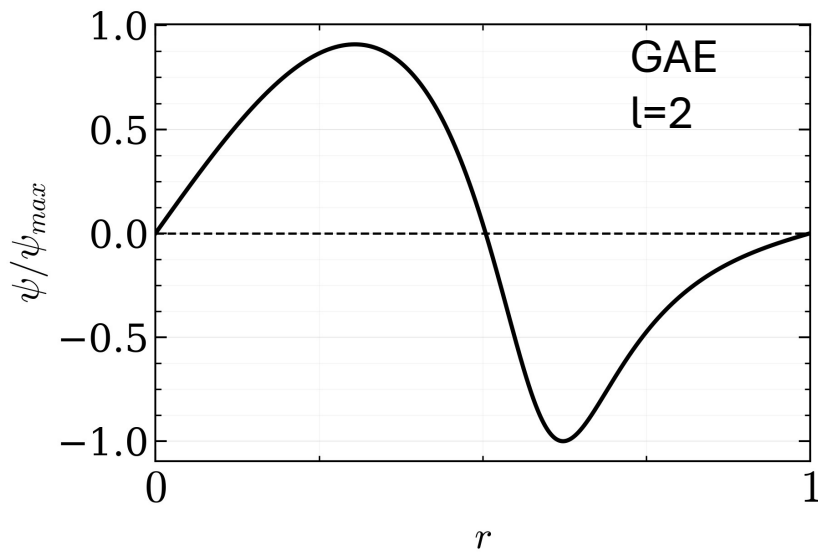
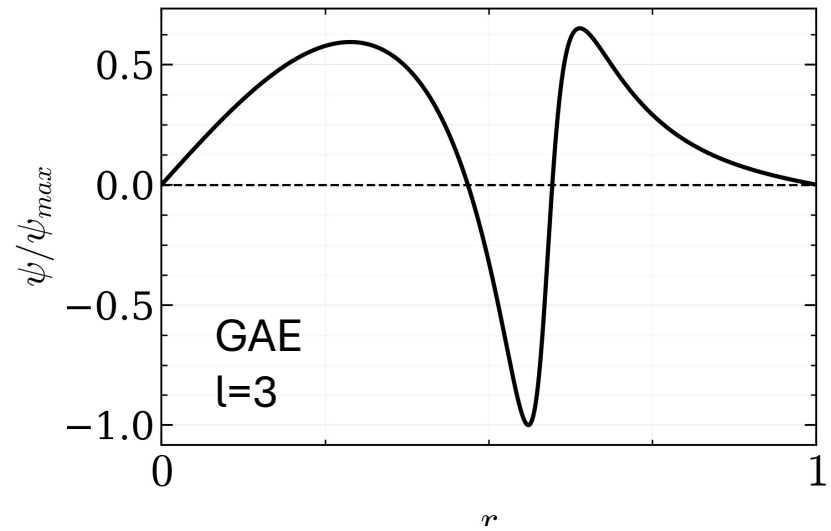
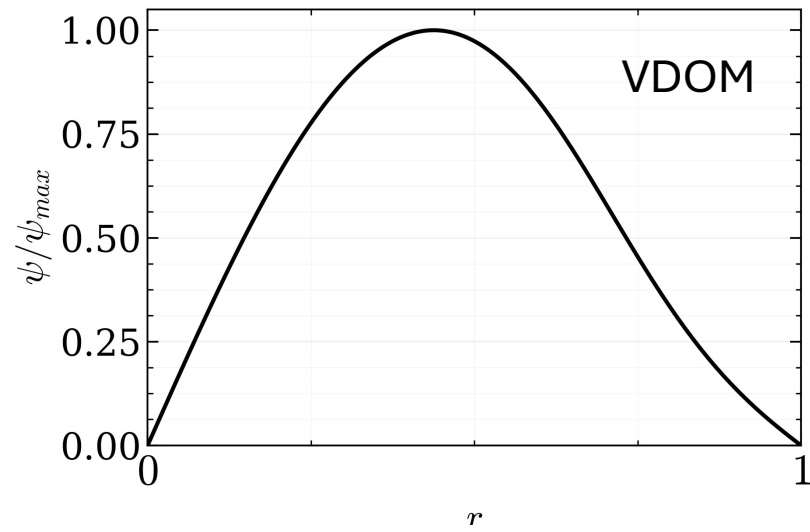


$$r_w = 2$$

Dependence
of the mode
frequency on
plasma-wall
distance



Simulations with $r_w = 1 \rightarrow$ internal modes



Conclusions

- In an idealized straight tokamak configuration with circular cross section, the $l=1$ solution appears to be fundamentally different from the $l>1$ modes.

$l=1$ is an external mode, whose frequency depends sensitively on the plasma-wall distance; it can become ideally unstable if the elliptical elongation exceeds a critical threshold;

$l>1$ are mostly internal modes, with a frequency just below the Alfvén continuum spectrum.

Hence: a different name for $l=1$ (with vertical flow): it is a VDOM; $l>1$ are GAEs.

- Analytic theory of Vertical Displacement Oscillatory Modes (VDOM) successfully verified by NIMROD simulations in realistic tokamak geometry

-> see, e.g. T. Barberis et al, *Simulations of vertical displacement oscillatory modes and global Alfvén Eigenmodes in JET geometry*, NF 2024;

D. Banerjee et al, NIMROD MHD simulations of axisymmetric modes observed in the TCV tokamak, this conference, poster this afternoon (29 September).

- VDOM can be driven unstable by fast ions, adding a new possibility to the catalogue of Energetic Particle modes.
- Saturation of VDOM due to fast particle nonlinearities -> see S. Cavallero, this conference, poster this afternoon (29 September).
- VDOM and/or GAE driven by fusion alpha particles can explain the observation of $n=0$ modes in JET DT3 campaign -> expect these modes to become important in future burning plasma experiments.

Supplementary slides

Analytic theory

Resistive Wall – Ideal Plasma - Cubic dispersion relation

$$\gamma^3 + \gamma^2 \frac{1}{\tau_\eta} \frac{1}{1 - \hat{e}_0 D} + \gamma \omega_0^2 + \omega_0^2 \frac{1}{\tau_\eta} \frac{1}{1 - D} = 0$$

$$\tau_\eta \rightarrow \infty$$

$$\gamma = -i\omega$$

τ_η : resistive wall
time

$$e_0 = \frac{b^2 - a^2}{b^2 + a^2}: \text{ellipticity parameter; } \hat{e}_0 = \frac{b}{a+b} e_0$$

$$D\left(\frac{b}{b_w}, \kappa\right) = \frac{\kappa^2 + 1}{(\kappa - 1)^2} \left\{ 1 - \left[1 - \frac{\kappa^2 - 1}{\kappa^2} \left(\frac{b}{b_w}\right)^2 \right]^{1/2} \right\}: \text{geometrical wall parameter; } \kappa = \frac{b}{a}$$

$$\omega_0 = \left[\frac{D-1}{1 - \hat{e}_0 D} \right]^{\frac{1}{2}} \gamma_\infty: \text{oscillation frequency (for } D > 1); \gamma_\infty = \frac{2ab}{a^2 + b^2} \left(1 - \frac{a}{b}\right)^{1/2} \tau_A^{-1}: \text{no-wall growth rate (} D=0)$$

The wall position determines the stability and oscillation frequency

$$D = D(\kappa, \frac{b}{b_w})$$

$D = 1$ Marginal stability

$D < 1$ Ideal vertical instability

**$D > 1$ Oscillatory modes +
zero-freq. unstable resistive
solution**

In relevant conditions ($D > 1$), the solutions of the cubic dispersion relation (resistive wall) are:

$$\omega = \pm\omega_0 - i \frac{1}{2\tau_\eta} \frac{D(1 - \hat{e}_0)}{(D - 1)(1 - \hat{e}_0 D)}$$

$$\tau_\eta \omega_0 \gg 1$$

→

$$\gamma = \frac{1}{(D - 1)\tau_\eta}$$

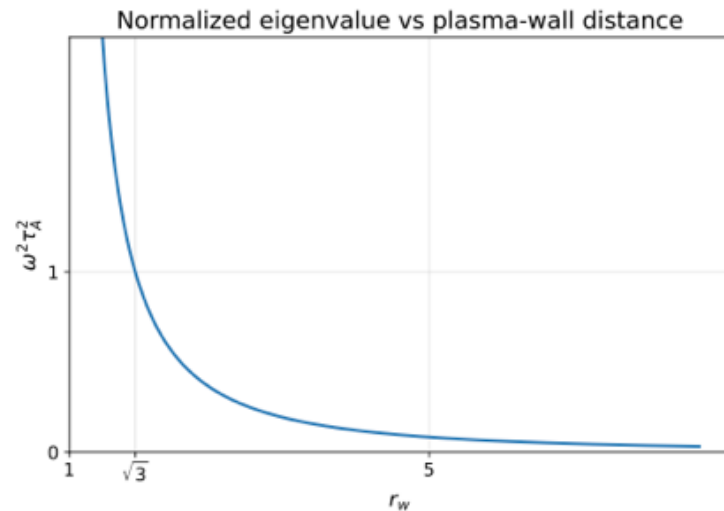
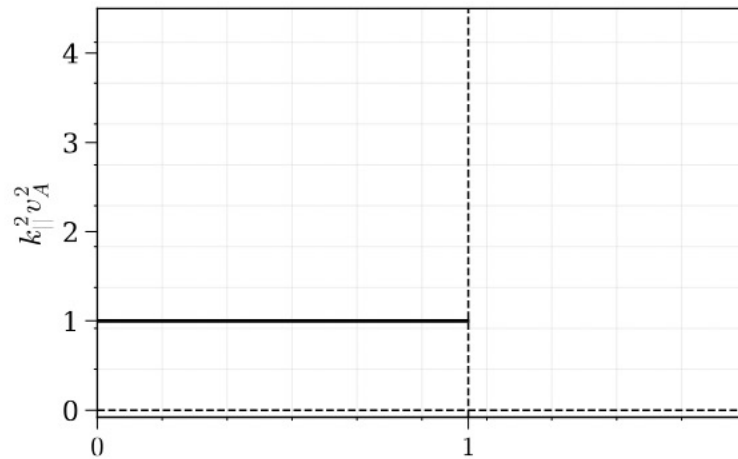
γ_η

- The ideal-MHD marginal stability condition against vertical displacement agrees with the pioneering work by Laval et al, PF 1974.
- The cubic dispersion relation for vertical displacement was obtained previously by several authors, see, e.g.,
 - J. Wesson, Hydromagnetic stability of tokamaks, Nuclear Fusion (1978); J. Wesson, Tokamaks (Clarendon, Oxford, 4th Edition, 2011).
 - S.C. Jardin, and D.A. Larrabee, Feedback stabilization of rigid axisymmetric modes in Tokamaks, Nucl. Fusion, 22 (1982).
 - D. Pfefferlé and A. Bhattacharjee, Algebraic motion of vertically displacing plasmas, Phys. Plasmas, 25 (2018).
 - K.E.J. Olofsson, Fast calculation of the tokamak vertical instability, Plasma Phys. Control. Fusion, 64 (2022).
 - A. Fukuyama, Seki, S., Momota, H., and Itatani, R., Positional Instabilities in a Tokamak with a Resistive Shell, Japanese Journal of Applied Physics, 14 (1975).
- + many others.
- However, these articles focussed on the growth and control of the usual, non-rotating VD resistive wall mode, while the VDOM and its related physics was not considered.

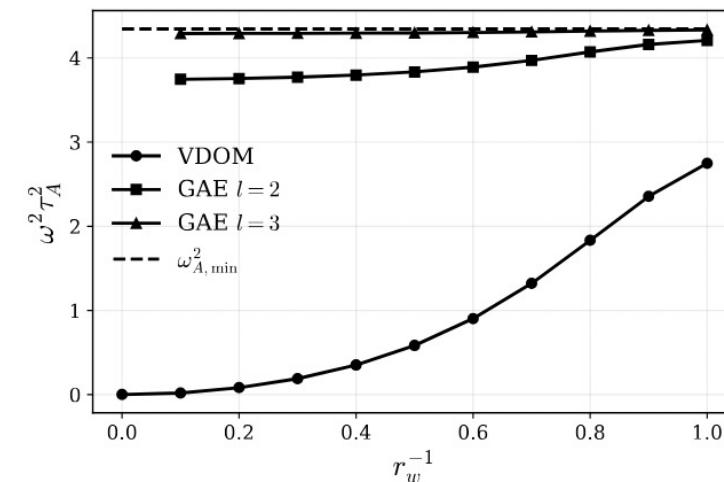
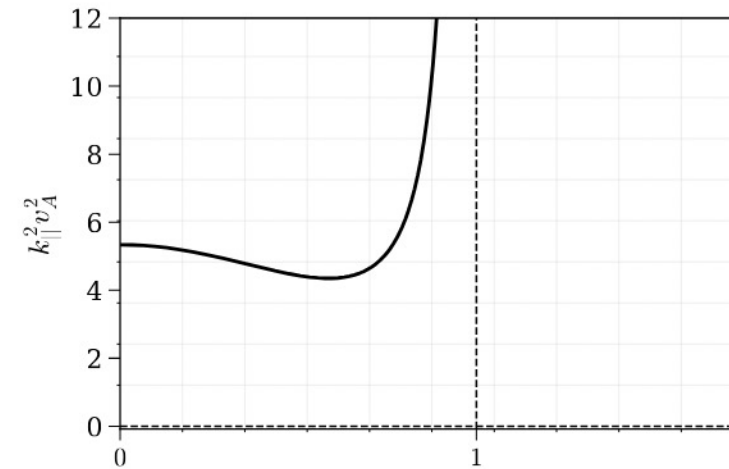
Continuum damping of VDOM

- It happens when, as the wall is brought closer to the plasma boundary, ω^2 can exceed the minimum of $k_{\parallel} v_A$.
- This does not happen for the latter equilibrium, but it happens for the former (shearless) equilibrium.

Shearless equilibrium – constant density



Equilibrium with shear and non-constant density



JET DT3 campaign (2023) n=0 are clearly driven by fusion alphas

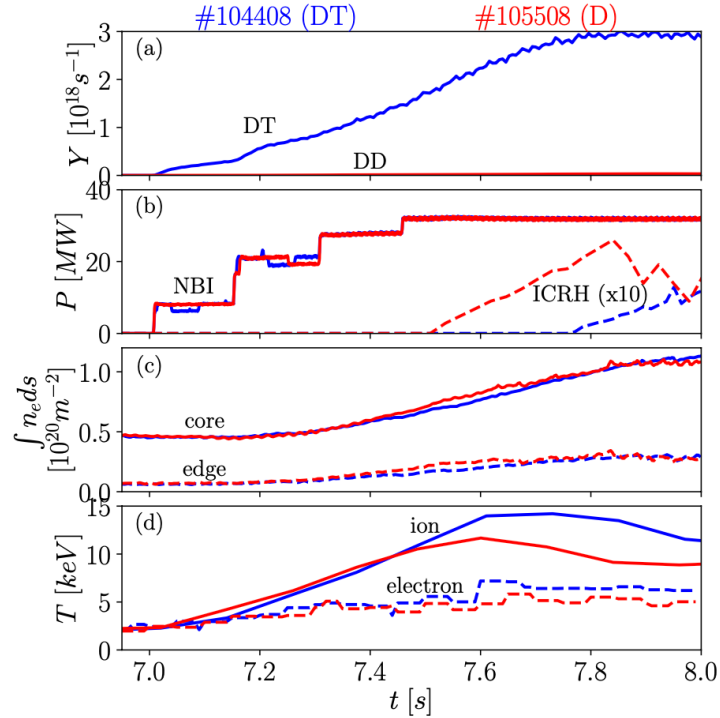


FIG. 1. Time evolution of plasma parameters for D-T pulse #104408 (blue) and pure D pulse #105508 (red): (a) neutron yield, (b) auxiliary heating power, (c) line integrated electron density, and (d) ion and electron temperature at $R = 3.3 \text{ m}$.

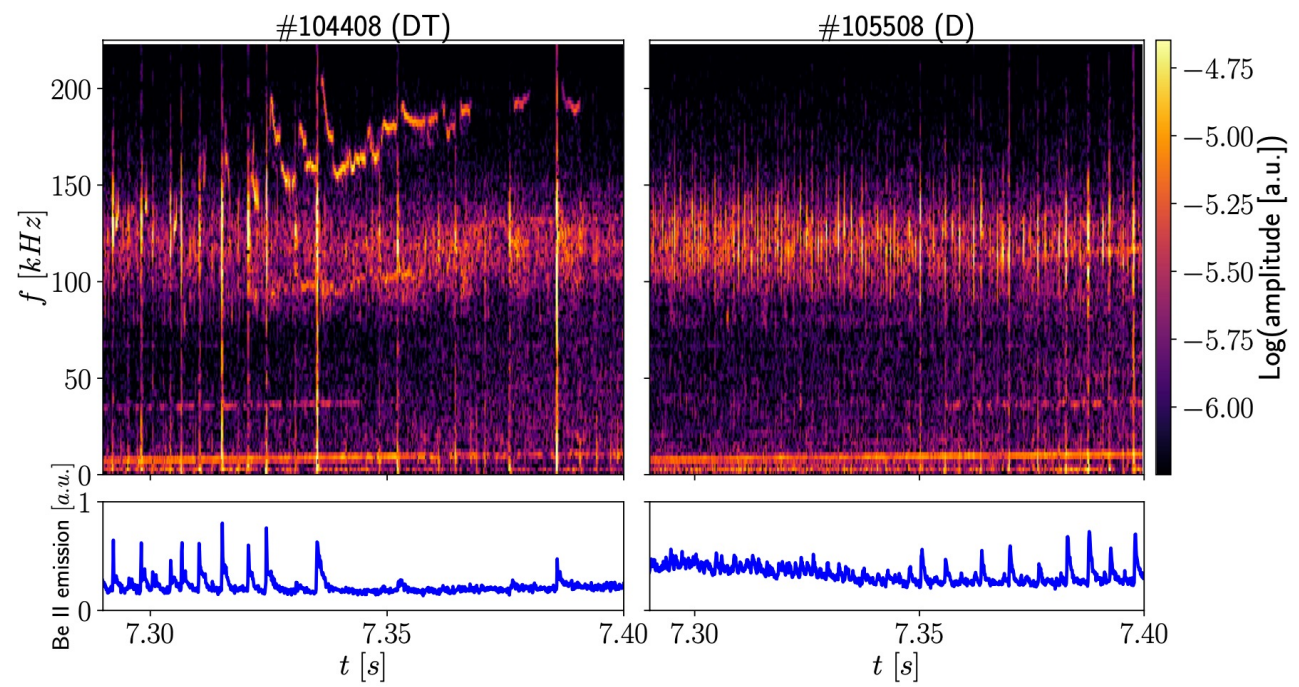


FIG. 2. Top: spectrogram of D-T shot #104408 and pure D shot #105508 showing the amplitude of the perturbed magnetic field. Bottom: Be II emission at the divertor.

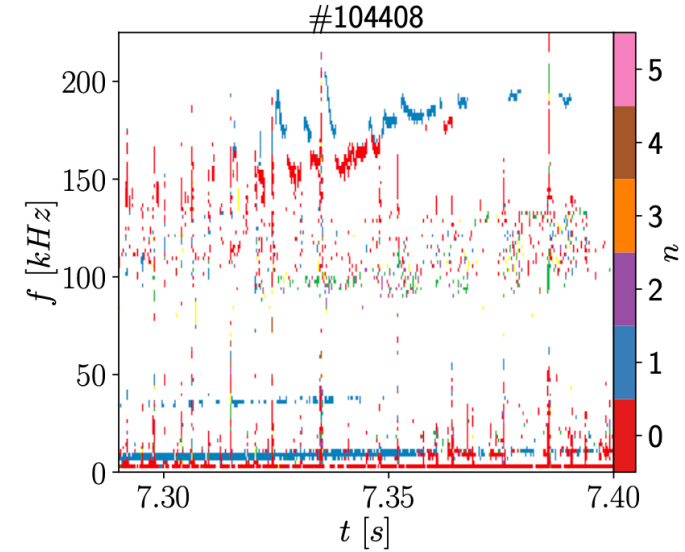


FIG. 3. Phase spectrogram of magnetic data showing toroidal mode numbers n of coherent signals in pulse #104408.

TCV, Lausanne

Figure 1. Low density, $I_p = 120$ kA, $B_T = 1.3$ T, positive triangularity discharge #72101 with NBI-1 injection in counter- I_p direction. Spectrogram of radial-component low temperature co-fired ceramic (LTCC) magnetic sensors [25].

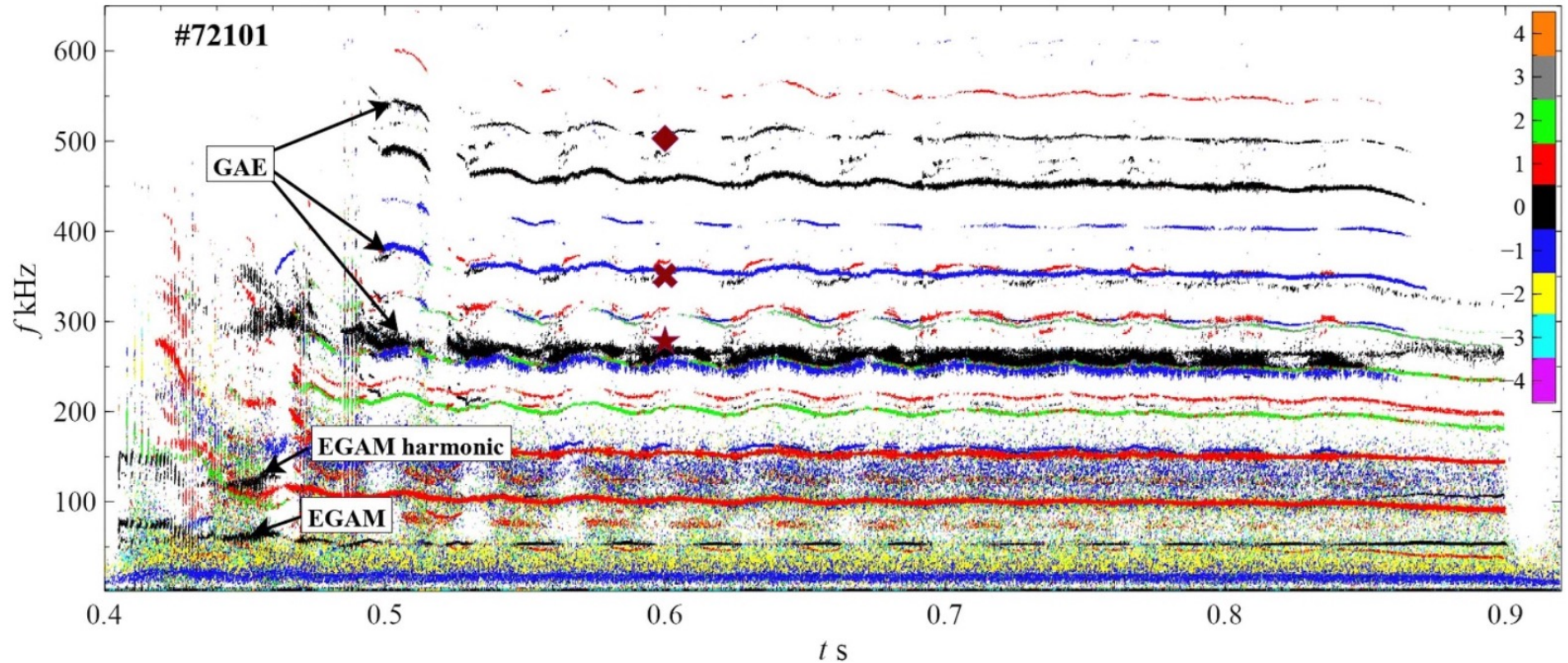
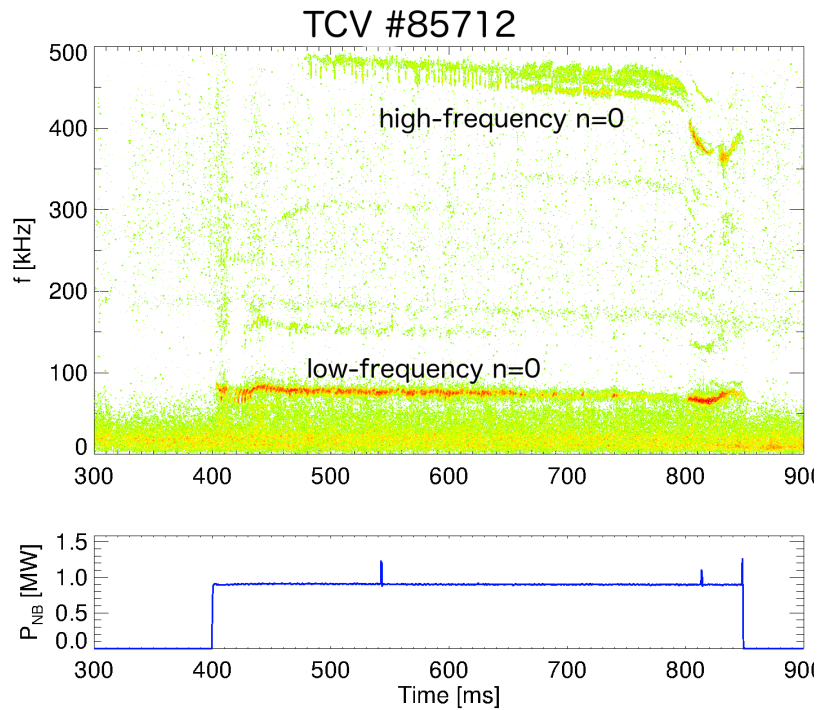
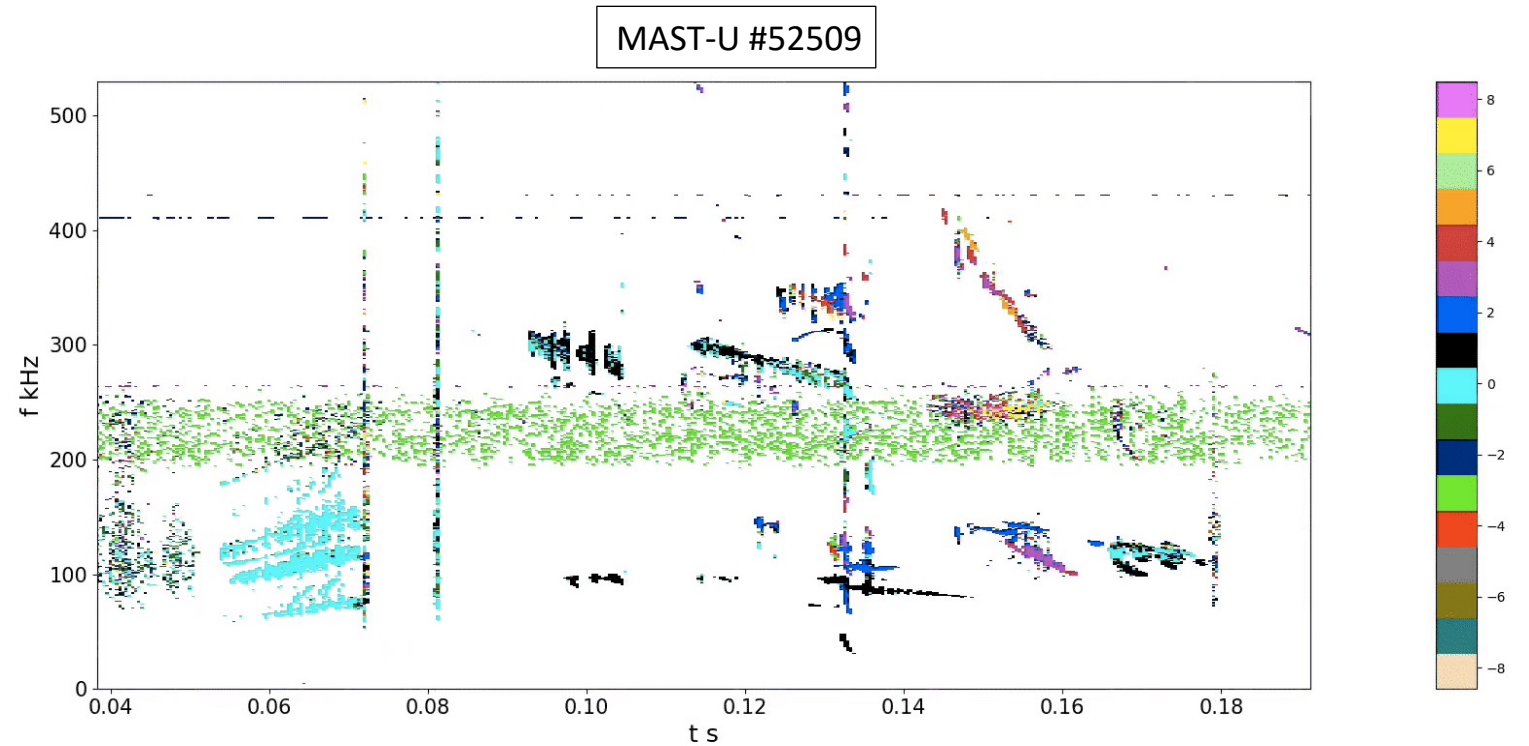


Figure 2. Toroidal mode numbers n evaluated by radial-component magnetic LTCC coils in TCV pulse #72101. Stars mark the axisymmetric GAEs calculated in figure 5. Reproduced from [4]. © IOP Publishing Ltd. All rights reserved.

High-frequency (Alfvén) $n=0$ modes have also been observed on TCV and on MAST-U



Example of NBI-heated TCV discharge with simultaneous destabilization of low- and high-frequency $n=0$ modes.



Example of NBI-heated MAST-U discharge #52509 with destabilization of low- and high-frequency $n=0$ modes. Traces corresponding to $n=0$ modes are in light blue.

On the drive of $n_\phi = 0$ modes by ICRF accelerated ions in a tokamak

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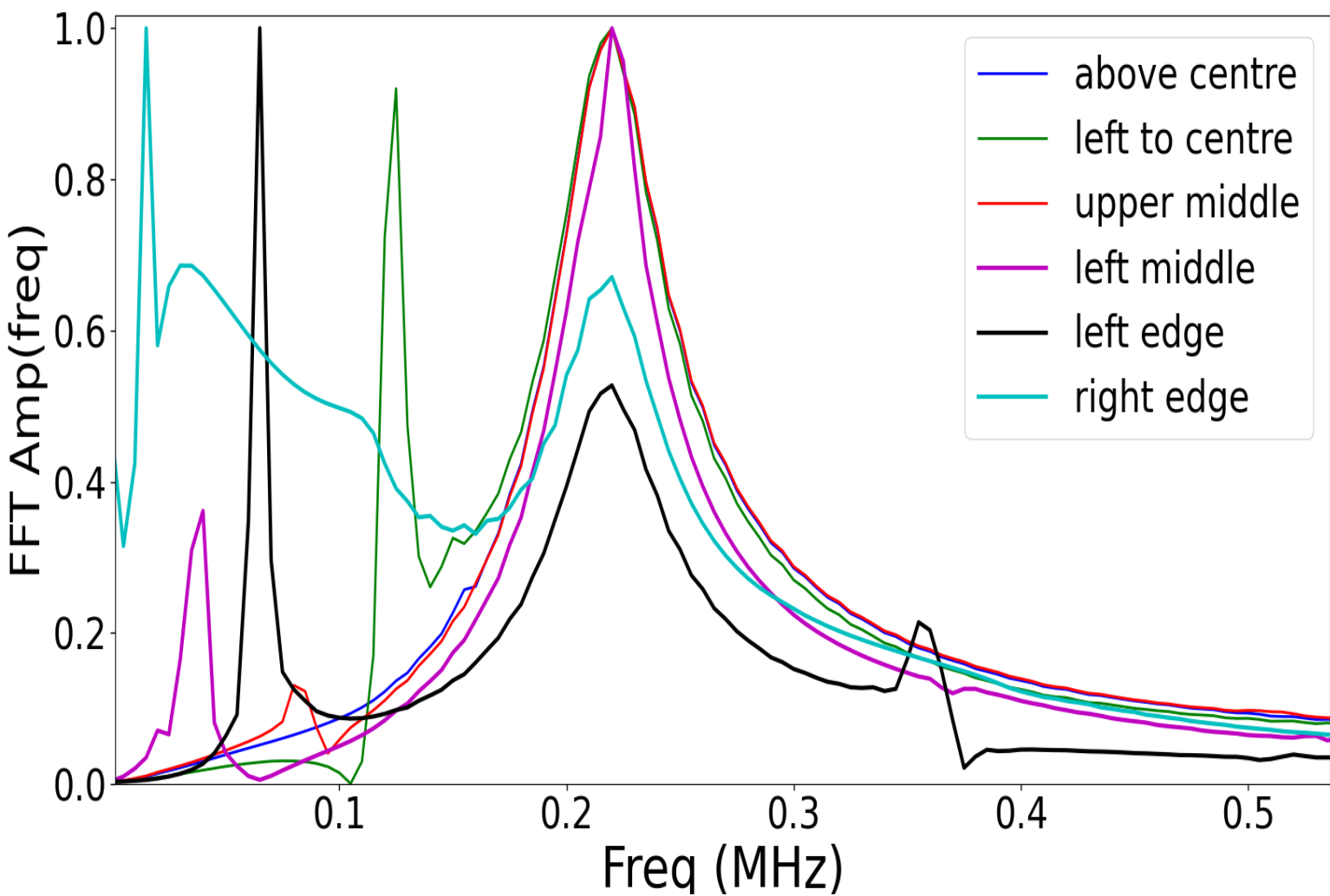
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Abstract

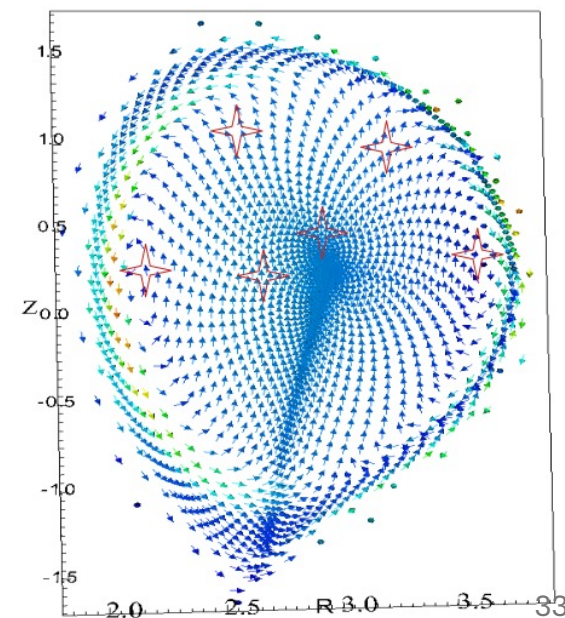
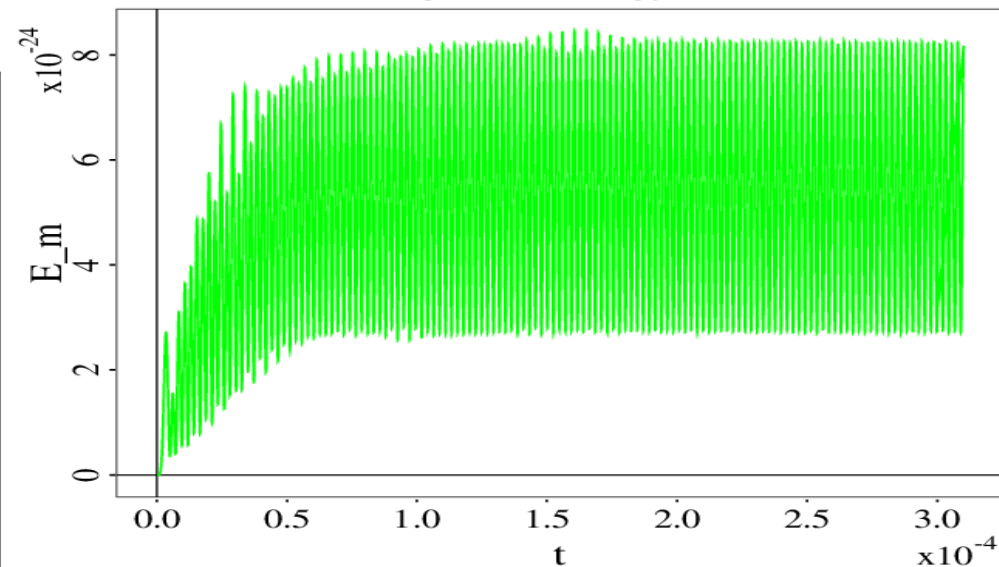
Energetic ions have the potential to drive instabilities with toroidal mode number $n_\phi = 0$ in tokamak fusion plasmas. A necessary condition is that their distribution function is either anisotropic or exhibits a region of phase space with a positive slope in the energy direction (so-called ‘bump-on-tail’ distribution). Here an exploration of both possibilities is presented for Ion Cyclotron Range of Frequency (ICRF) accelerated energetic ions. It is found that ion cyclotron resonance layers placed on the high-field side of the magnetic axis provide the most conducive conditions for the drive of vertical $n_\phi = 0$ modes. We also discuss to what extent sawtooth redistribution of the ICRF accelerated ions can transiently lead to distribution functions that are locally inverted in the energy direction. However, the latter effect appears to be less efficient in driving $n_\phi = 0$ vertical modes than velocity space anisotropy for high-field-side resonances.

Probing a VDOM discrete normal mode (JET equilibrium)

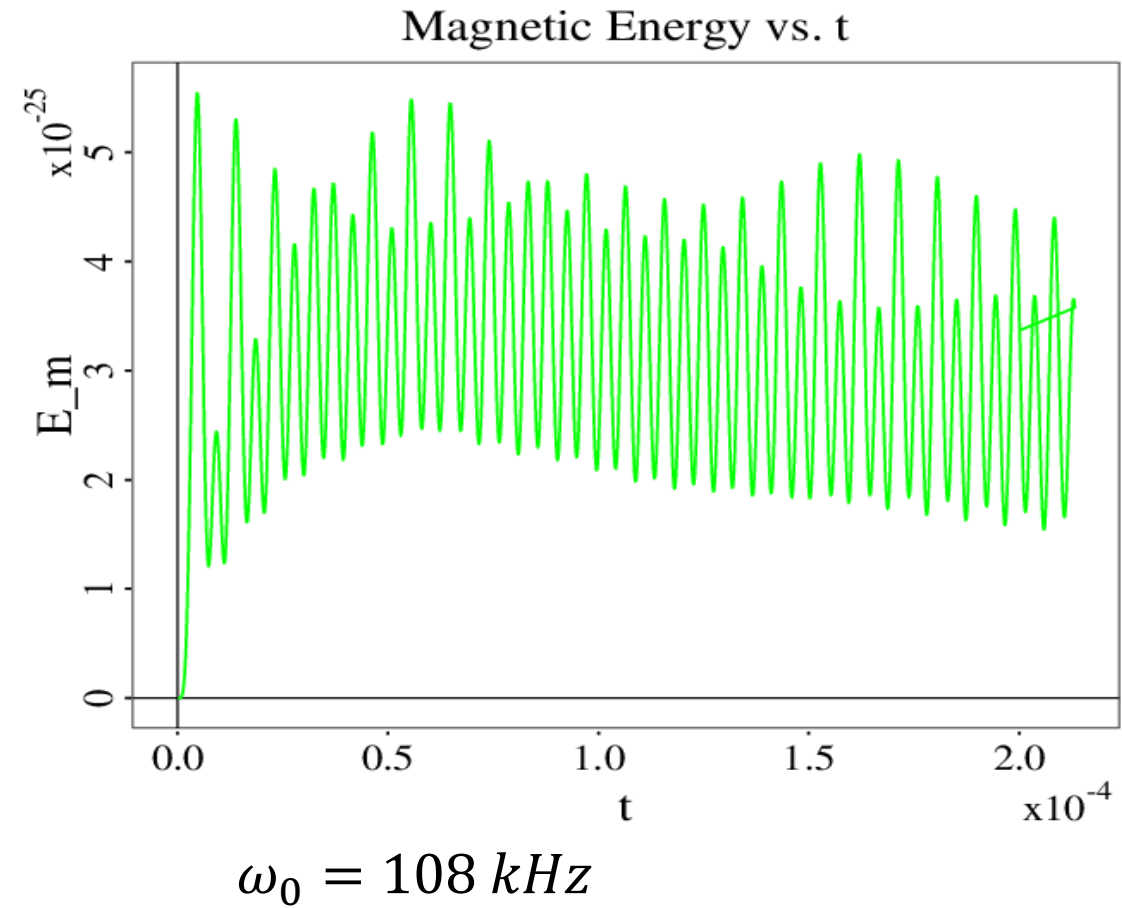
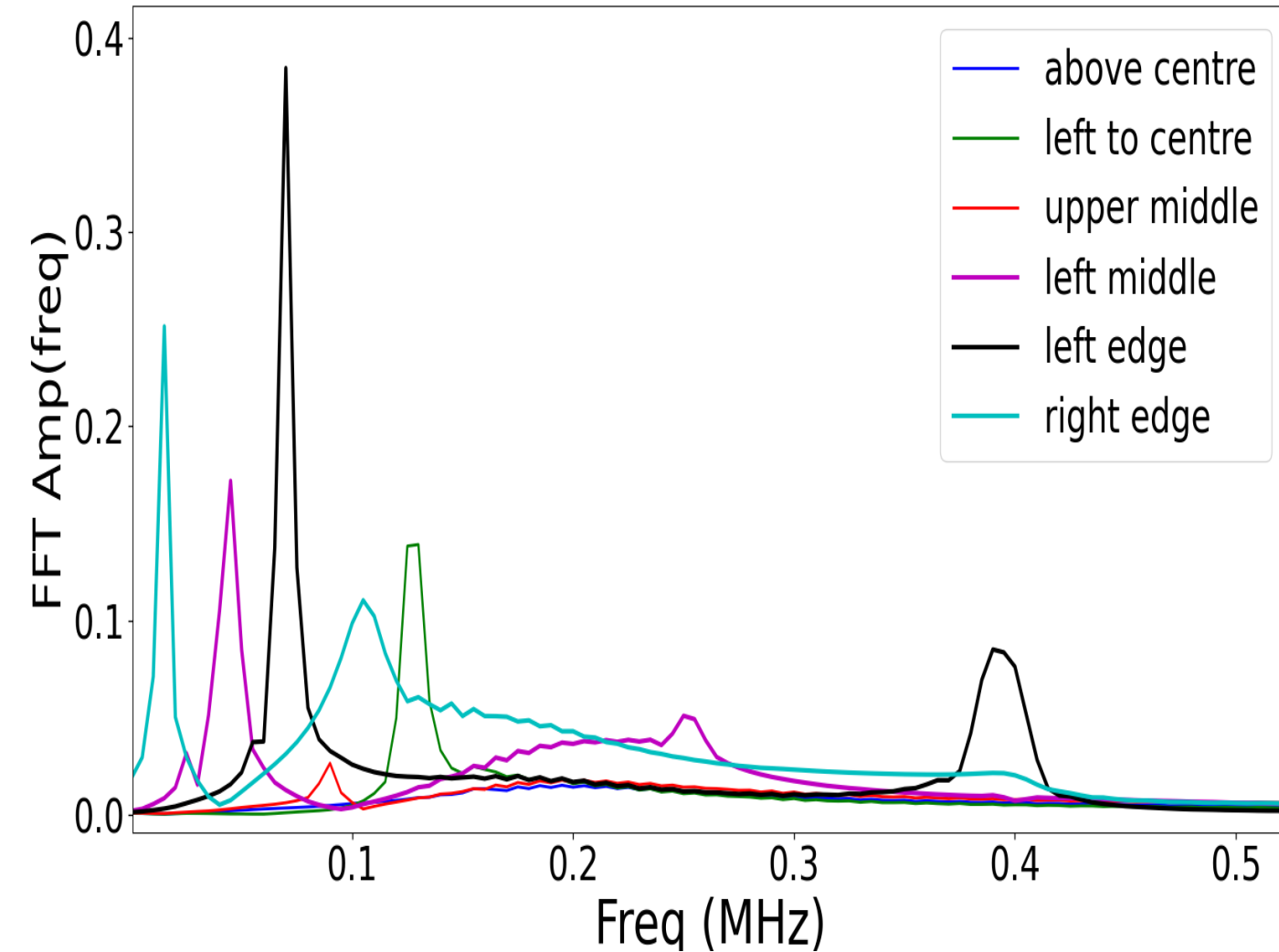


$$\omega_0 = 217 \text{ kHz}$$

Magnetic Energy vs. t

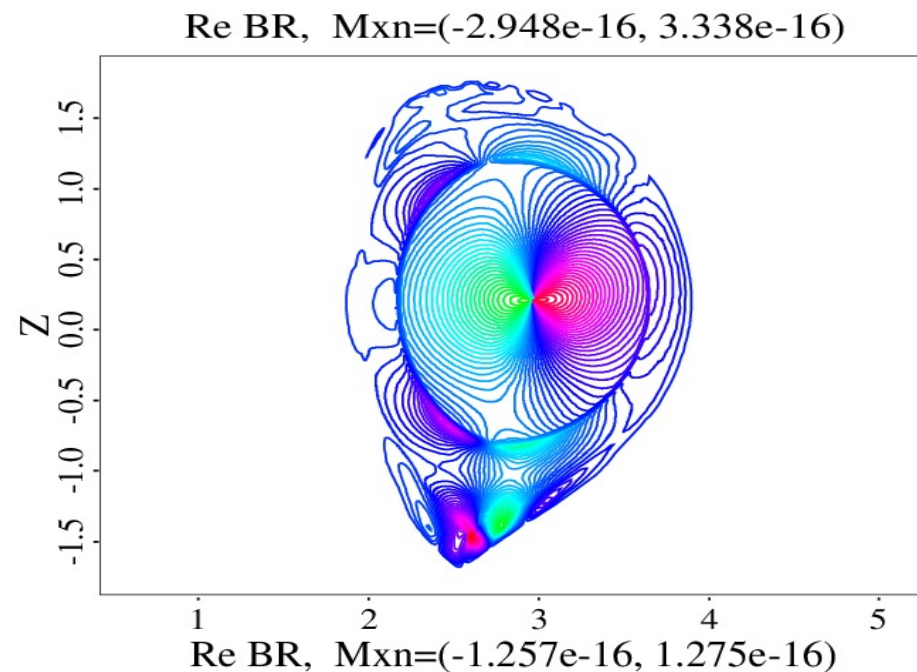
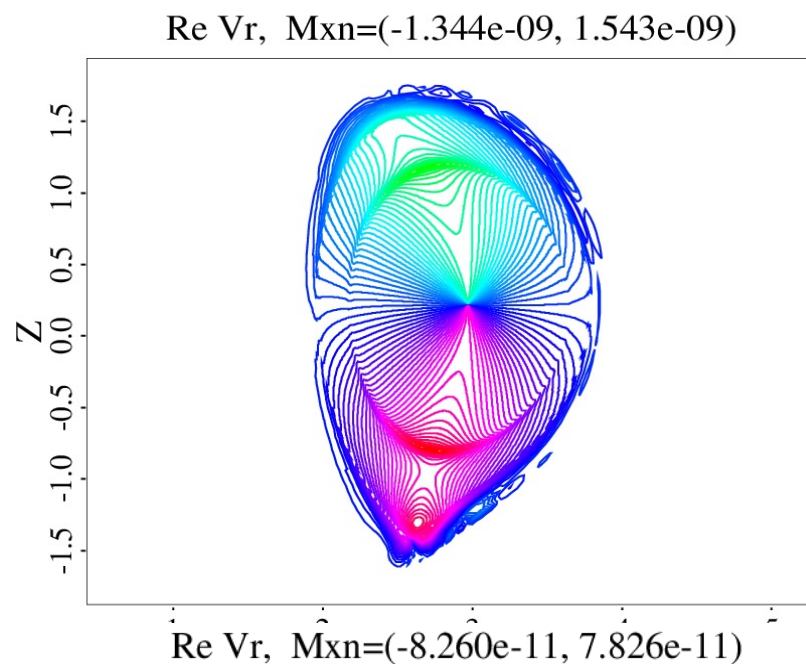


Continuum Alfvén quasi-mode: different probes are not synchronized



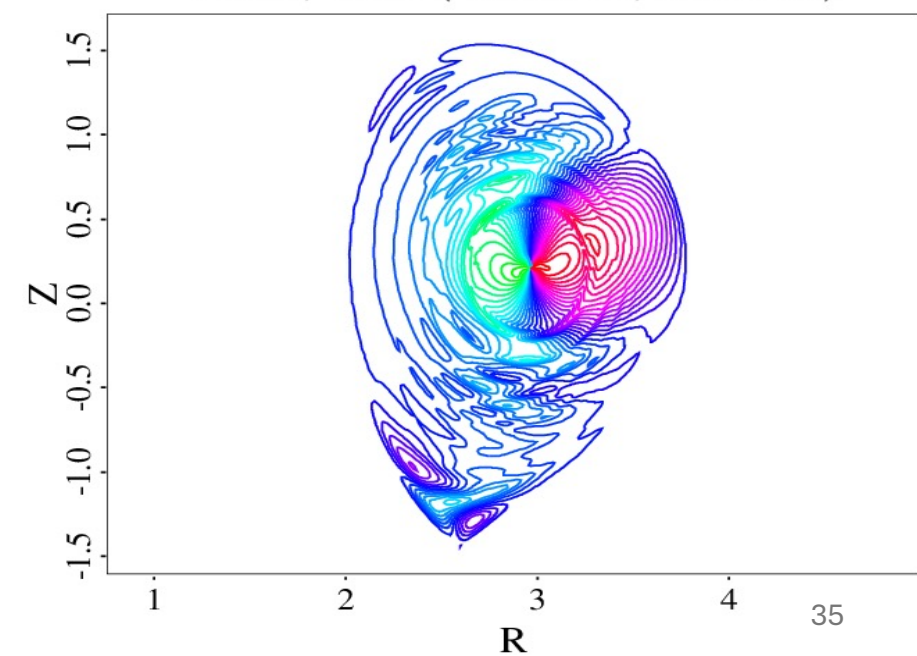
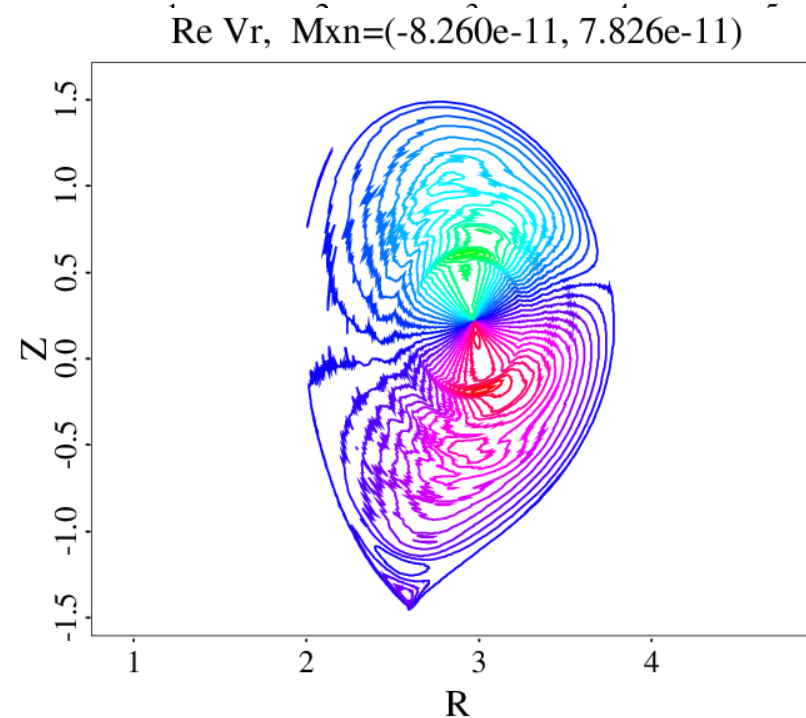
Upper panels: VDOM

different probes are
synchronized



Lower panels: continuum
quasi-mode

different probes are not
synchronized



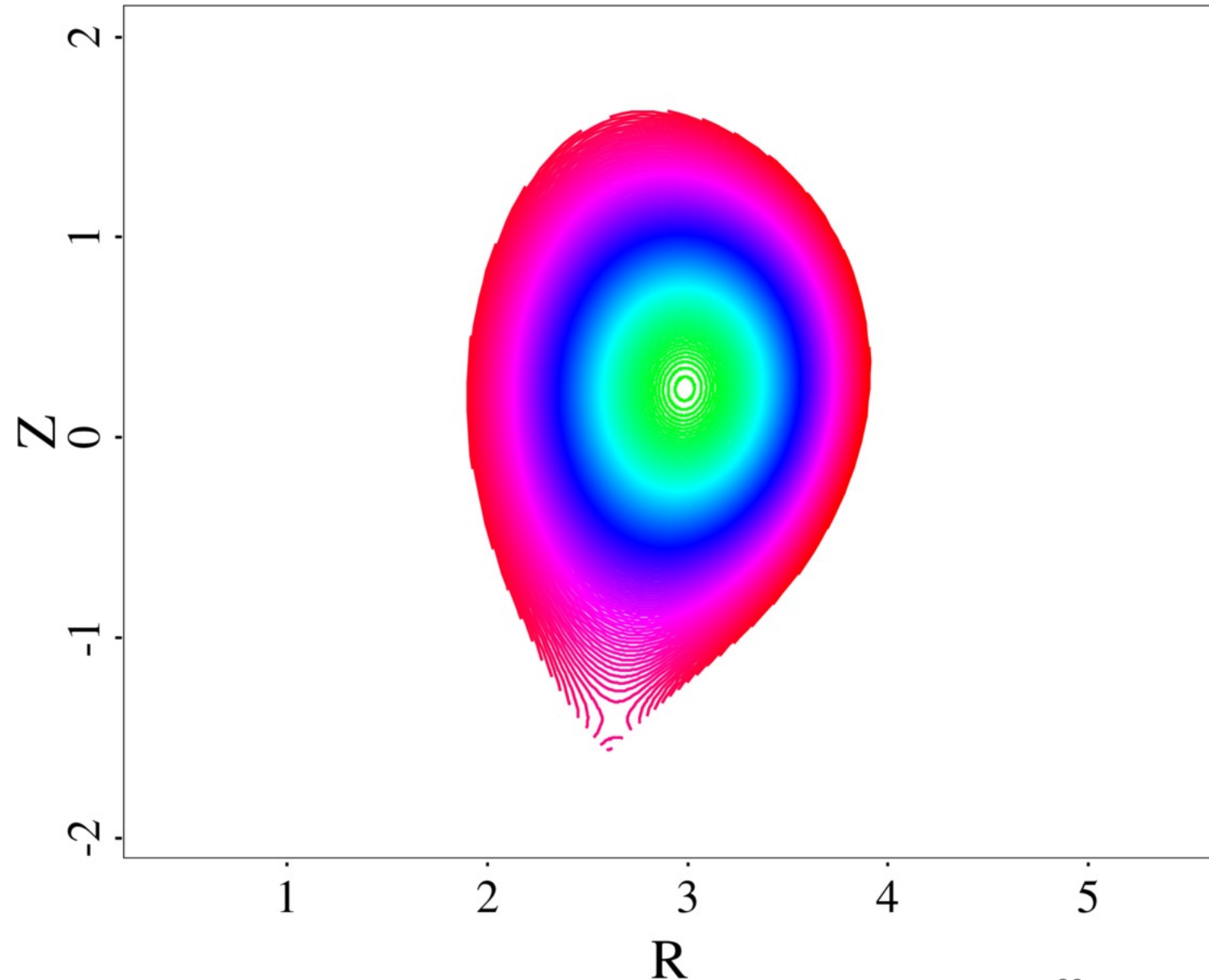
Nimrod simulations of JET discharges

Pressure and current density profiles from EFIT equilibrium reconstruction of shot #102371 at $t = 51.000$

Look for stable oscillations
→ Minimize damping sources

Single null configuration with non-zero triangularity
→ one X-point is in the domain (but in the halo region) while a second X-point still lies outside.

Tommaso Barberis et al, NF 2024

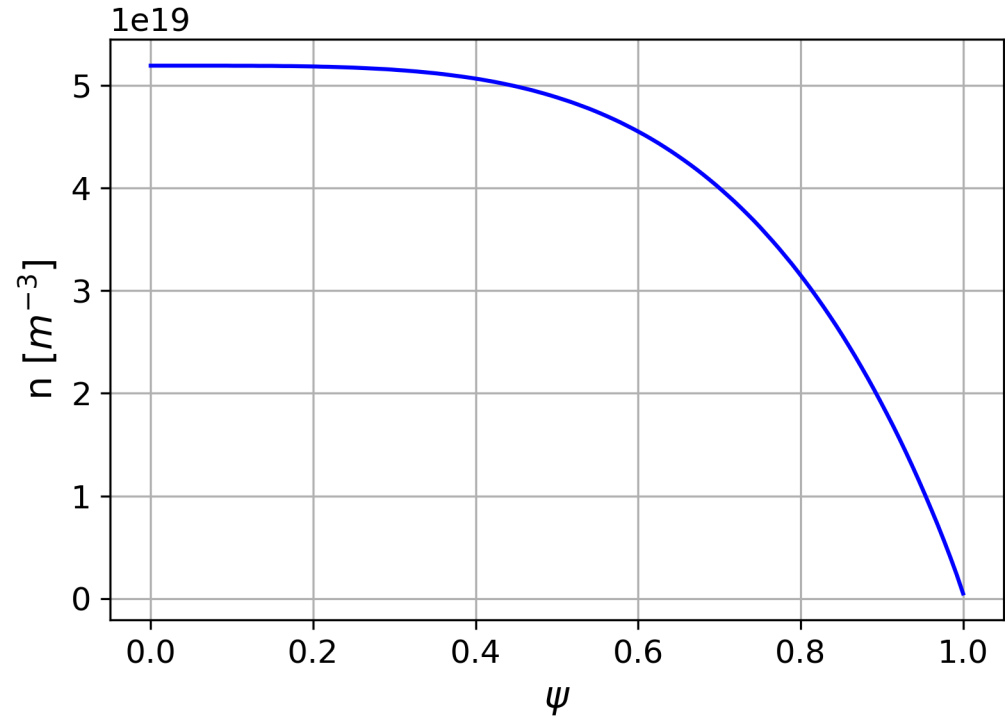
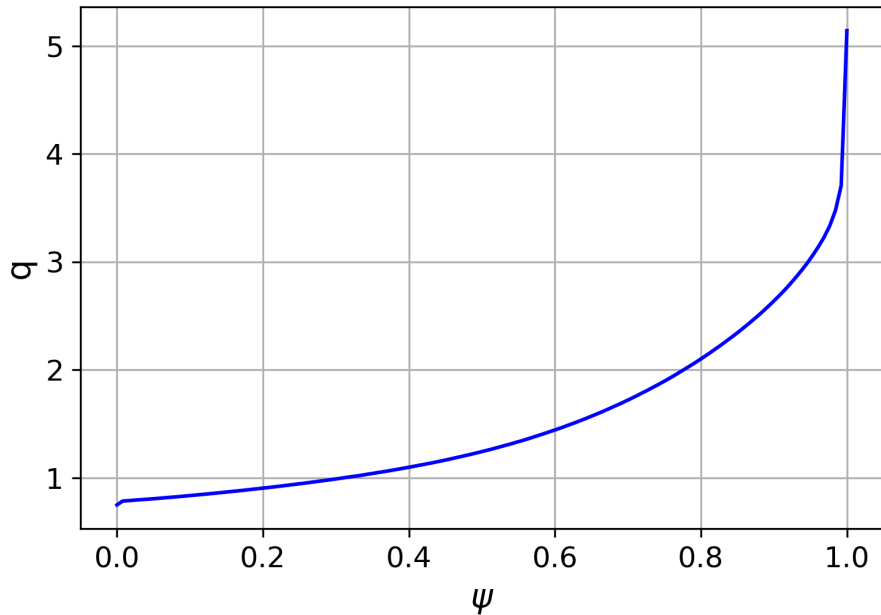
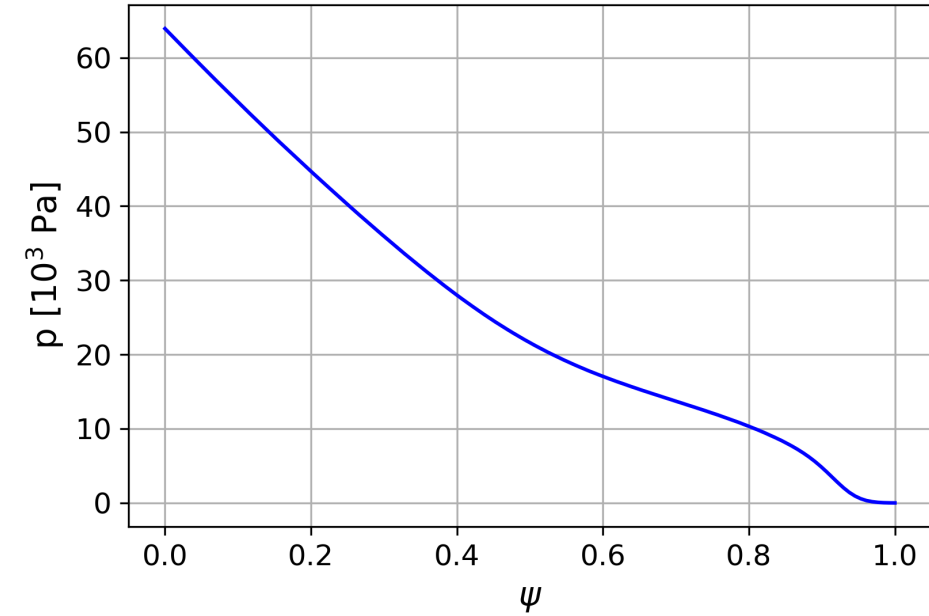


Realistic Equilibrium profiles

→ current and plasma density profiles will impact the analytic model mode structure

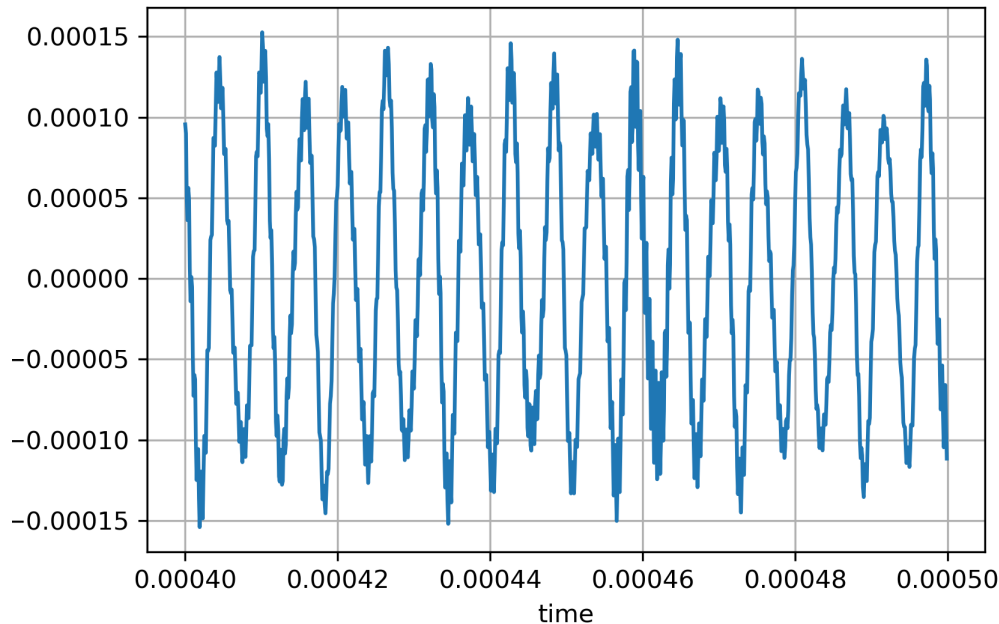
← Left Safety factor and pressure profiles up to separatrix

Right → Density profile up to the separatrix.
Halo region density uniform with $n_{halo} = 5 \times 10^{17}$



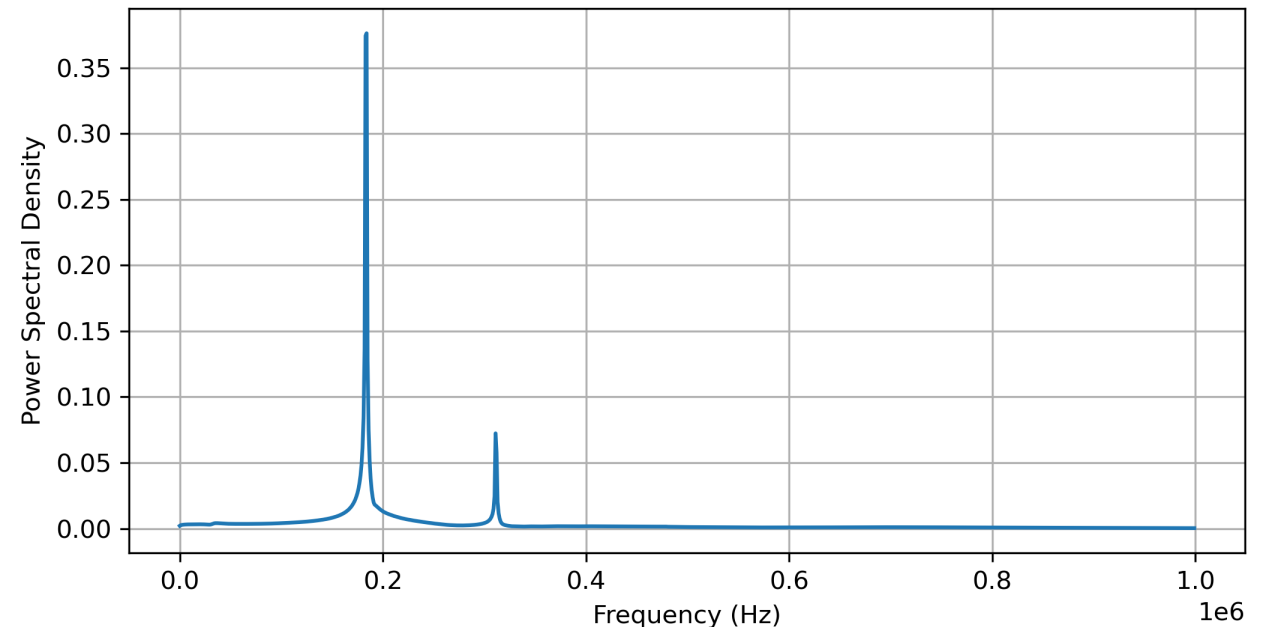
Stable oscillations found when perturbing the equilibrium with a «vertical push» of the plasma

Magnetic energy time trace

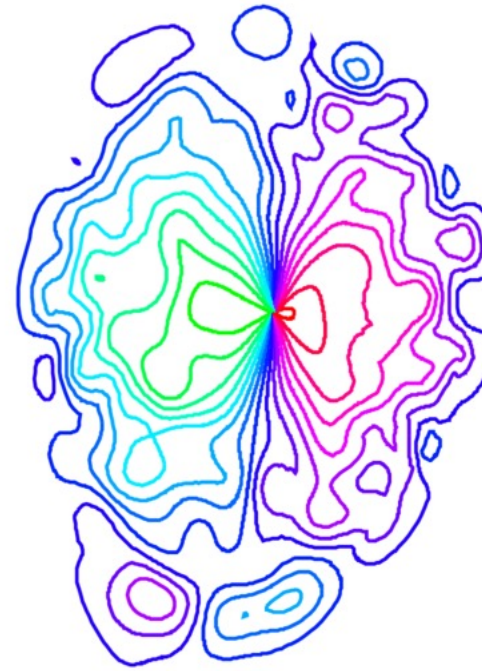
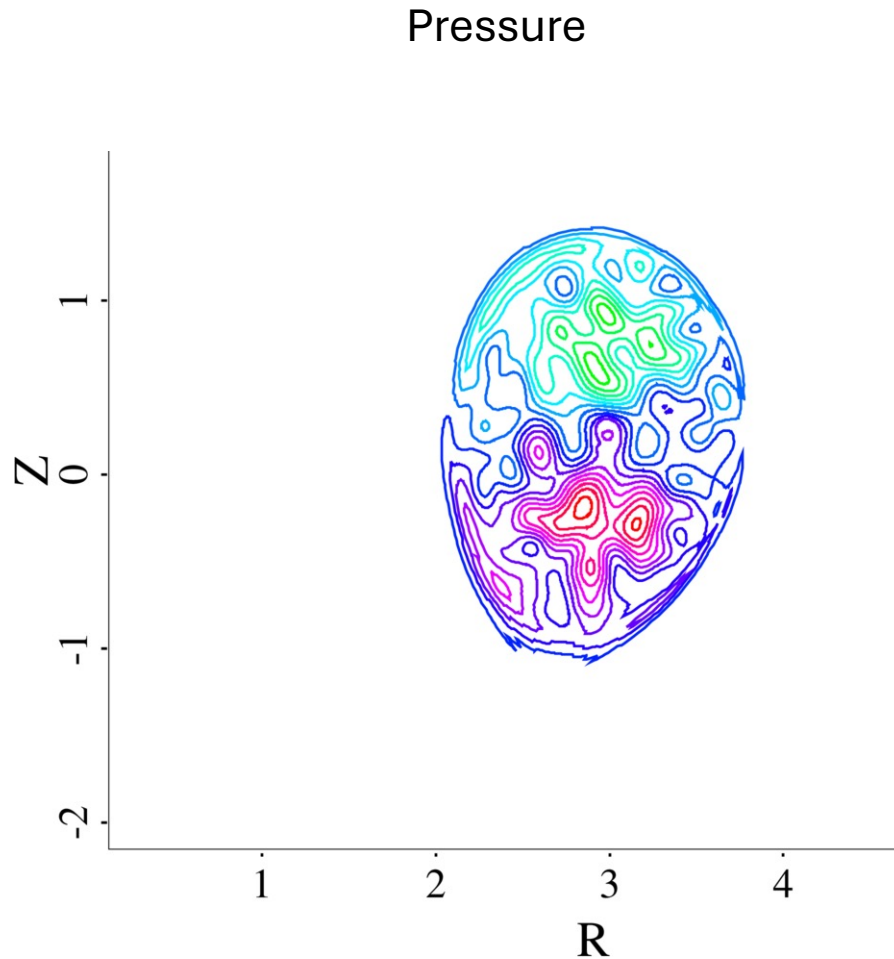


FFT signal shows two distinct peaks:
Dominant peak at $\sim 184\text{ kHz}$ and
secondary peak at $\sim 311\text{ kHz}$

Dominant Frequency: 184000.00, Hz



Hints of an up-down symmetric pressure perturbation and $m = 1$ mode structure

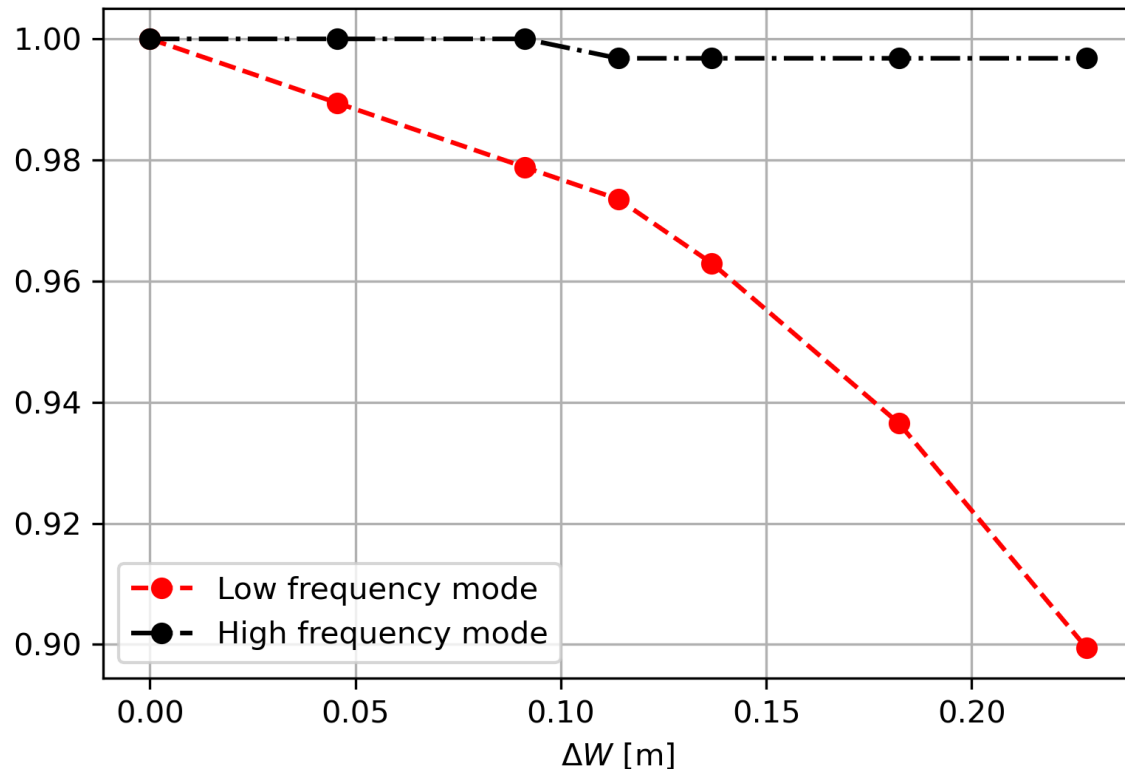


Perturbation in B norm.



Perturbation in B tang.

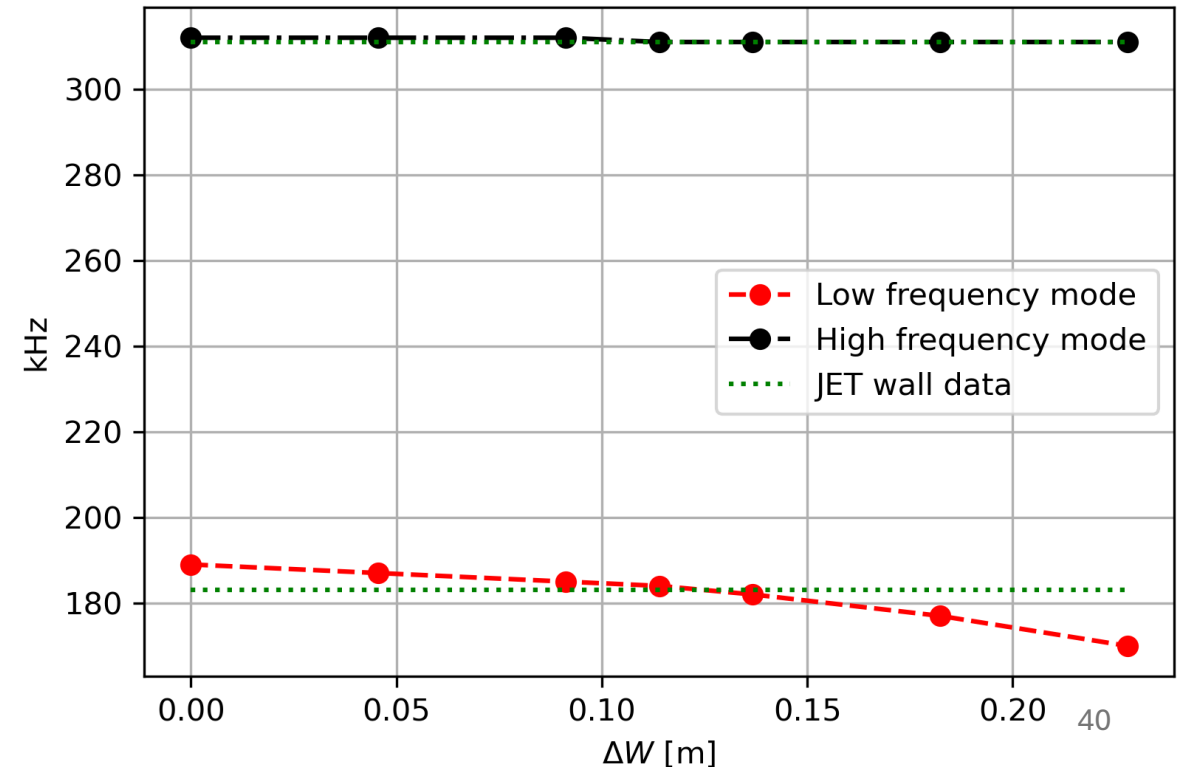
THE TWO FREQUENCIES SHOW DIFFERENT DEPENDENCE WITH RESPECT TO WALL POSITION



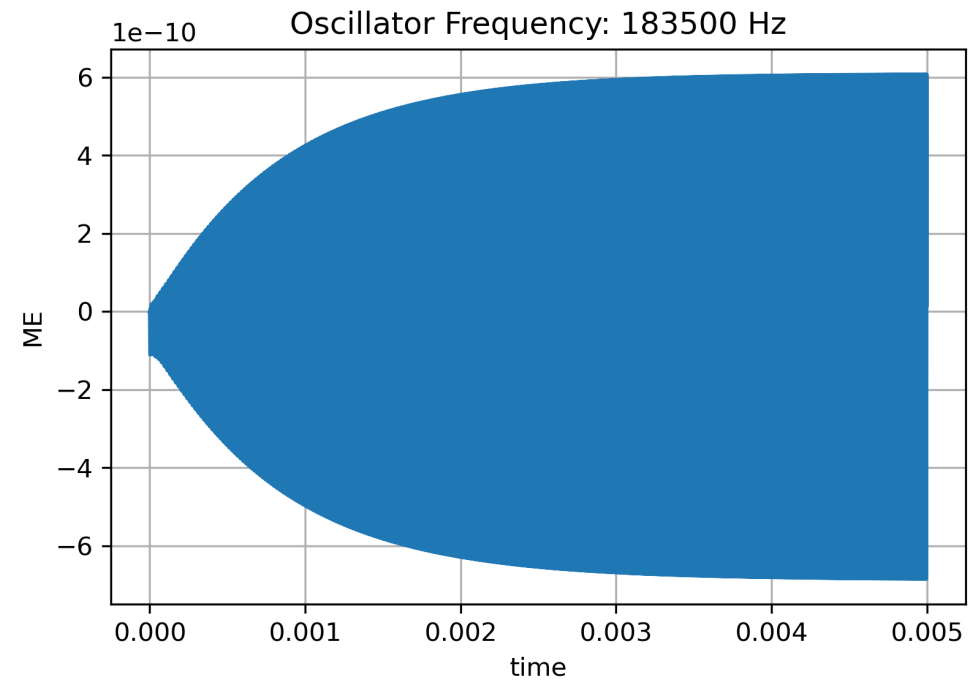
Frequency change of the two modes as a function of the wall distance in terms of the plasma boundary-wall distance ΔW .

High frequency mode (GAE) ~ insensitive (change of 2 kHz)

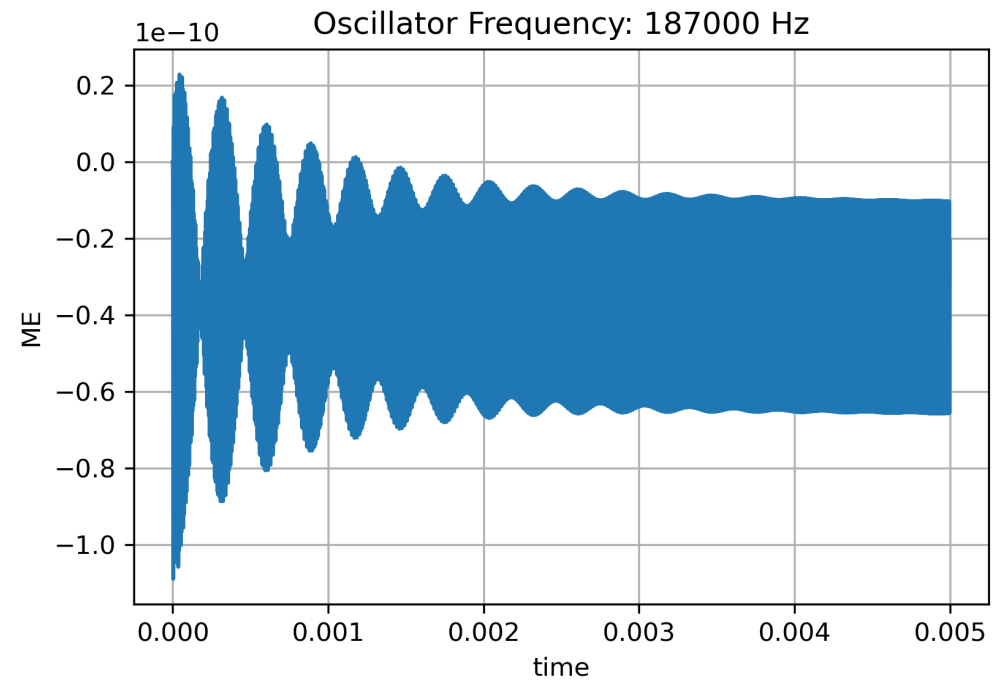
Low frequency mode (VDM) shows a stronger dependence (189 kHz – 170 kHz).



Resonant case:
Growing & saturation



Off-resonant case:
Beating & saturation
 $f_{beat} = |f_{osc} - f_{mode}|$



In order to study each mode independently we consider a different kind of initial perturbation: Oscillator in T

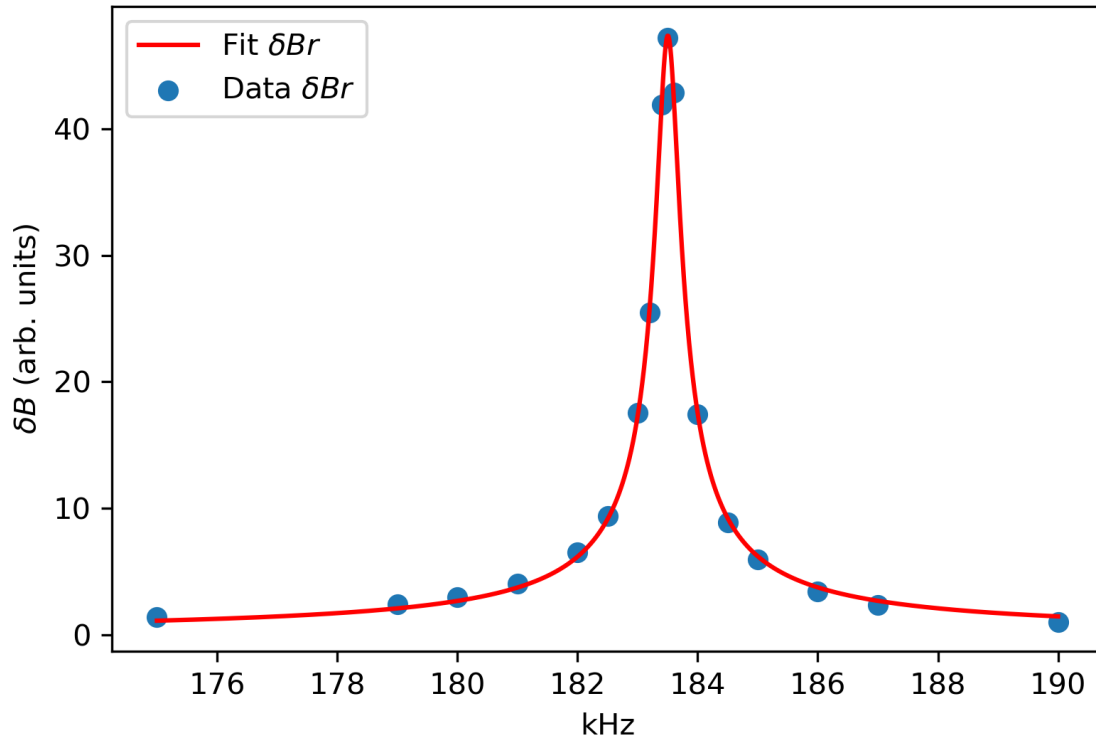
Extra term in the temperature evolution $\propto \sin(\omega_0 t) * \exp\left(\frac{(r-r_0)^2 + (z-z_0)^2}{\Delta^2}\right)$

Data fitted with $|Y| \propto \frac{C}{\sqrt{(\omega - \omega_0)^2 + D^2}}$ assuming resonance condition $Y \propto \frac{C}{(\omega - \omega_0) + iD}$

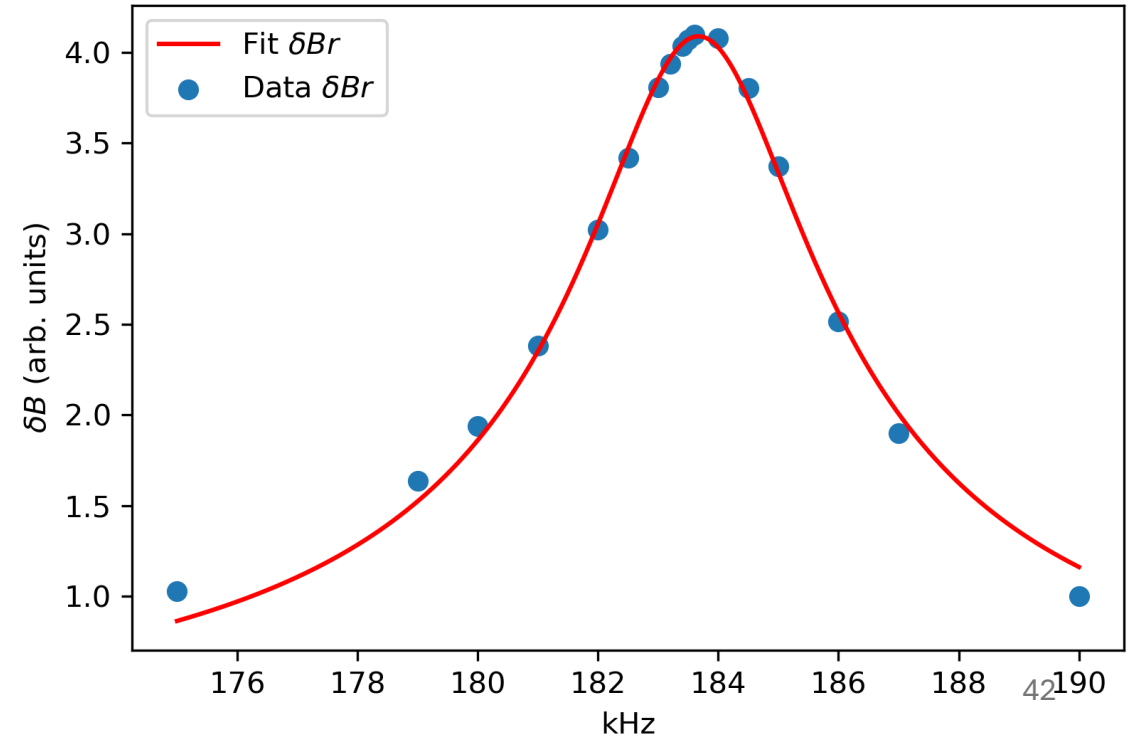
20x viscosity



Fitted resonance: $\omega_0 = 183.51$, $D = 1.24$

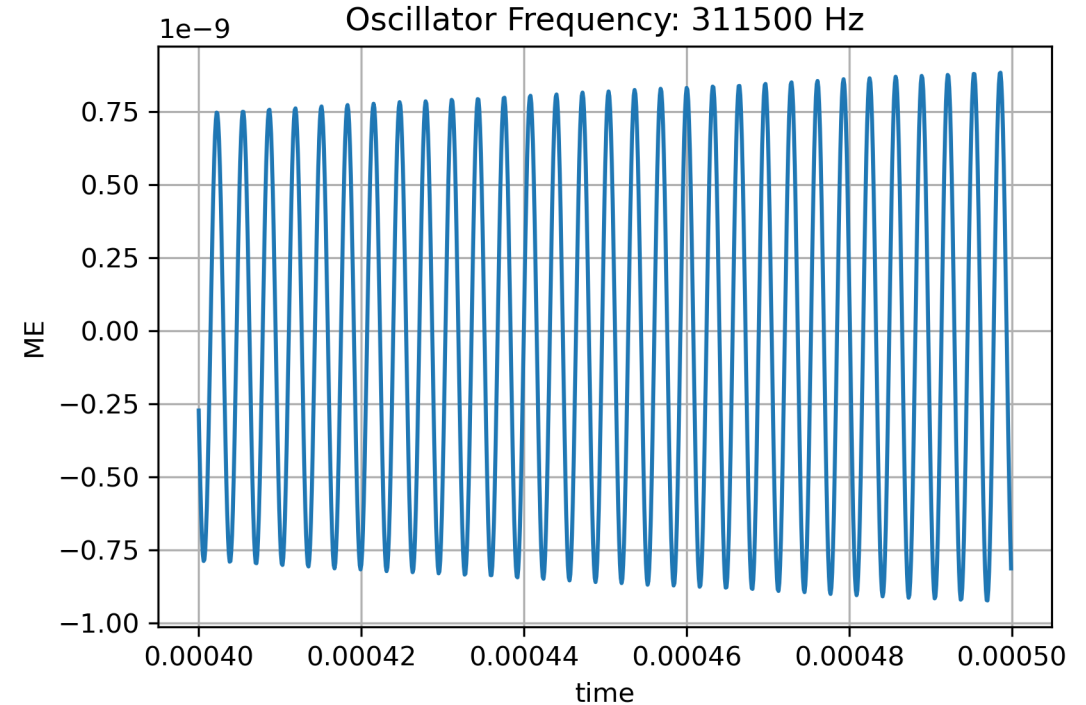
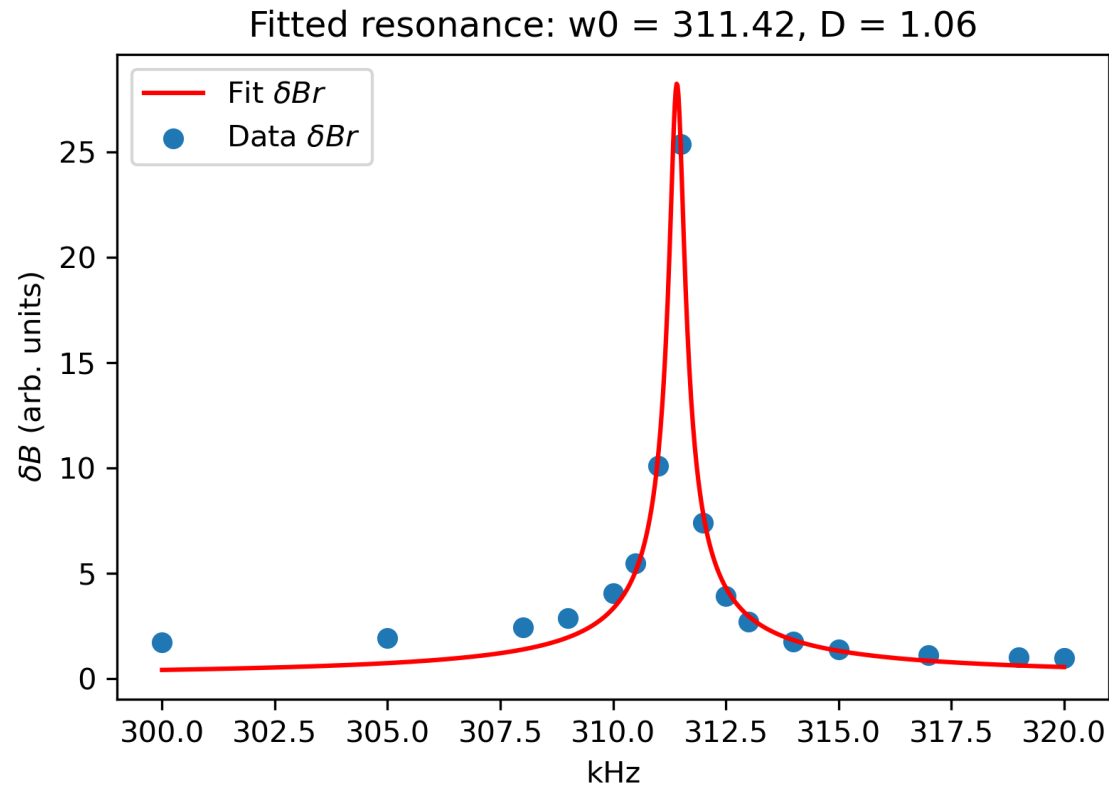


Fitted resonance: $\omega_0 = 183.67$, $D = 11.79$

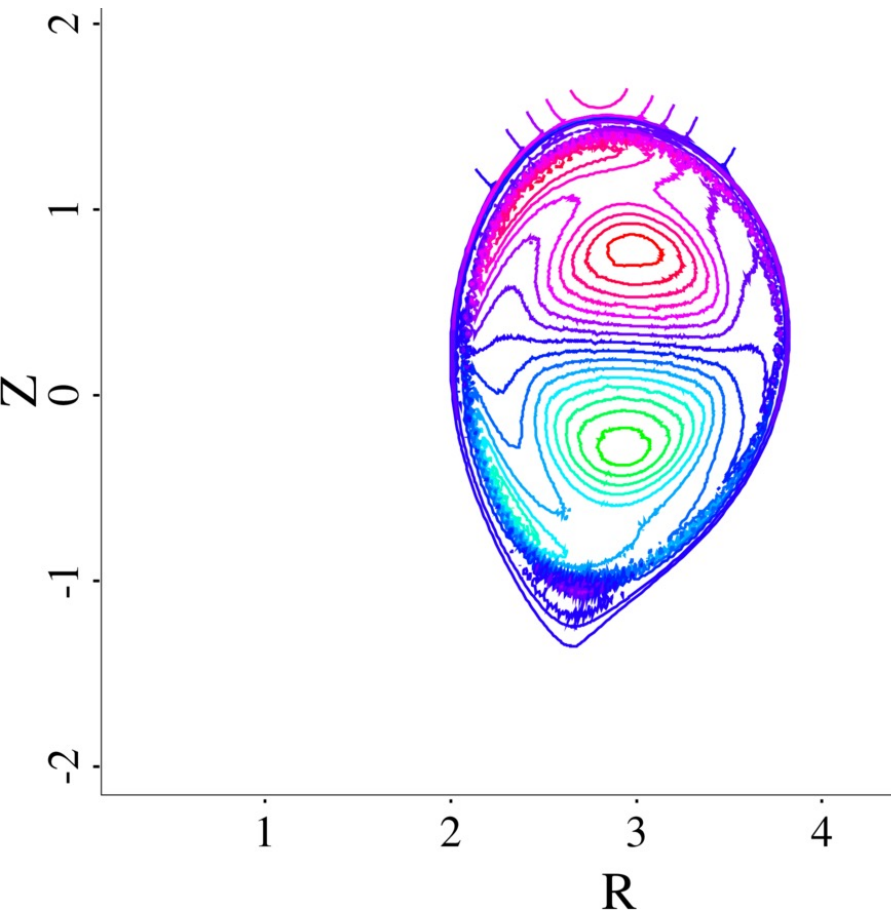


B

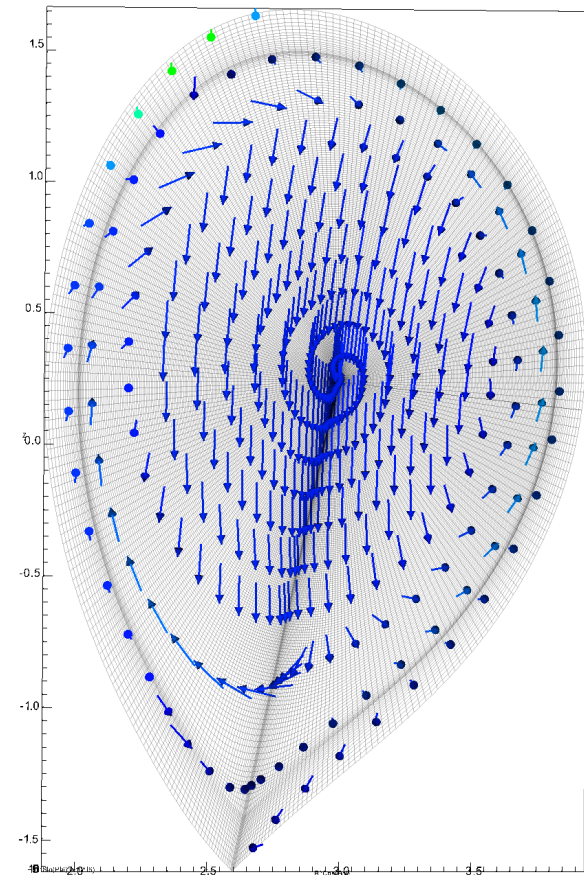
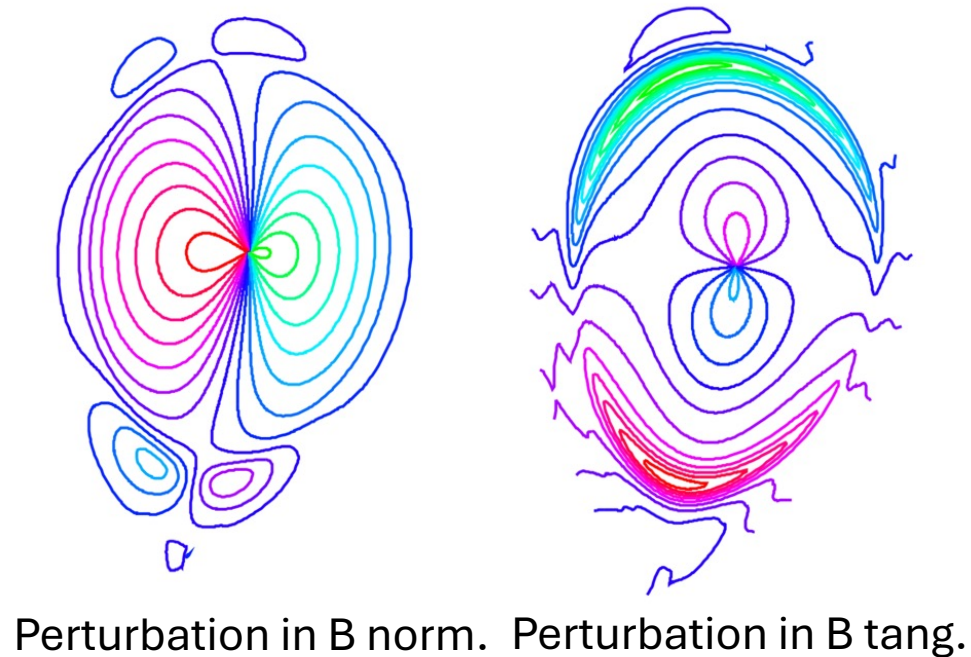
With the oscillator we can investigate the two modes – VDOM and GAE – independently.



VDOM mode structure: up-down symmetric pressure perturbation and $m = 1$ mode structure

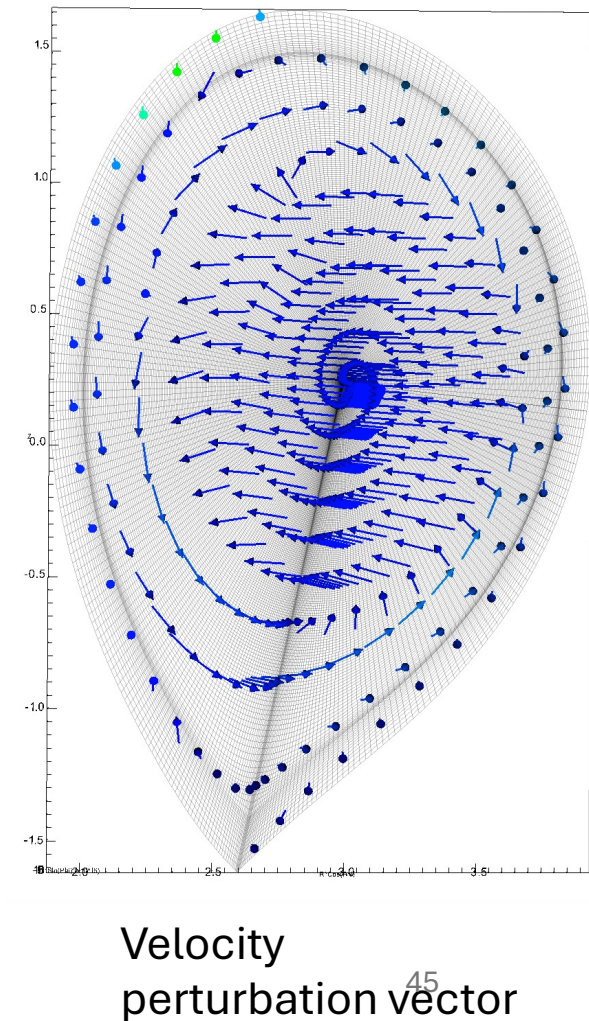
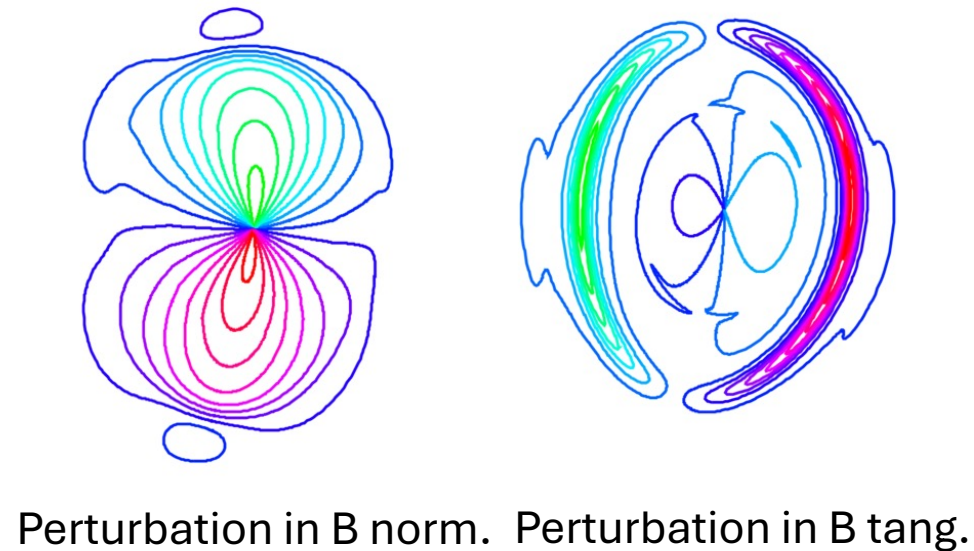
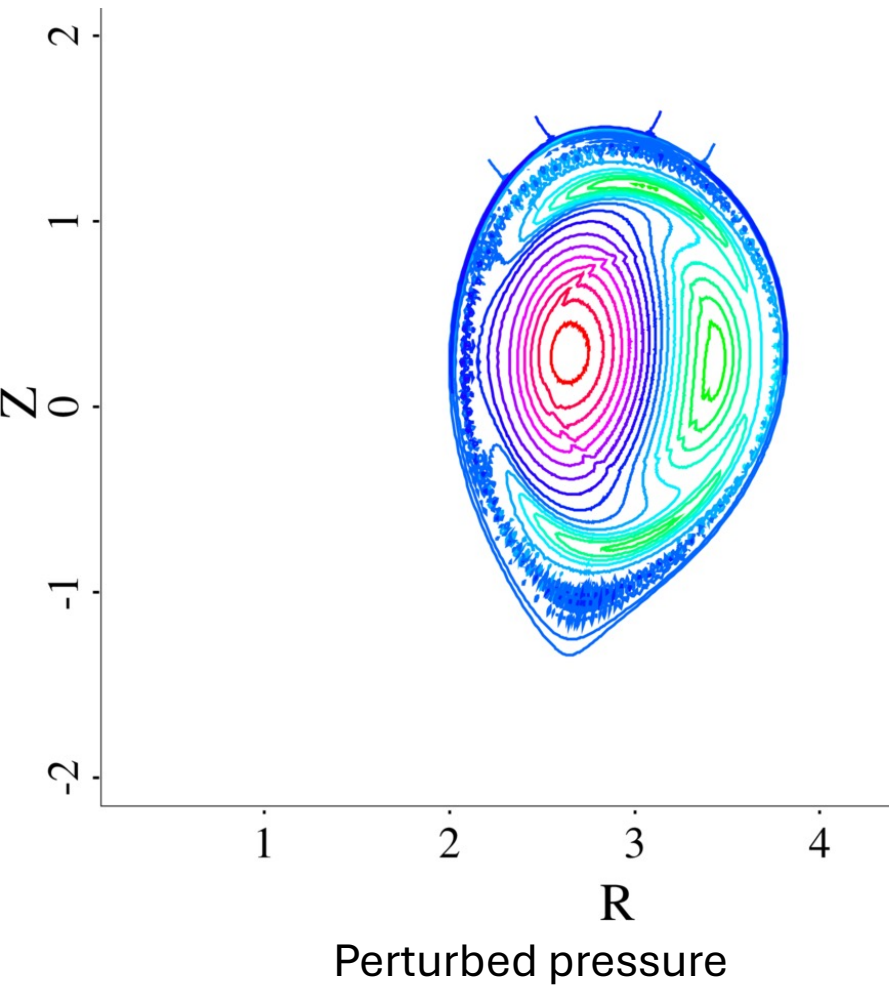


Perturbed pressure



Velocity perturbation
vector plot

GAE mode structure: «horizontal» pressure perturbation and $m = 1$ mode structure

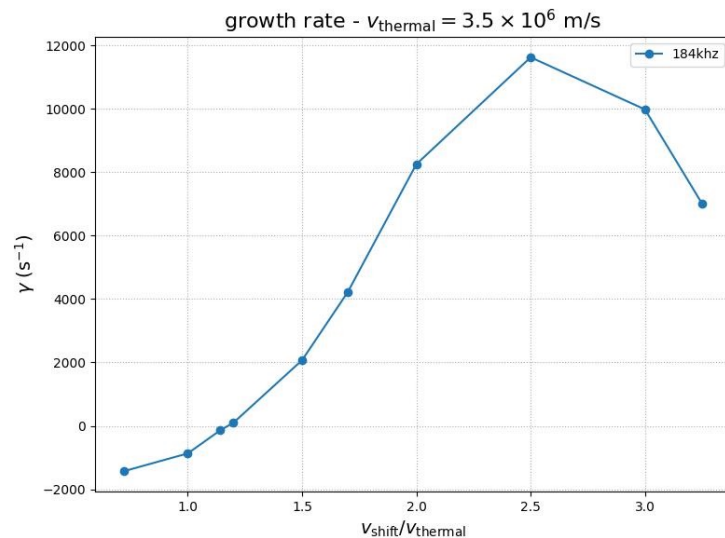


A new fast-ion module has been implemented in NIMROD

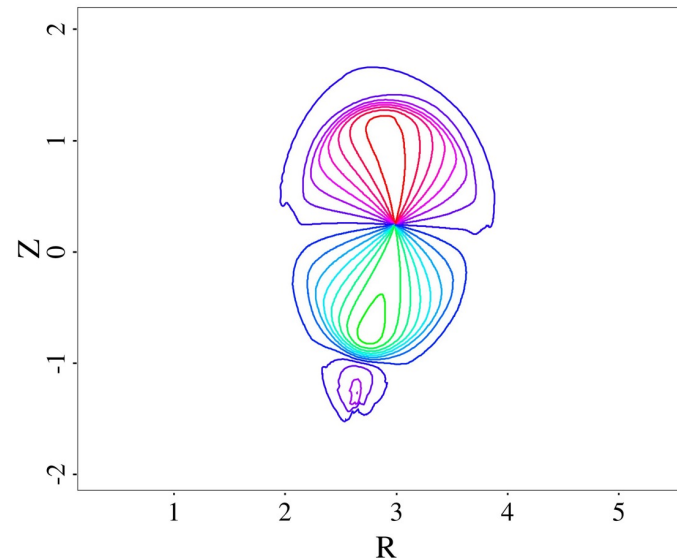
Analytic distribution function:

$$F_h = C v_{therm}^{-3} \exp \left(-\frac{P_\phi}{\psi_n} - \frac{(v_{\parallel} + v_{shift})^2 + v_{\perp}^2}{v_{therm}^2} \right)$$

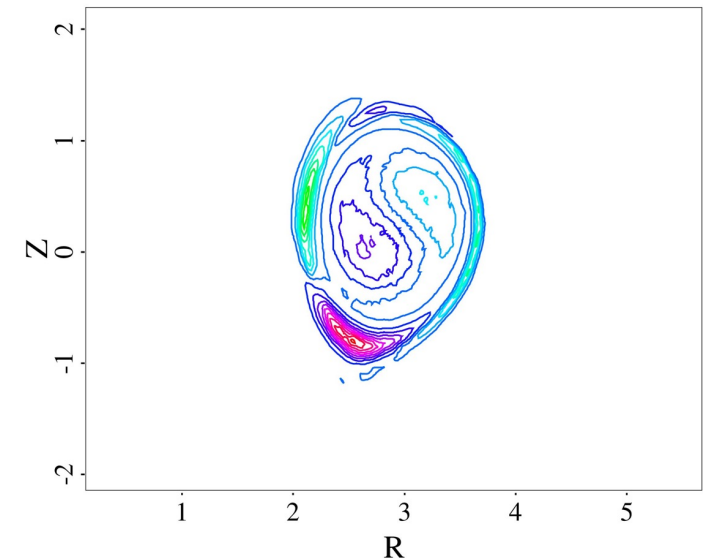
Numerical results:



Linear growth rate as a function of the normalized velocity shift.



Contours of the normal component of the plasma flow velocity.



Contours of the fast particle density, n_h .

Open questions

- Impact of magnetic X-points
- Continuum damping of VDOM: when and how? Difference between L-mode and H-mode scenarios.
- Nonlinear saturation of fast-ion-driven $n=0$ modes.

**n=0 modes
are *resonant*
at divertor
X-points**

Impact of magnetic X-points not fully understood yet

$$\mathbf{B} \cdot \nabla \xi = i(n/R)B_T \xi + i B_p(\psi, \theta) \sum_m (m/r) \xi_m = 0$$

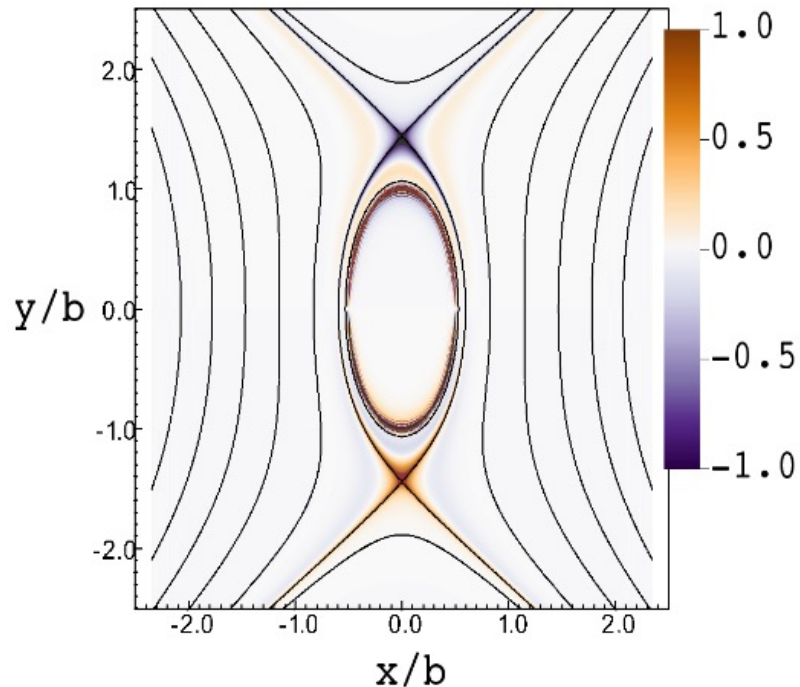
Resonance condition at X-points
satisfied for n=0 and any m

because $B_p(\psi, \theta)=0$ on the X-points.

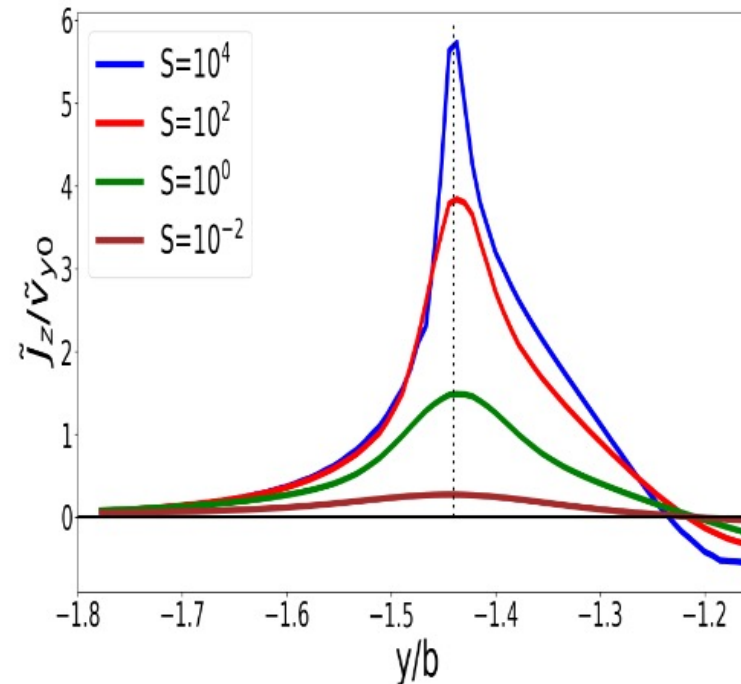
The toroidal field line going through a magnetic X-point is resonant to n=0 perturbations!

X-point current sheets in numerical simulations

NIMROD simulations of $n=0$ vertical
displacements
in a straight tokamak equilibrium with
elliptical cross section.



(a) $\tilde{J}_z$



(b) Normalized $\tilde{J}_z$ profile across lower X-point

See also

I. Krebs, F. G. Artola, C. R. Sovinec, S. C. Jardin, K. J. Bunkers, M. Hoelzl, and N. M. Ferraro, Phys. Plasmas 10, 930 (2020).