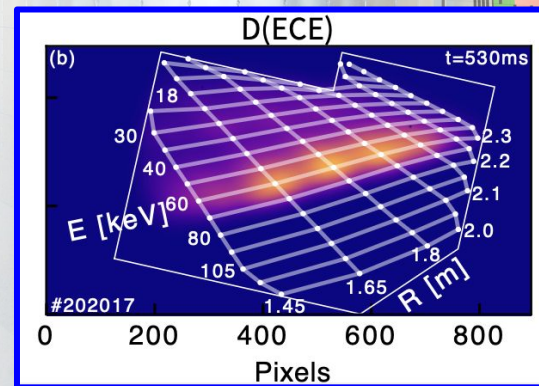


by
K.J. Callahan, M.A. Van Zeeland, W.W. Heidbrink, J.B.
Lestz, J. Rueda-Rueda, X.D. Du, S.R. Haskey, Q. Pratt,
M. Austin, L. Liu, E.M. Bass, T. Neiser

Sept 30th, 2026 11:20-11:55 am



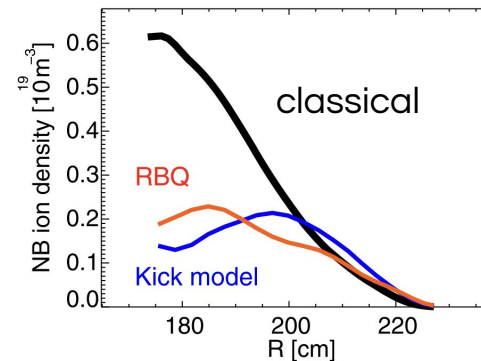
Physics codes describing the interaction of energetic-particles & Alfvén waves and their resulting transport come in many different flavors and fidelities

Heidbrink, PoP 15
(2008) 055501

Alfvén Waves

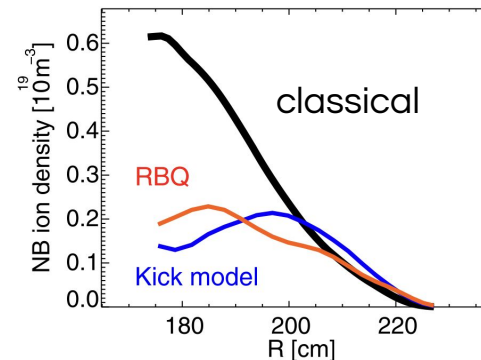
Resonant drive
Transport

Fast Ions

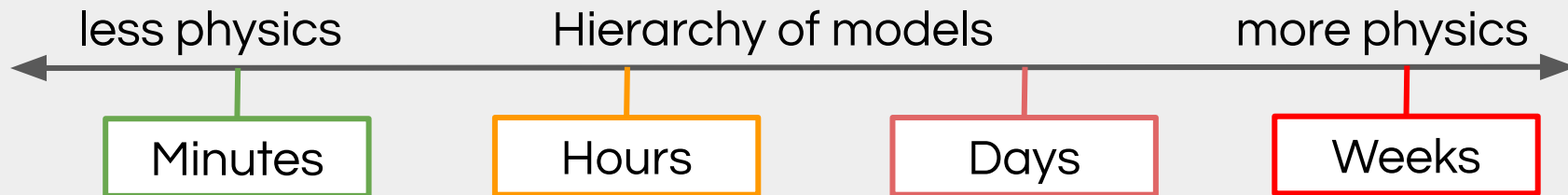


Gorelenkov, N. N., et
al. *PoP* 26.7 (2019)

Physics codes describing the interaction of energetic-particles & Alfvén waves and their resulting transport come in many different flavors and fidelities



Gorelenkov, N. N., et al. *PoP* 26.7 (2019)



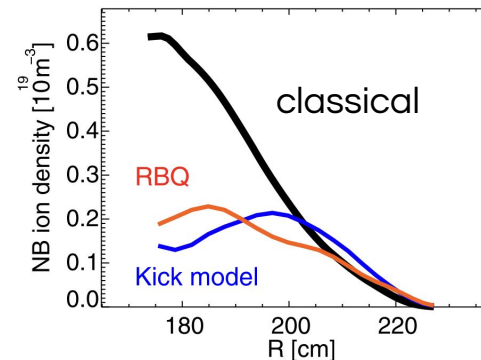
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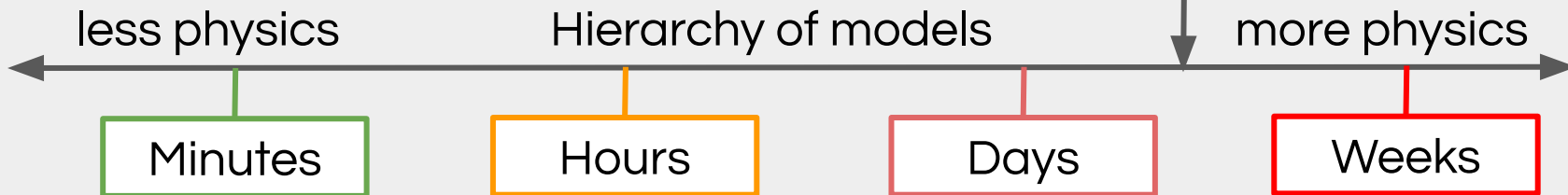
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Gorelenkov, N. N., et al. *PoP* 26.7 (2019)



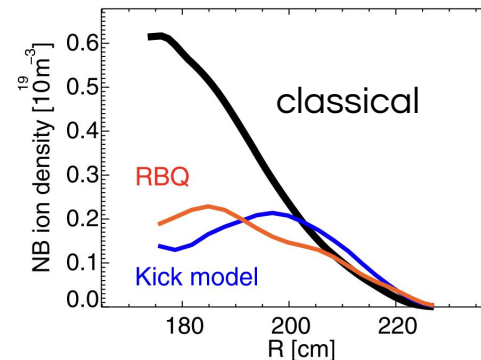
Physics codes describing the interaction of energetic-particles & Alfvén waves and their resulting transport come in many different flavors and fidelities

Heidbrink, PoP 15
(2008) 055501

Alfvén Waves

Resonant drive
Transport

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Gorelenkov, N. N., et al. *PoP* 26.7 (2019)

TGLF-EP*

RBQ* or KICK**

less physics

Hierarchy of models

more physics

Minutes

Hours

Days

Weeks

Physics codes describing the interaction of energetic-particles & Alfvén waves and their resulting transport come in many different flavors and fidelities

Heidbrink, PoP 15
(2008) 055501

Alfvén Waves

Resonant drive
Transport

Fast Ions

D_{ECE}^{**}

1-2 mins of wall time for
analysis of a whole discharge

TGLF-EP*

* predictive
** interpretive

RBQ* or KICK**

less physics

Hierarchy of models

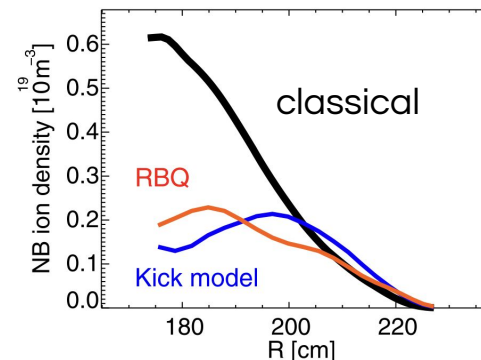
more physics

Minutes

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Days

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Gorelenkov, N. N., et
al. *PoP* 26.7 (2019)

The D_{ECE} empirical model contains several simplifying assumptions

Transport is assumed to be ...

- a) Local (flux tube like)
- b) Purely Radially Diffusive
- c) Independent of ion energy and pitch angle
- d) Flux surface symmetric

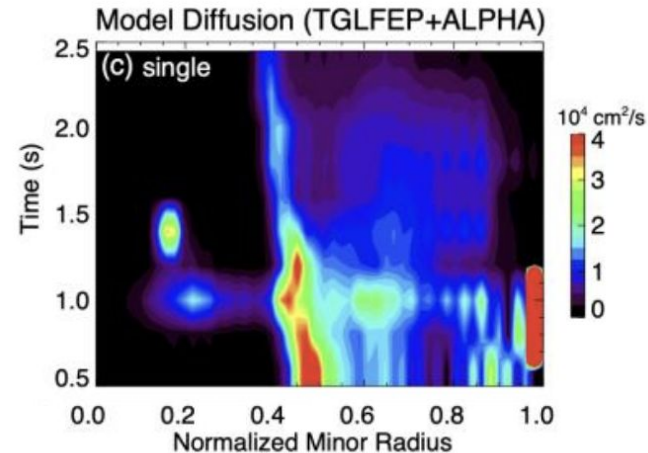
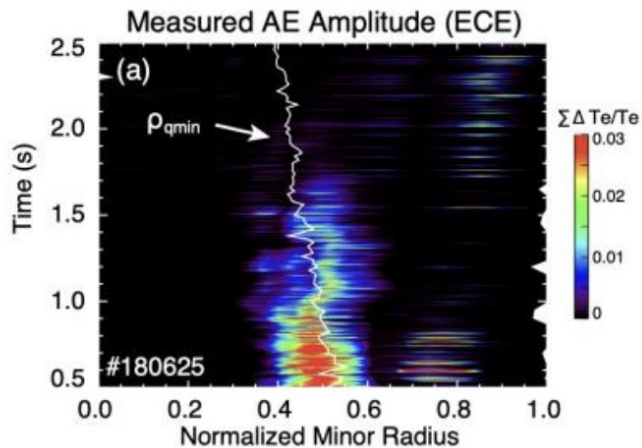
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$$D_{ECE}(\rho, \tau) = D_0 \frac{dT}{T}(\rho, \tau)$$

C.S. Collins et al. Proc. 28th IAEA Fusion Energy Conf 2021



The D_{ECE} empirical model is validated by 2 methods

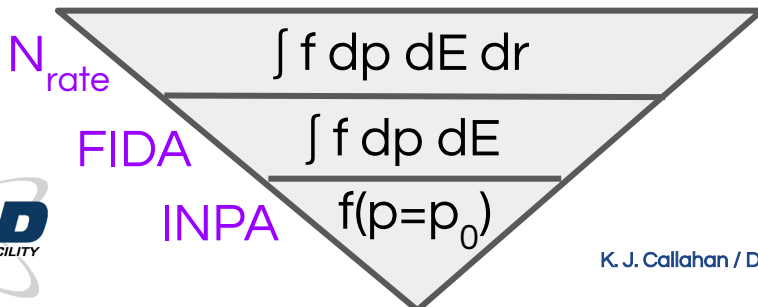
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- c) Independent of ion energy and pitch angle
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$$D_{ECE}(\rho, \tau) = D_0 \frac{dT}{T}(\rho, \tau)$$

Method 1

Primacy Hierarchy



Method 2

DIII-D AE database statistics

W_{MHD} & N_{rate}

K.J. Callahan, et al. PoP 33, 092511 (2026)

Outline

1. Model methodology

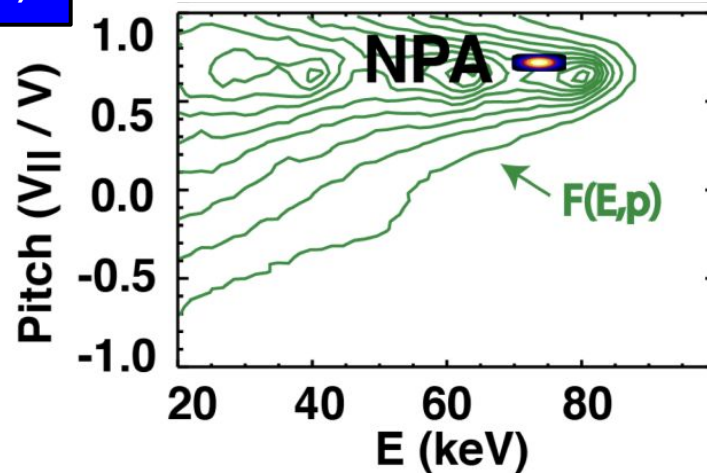
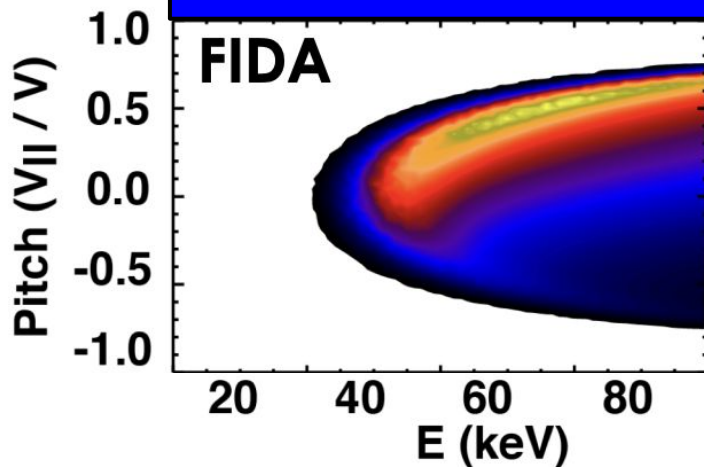
$$D_{ECE}(\rho, \tau) = D_0 \frac{dT}{T}(\rho, \tau)$$

Outline

1. Model methodology
2. Validation - Primacy Hierarchy
 - a. Volumetric - neutron rate
 - b. Spatial - Fast Ion Balmer Alpha (FIDA)
 - c. Phase space - Imaging Neutral Particle Analyzer (INPA)

$$D_{ECE}(\rho, \tau) = D_0 \frac{dT}{T}(\rho, \tau)$$

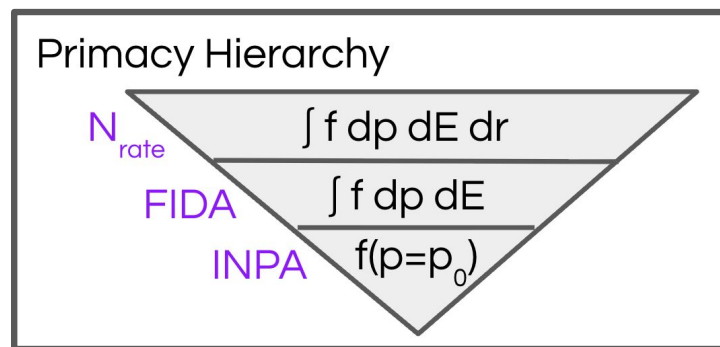
Heidbrink et al., PPCF 49, 1457 (2007)



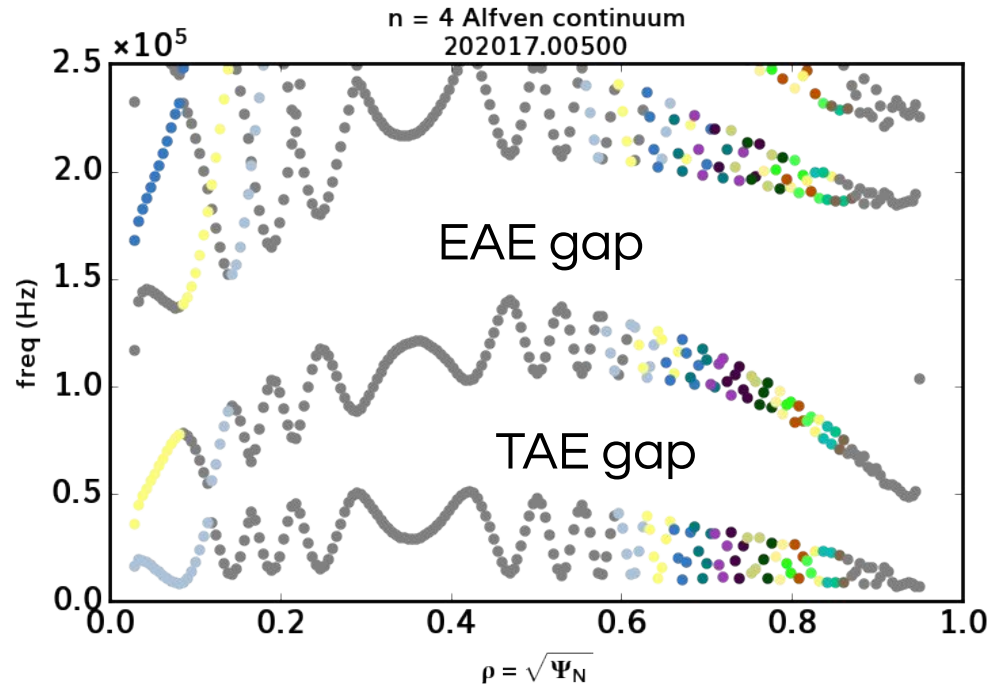
Outline

1. Model methodology
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 - a. Volumetric - neutron rate
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3. Validation - AE DIII-D Database
4. Discussion
 - a. Sat. regime & mixing length theory
 - b. Comparison to gyrokinetic modeling

$$D_{ECE}(\rho, \tau) = D_0 \frac{dT}{T}(\rho, \tau)$$



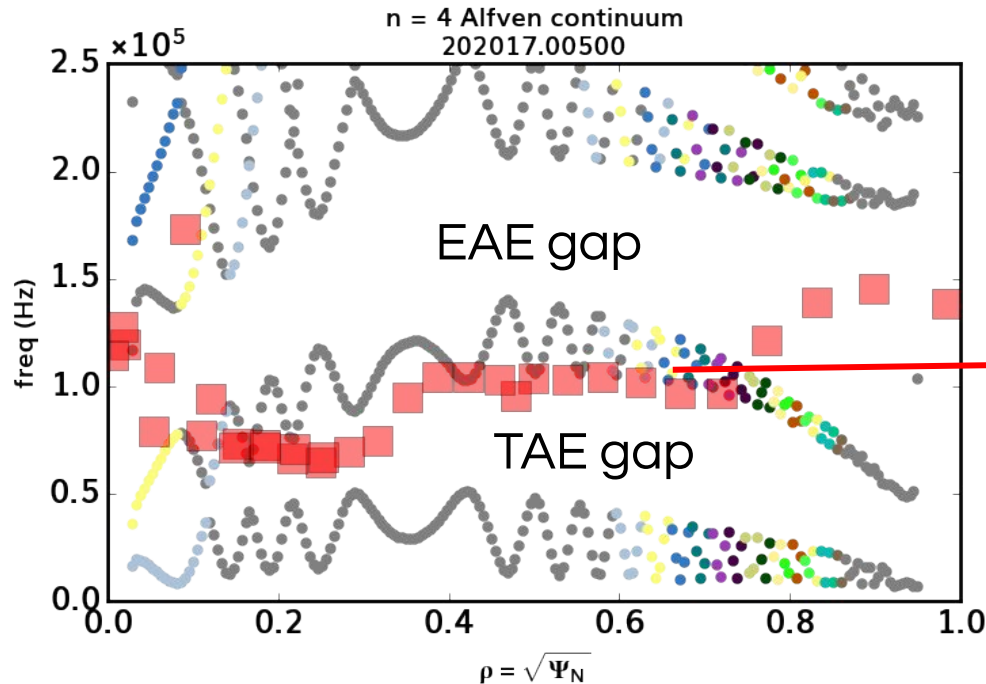
Methods: Fast ion transport modeling using cross spectral analysis of radially adjacent **Electron Cyclotron Emission** measurements



ECE diagnostic:

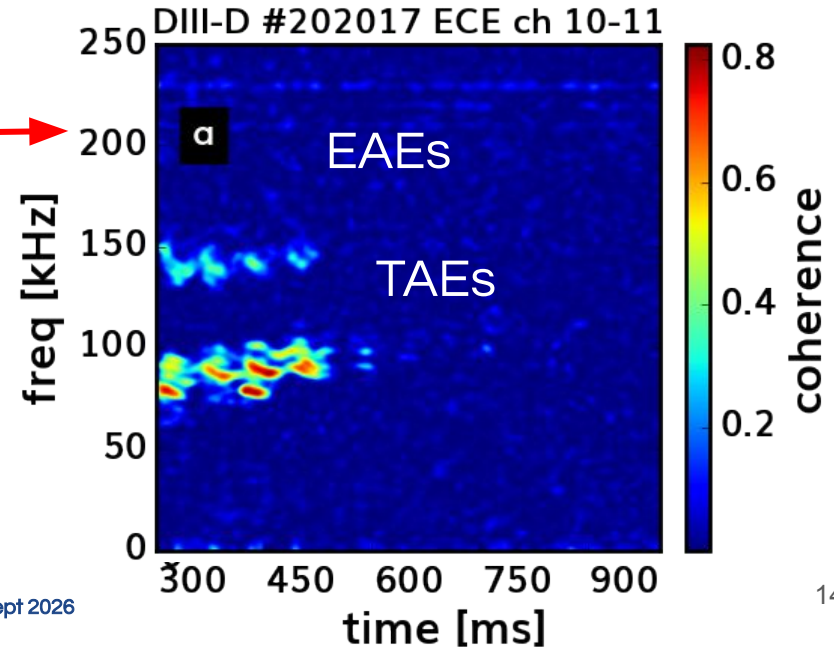
40 channels spanning $\rho = 0-1$
Sampling rate: 500 kHz

Methods: Fast ion transport modeling using cross spectral analysis of radially adjacent **Electron Cyclotron Emission** measurements

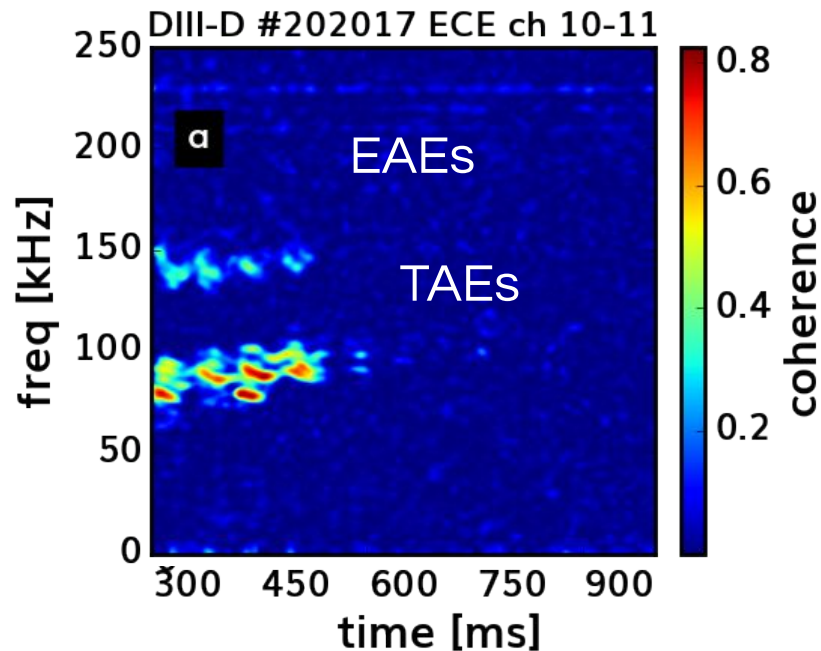


ECE diagnostic:

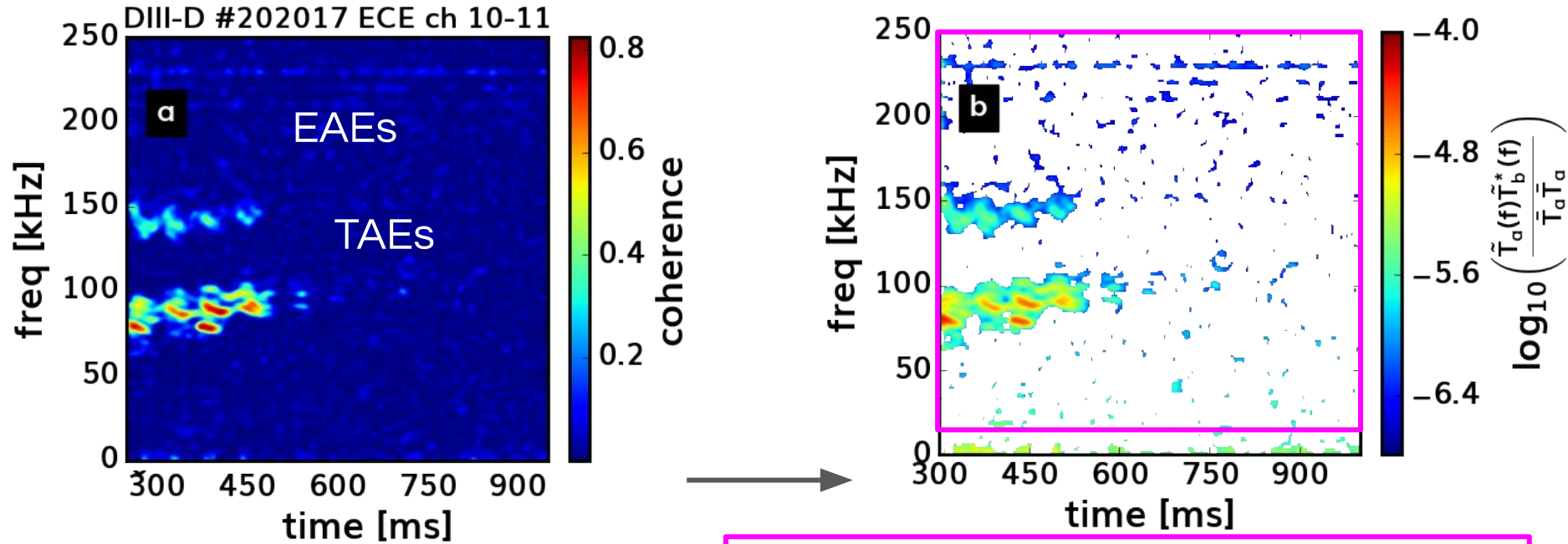
40 channels spanning $\rho = 0-1$
Sampling rate: 500 kHz



Methods: Coherent ECE fluctuations are extracted from experimental spectrograms using a closing morphology transformation methodology

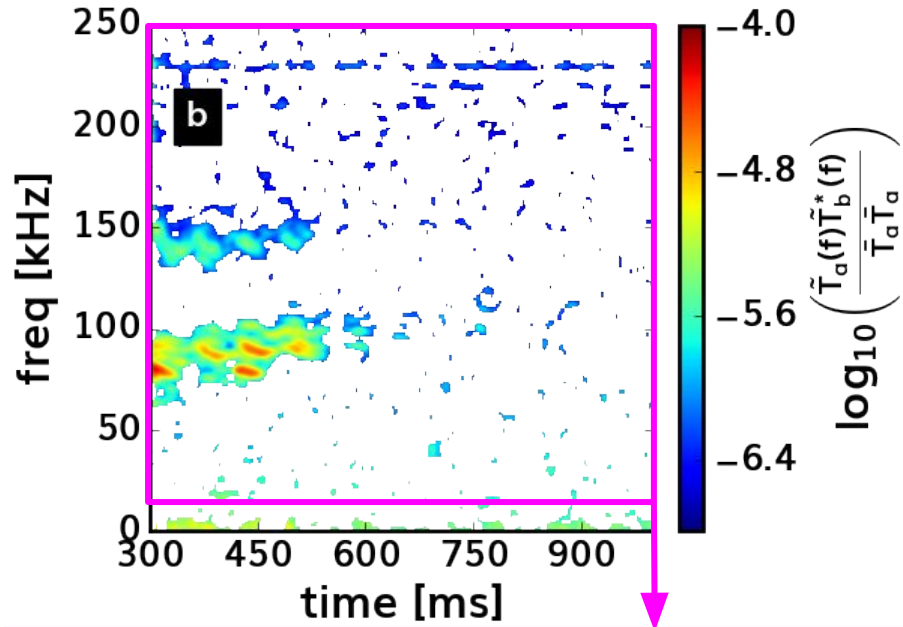


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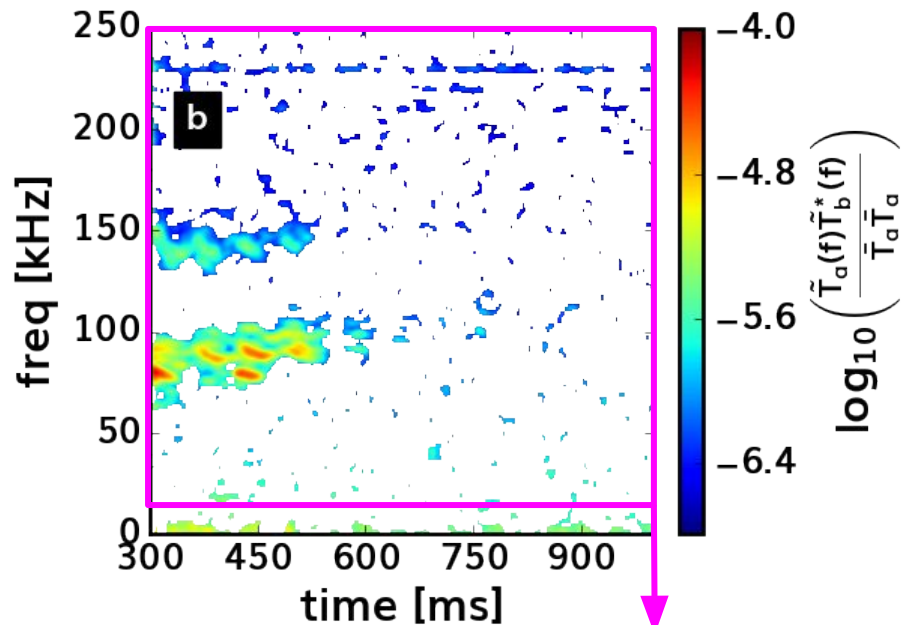
$$\frac{dT}{T}(\tau) = \sqrt{\int_{f_{tor,core}}^{f_{Nyquist}} \frac{|\tilde{T}_x(f, \tau) \tilde{T}_y^*(f, \tau)|}{\bar{T}_x(\tau) \bar{T}_y(\tau)} df}$$

Methods: Coherent ECE fluctuations are extracted from experimental spectrograms using a closing morphology transformation methodology

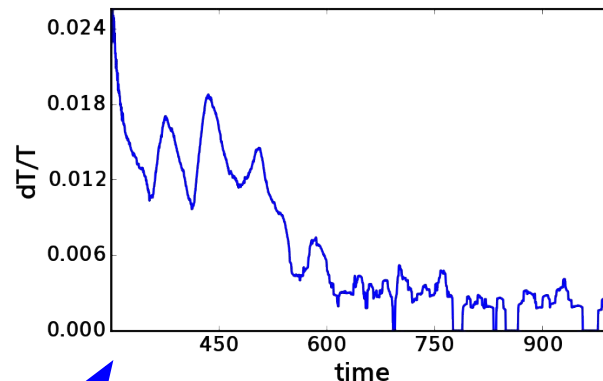


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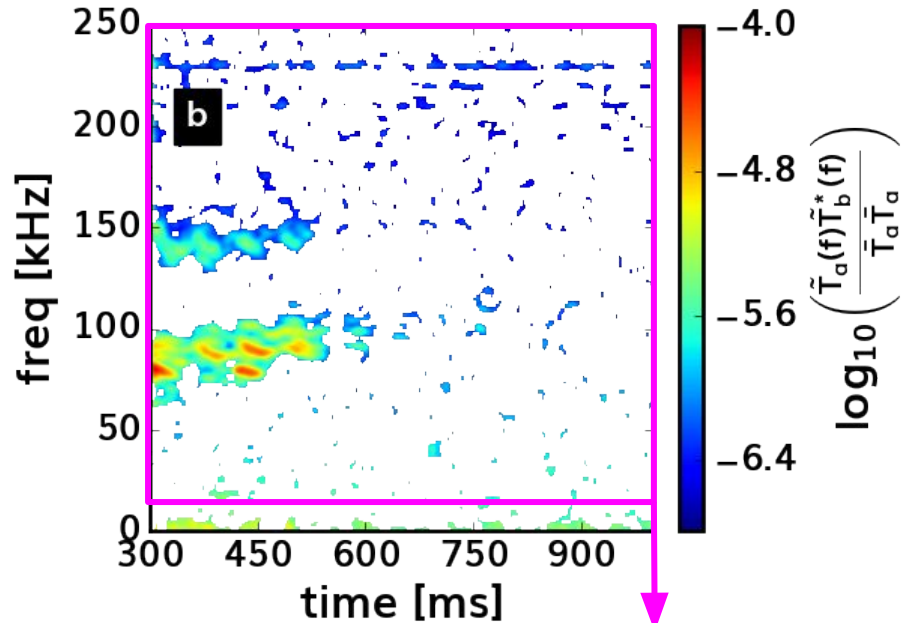


ECE ch 10-11

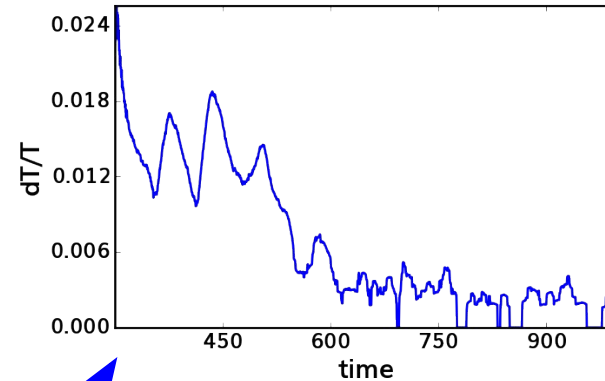


$$\frac{dT}{T}(\tau) = \sqrt{\int_{f_{tor,core}}^{f_{Nyquist}} \frac{|\tilde{T}_x(f, \tau) \tilde{T}_y^*(f, \tau)|}{\bar{T}_x(\tau) \bar{T}_y(\tau)} df}$$

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ECE ch 10-11

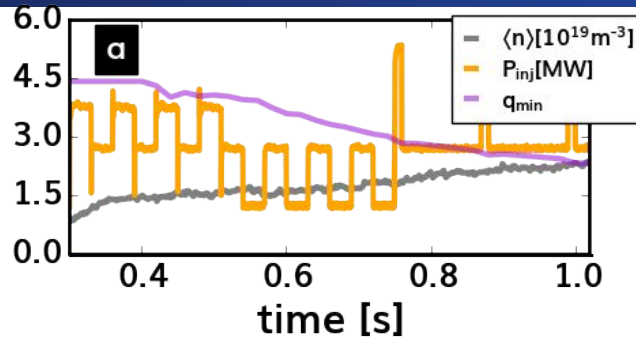


$$\frac{dT}{T}(\tau) = \sqrt{\int_{f_{tor,core}}^{f_{Nyquist}} \frac{|\tilde{T}_x(f, \tau) \tilde{T}_y^*(f, \tau)|}{\bar{T}_x(\tau) \bar{T}_y(\tau)} df}$$

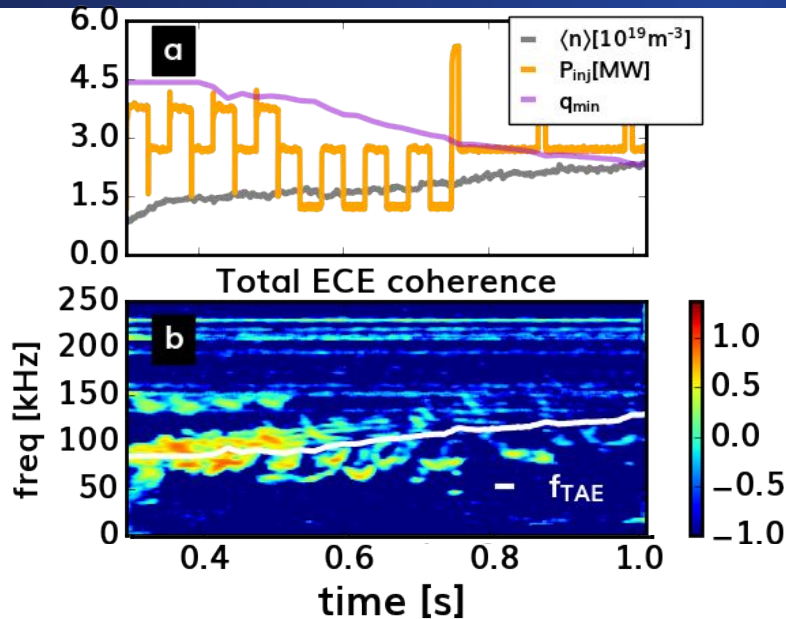
$$D_0 = 2.66 \times 10^6 \text{ cm}^2/\text{s}$$

$$D_{ECE}(\rho, \tau) = D_0 \frac{dT}{T}(\rho, \tau)$$

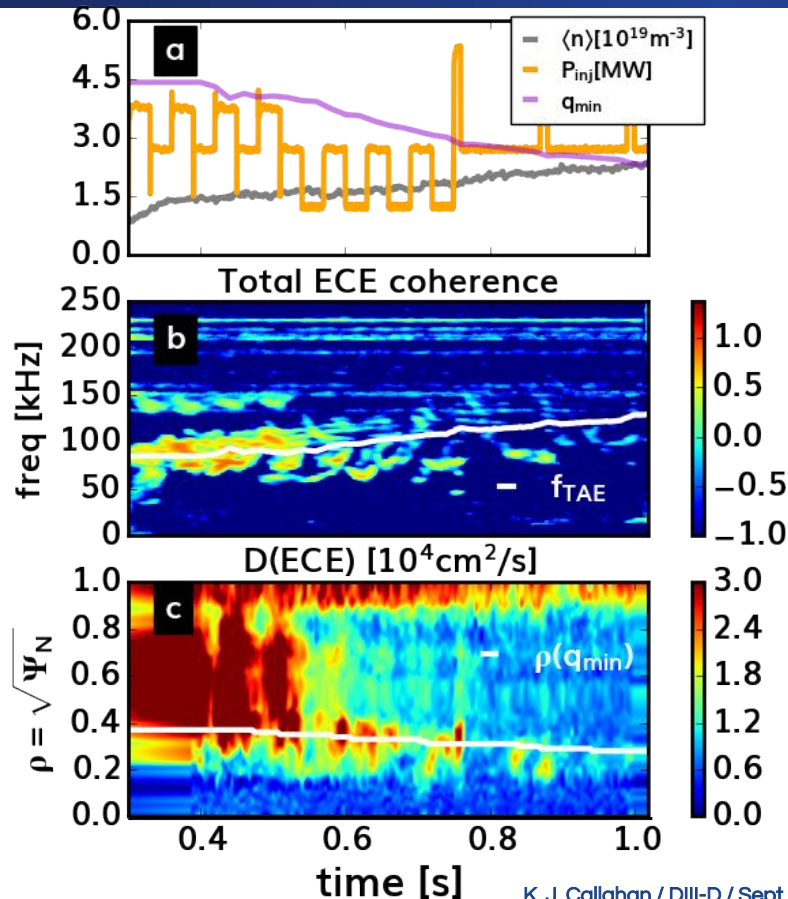
Validation 1a: Tested on DIII-D discharge #202017, with strong variation in fast ion transport, finds strong neutron rate agreement



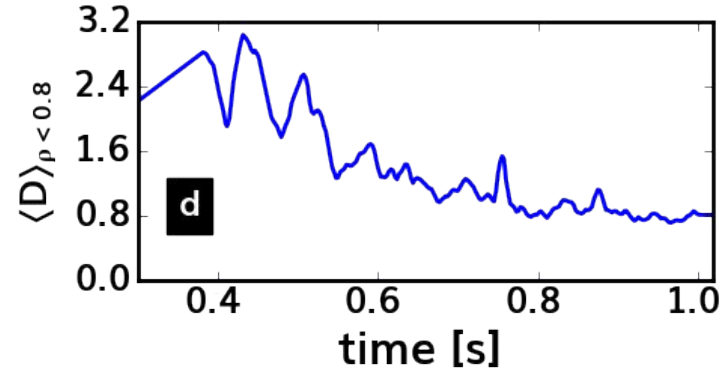
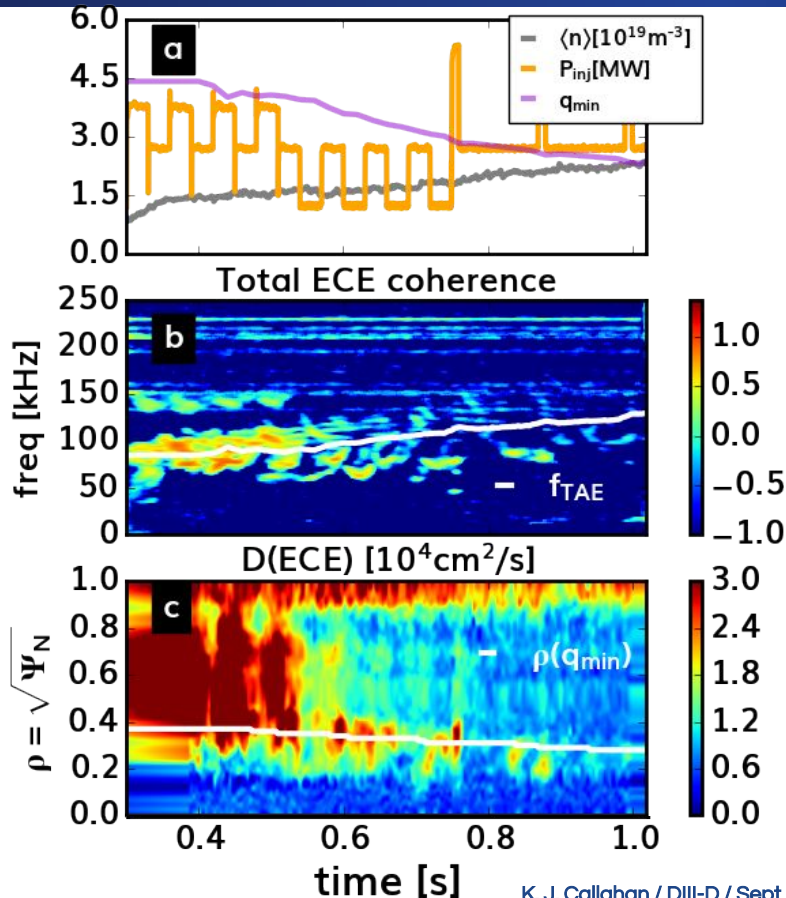
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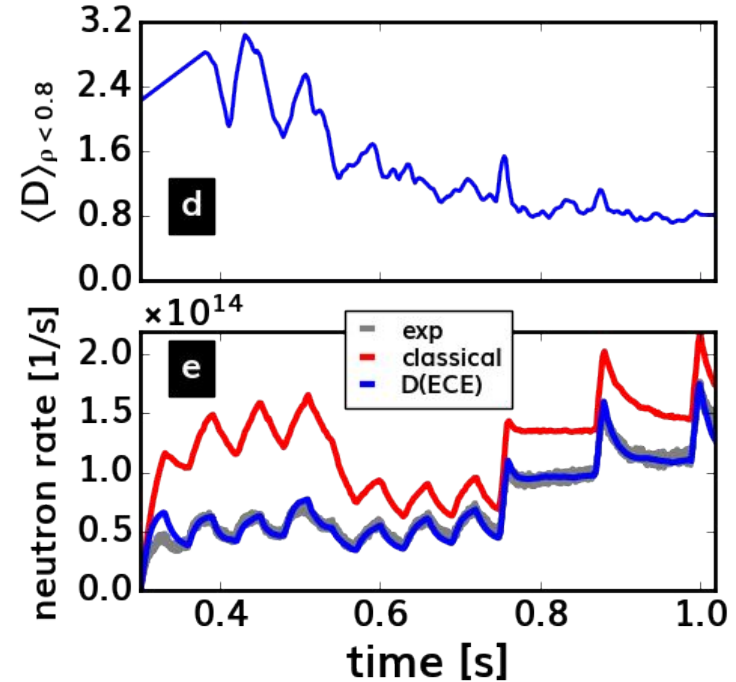
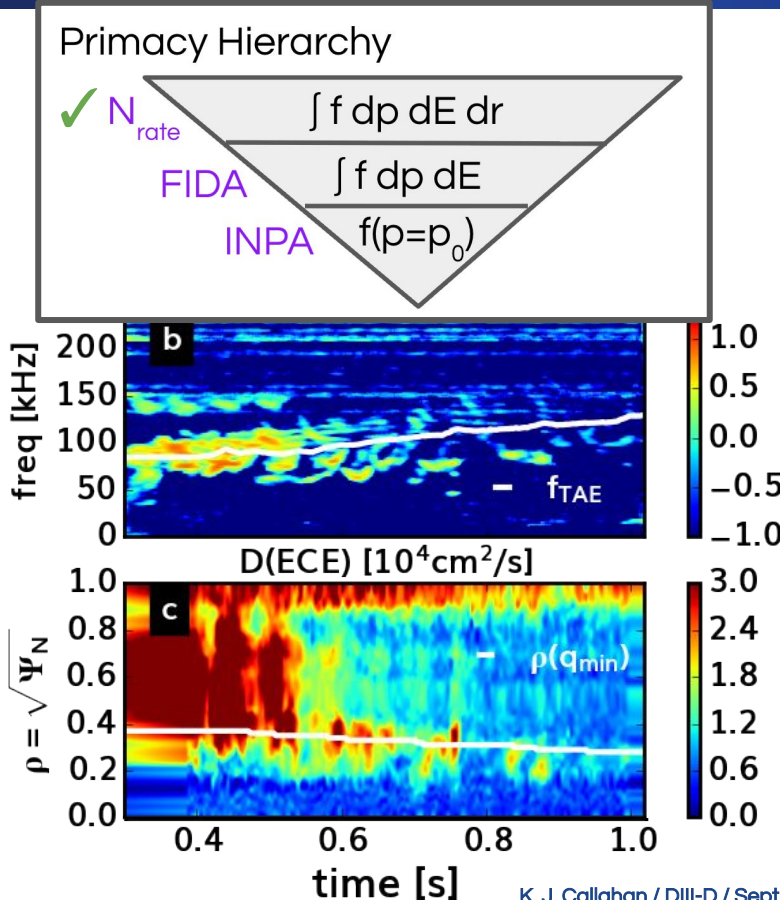
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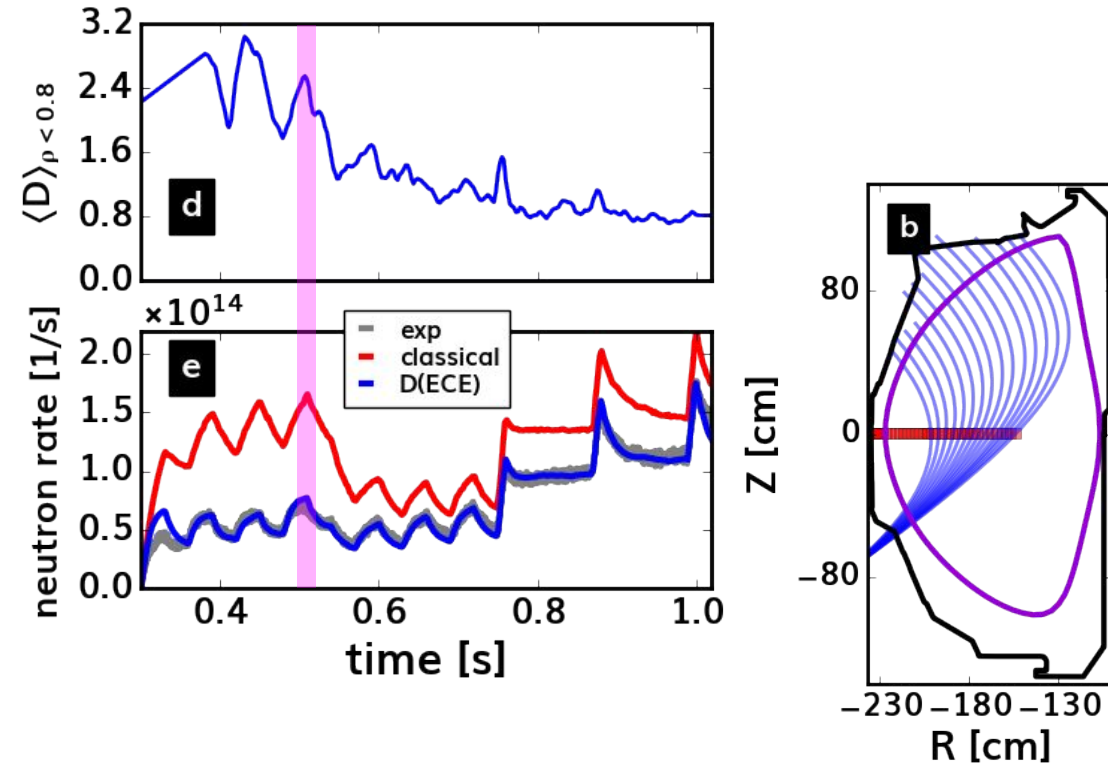


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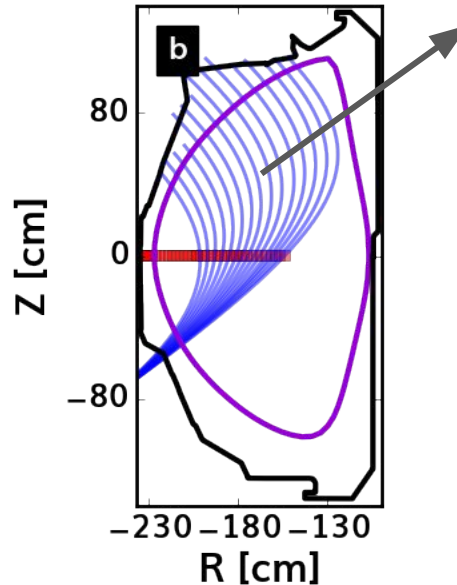
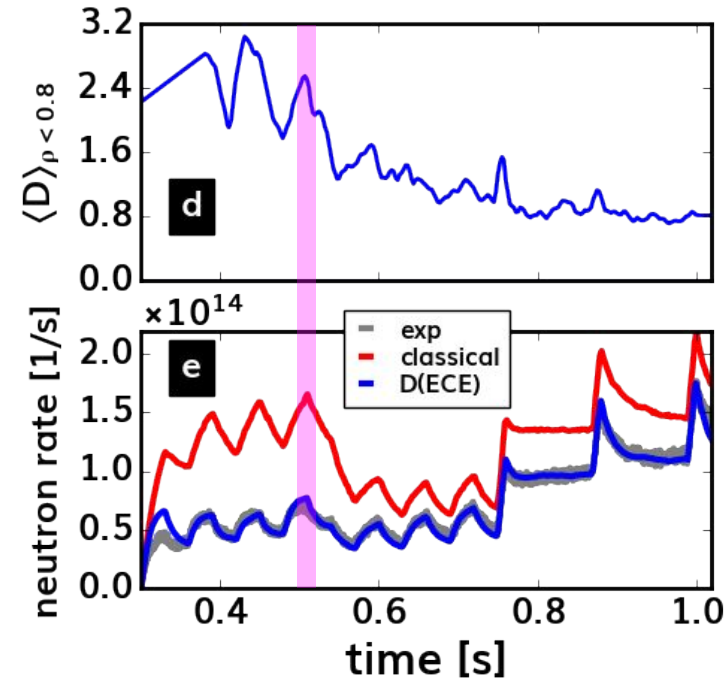


*classical transport = only slowing down and pitch angle scattering

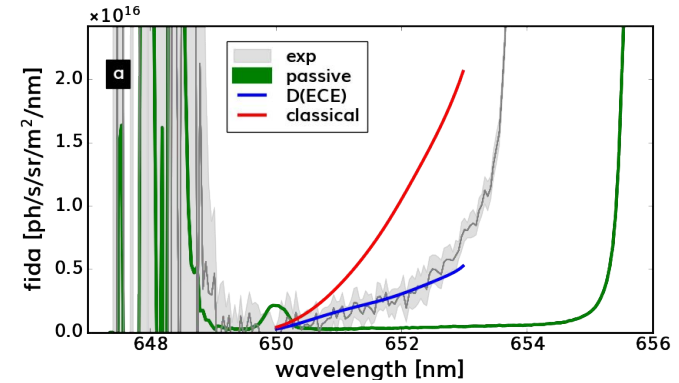
Validation 1b: Tested on Fast Ion Balmer Alpha (FIDA) data, with agreement within experimental measurement errors



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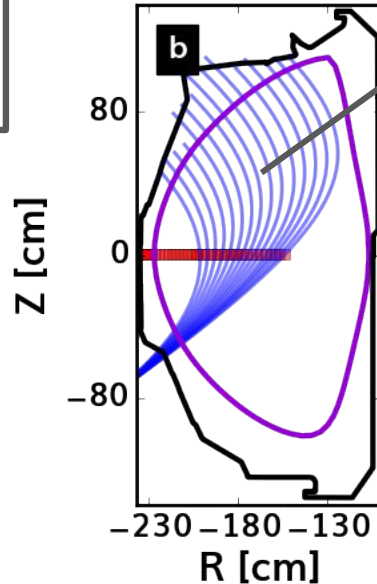
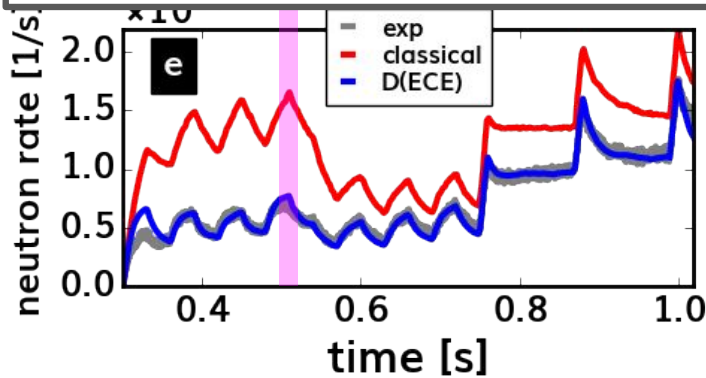
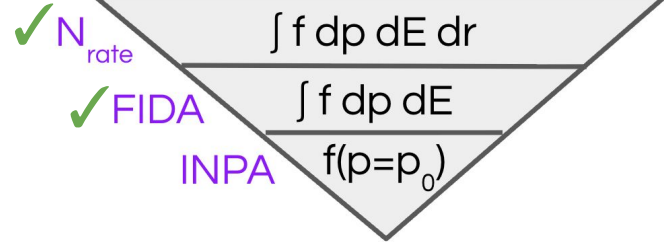


FIDA-channel F05, #202017, 500-530 ms, 650.0-653.0 nm

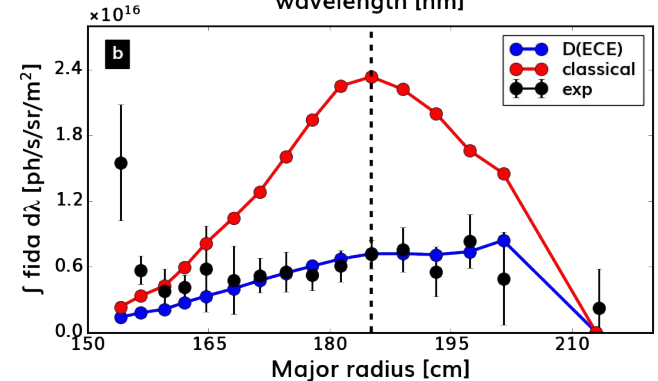
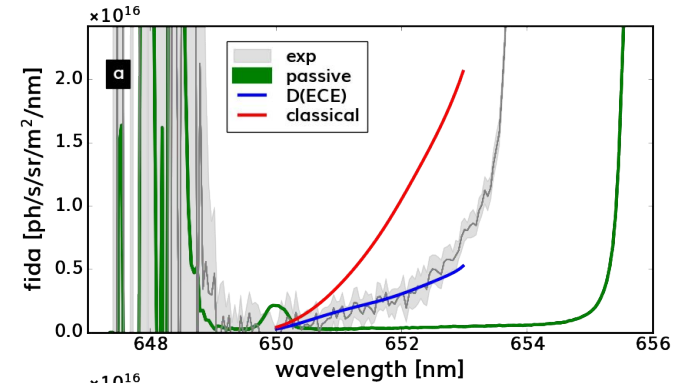


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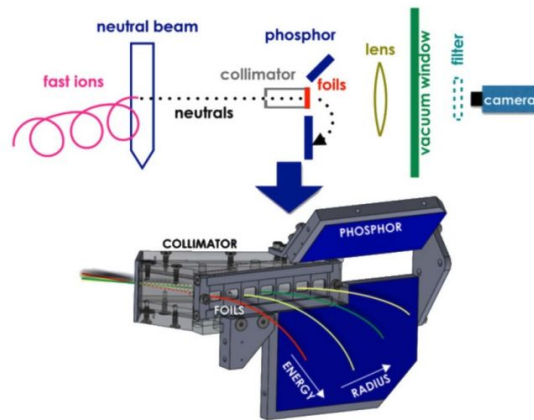
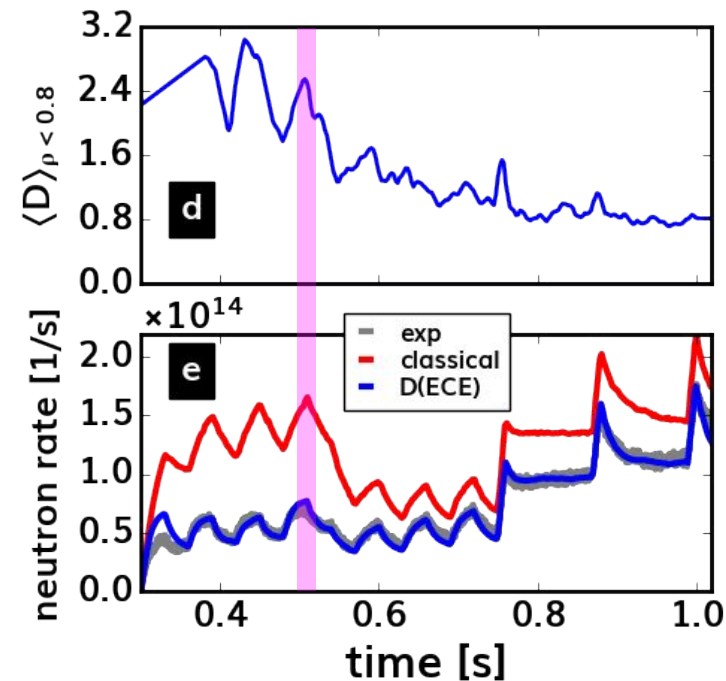
Primacy Hierarchy



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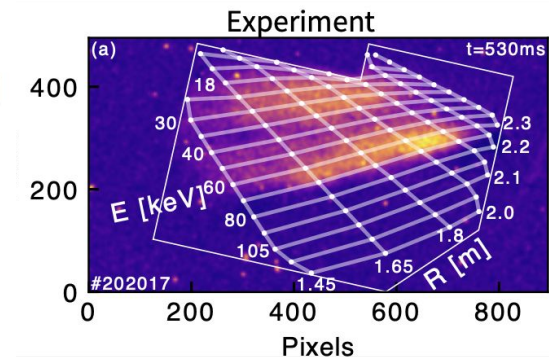
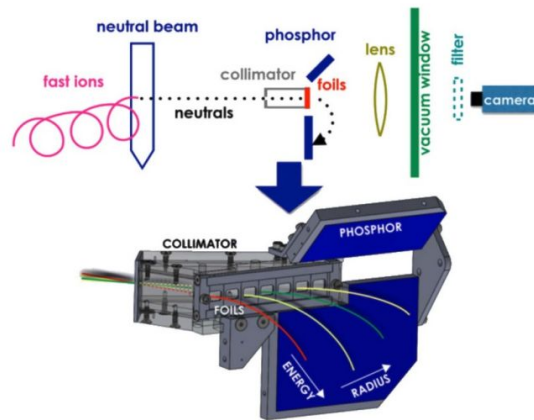
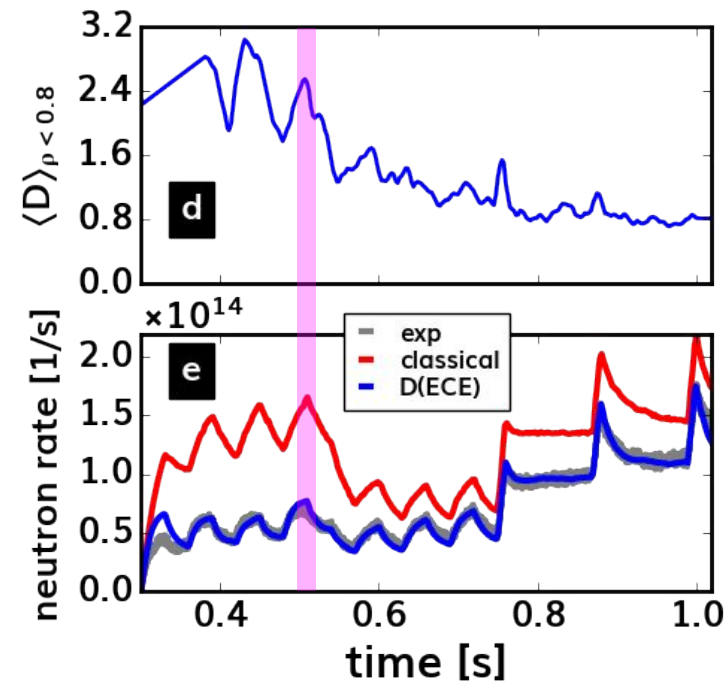


Validation 1c: Comparison of Imaging Neutral Particle Analyzer (INPA) data yields agreement at injection energies & disagreement at lower energies



Du, X. D., et al. NF 58.8 (2018): 082006.

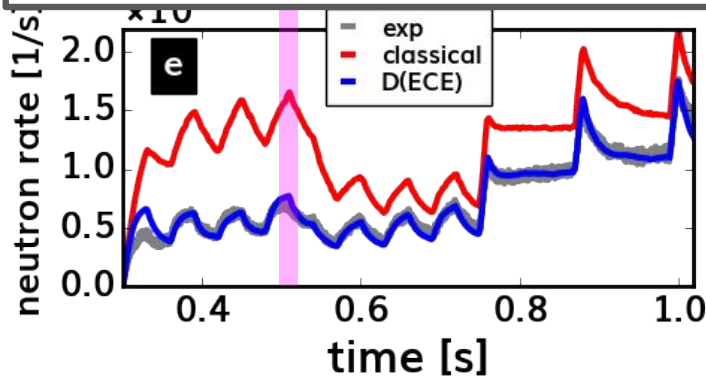
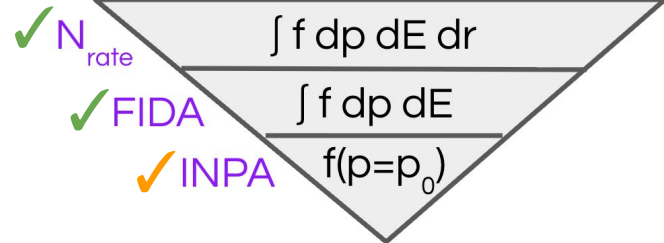
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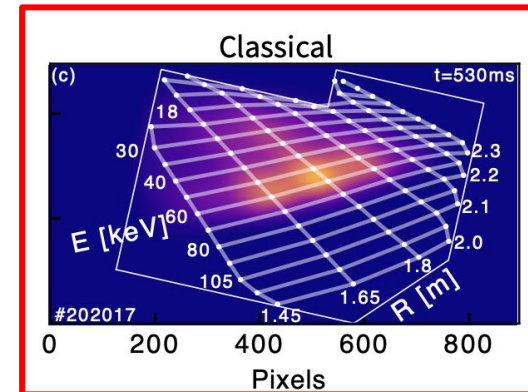
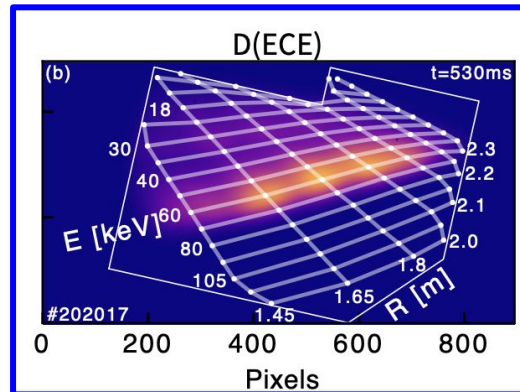
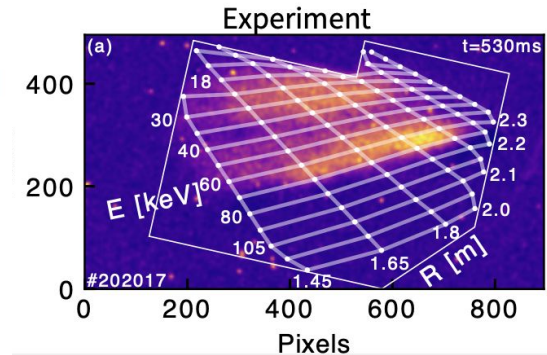
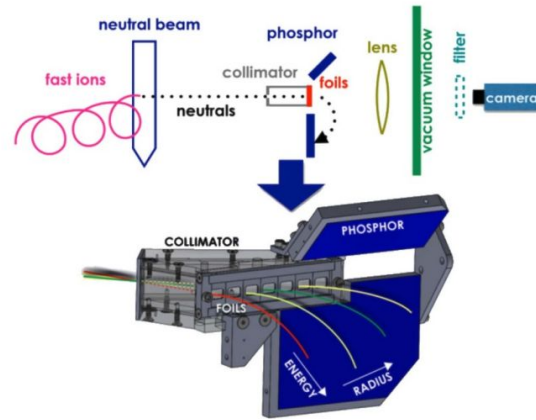
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Primacy Hierarchy



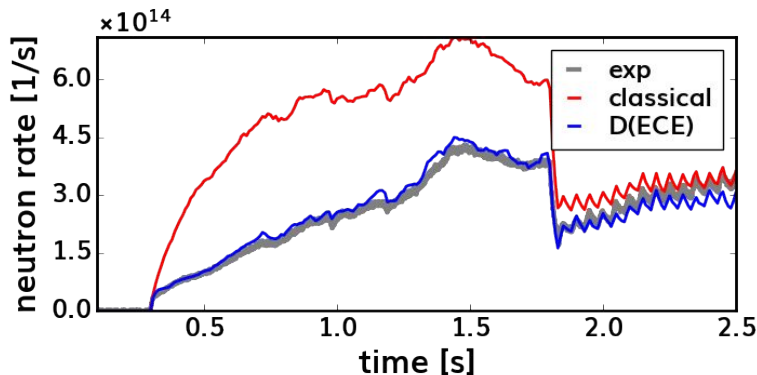
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D_{ECE} predicted neutron rates agree even in disparate scenarios

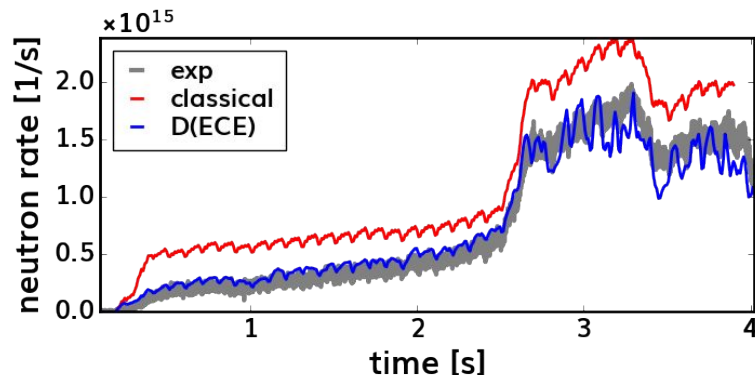
Ex: # 204232 (AE turbulence suppression) & #180619 (Steady State High q_{min})

Model effectiveness across discharges motivated a database level validation effort



X. Du - Experimental observation of microturbulence suppression induced by Alfvén Eigenmodes in DIII-D tokamak (10/01 - 9:35 am)

Du, X. D., et al. PRL 135.26 (2025): 265101.

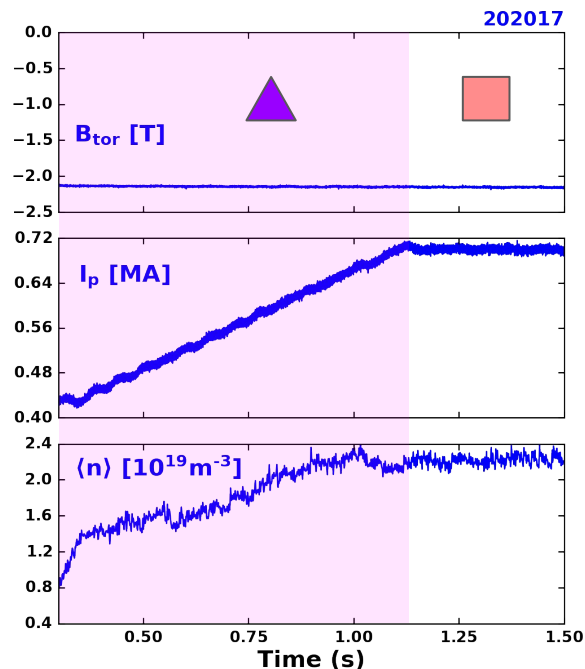


Collins, C.S., et al. Proc. 28th IAEA Fusion Energy Conf.(virtual event). 2021.

Validation 2: The D_{ECE} model developed is validated against a database of +40 discharges w/ strong MHD activity

Pointname	Description	Range
$ B_t $ [T]	Toroidal Field	1.3 $\rightarrow$ 2.1
I_p [MA]	Plasma Current	0.0 $\rightarrow$ 1.5
$\langle n_e \rangle [\times 10^{19} m^{-3}]$	Line-averaged density	0.5 $\rightarrow$ 5.0
P_{NBI} [MW]	Neutral Beam Power	0.5 $\rightarrow$ 10.0
P_{ECH} [MW]	Electron Cyclotron Power	0.0 $\rightarrow$ 3.2
δ	Triangularity	-0.6 $\rightarrow$ 0.7
κ	Elongation	1.1 $\rightarrow$ 1.8
$\sum_i f_i m_i$ [amu]	Fractional main ion mass	1.5 $\rightarrow$ 2.0

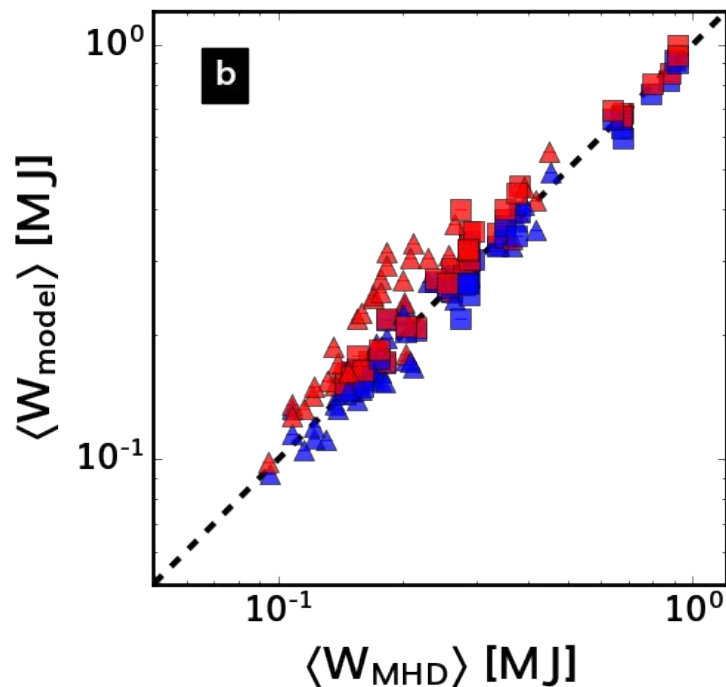
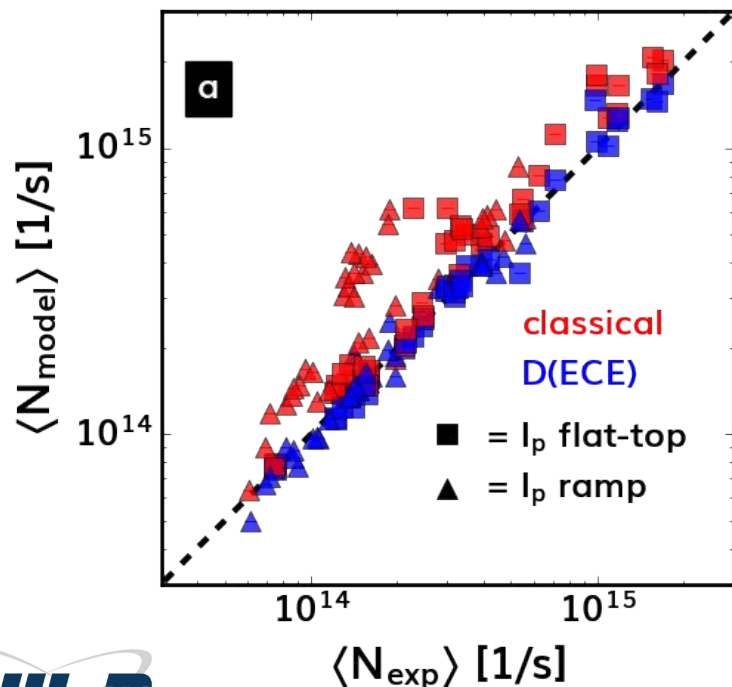
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κ	Elongation	$1.1 \rightarrow 1.8$
$\sum_i f_i m_i$ [amu]	Fractional main ion mass	$1.5 \rightarrow 2.0$

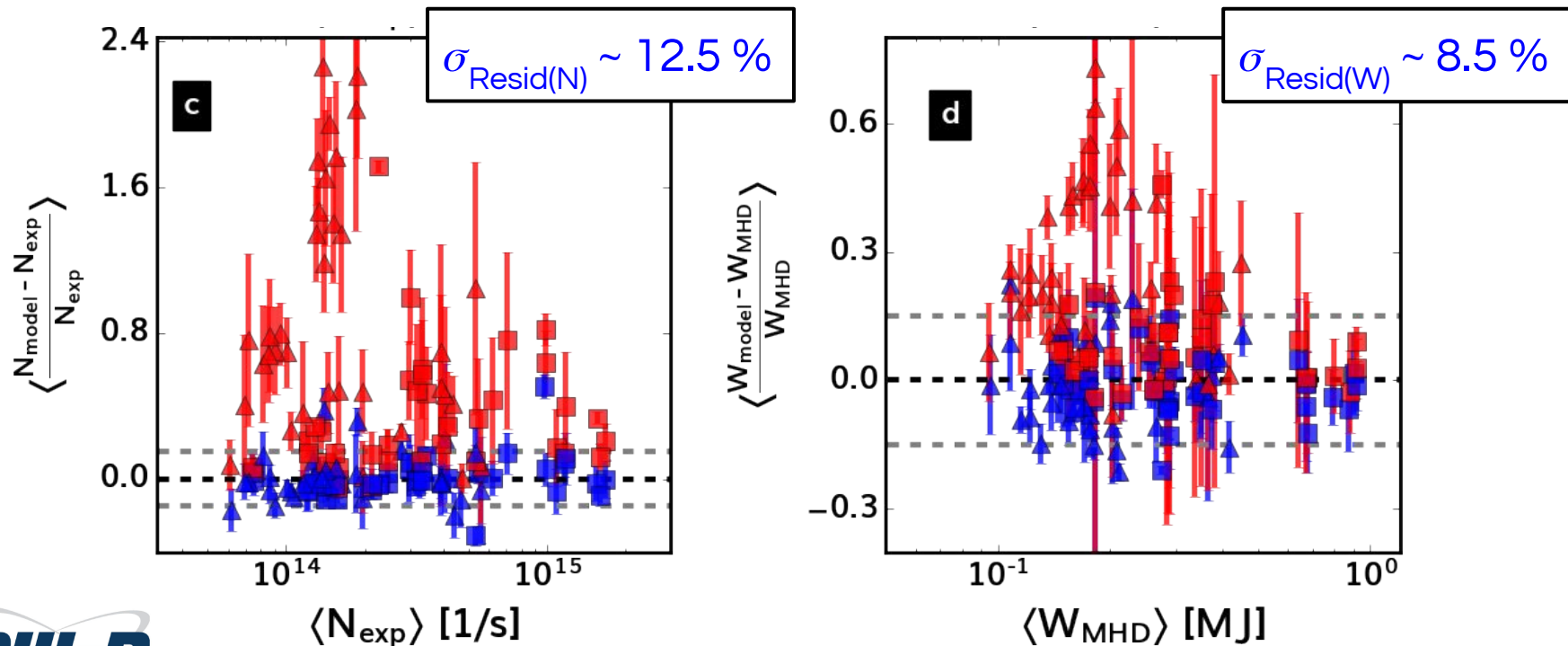
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DIII-D Alfvén Eigenmode Exp database

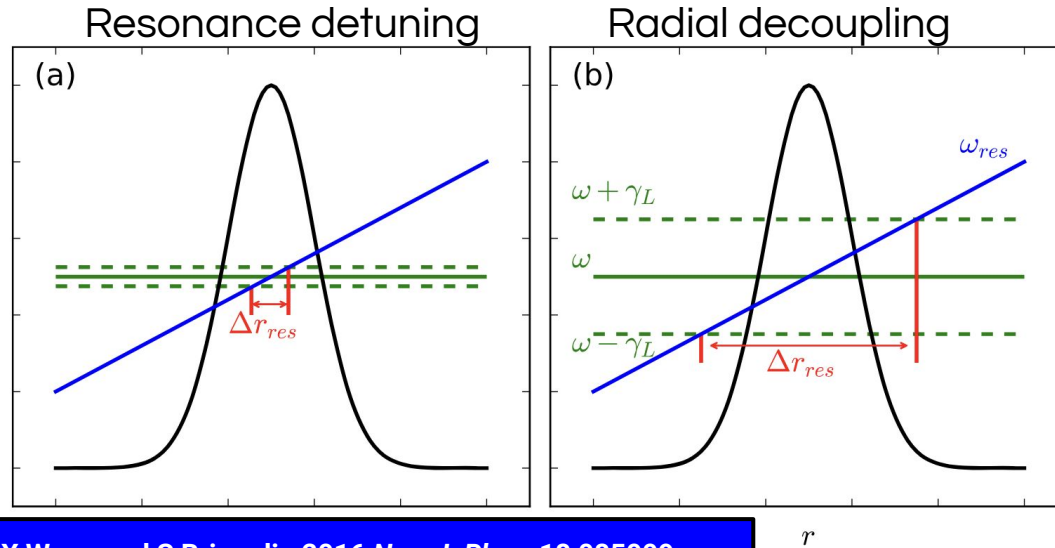


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DIII-D Alfvén Eigenmode Exp database

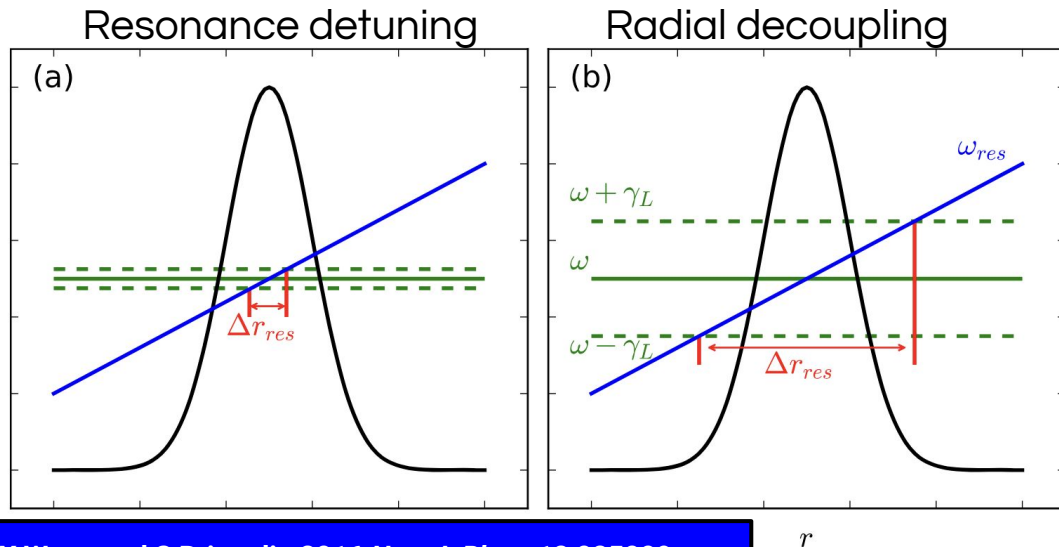


Discussion: On the possible origin of the fixed constant, a mixing length constant in the radial decoupling AE saturation regime

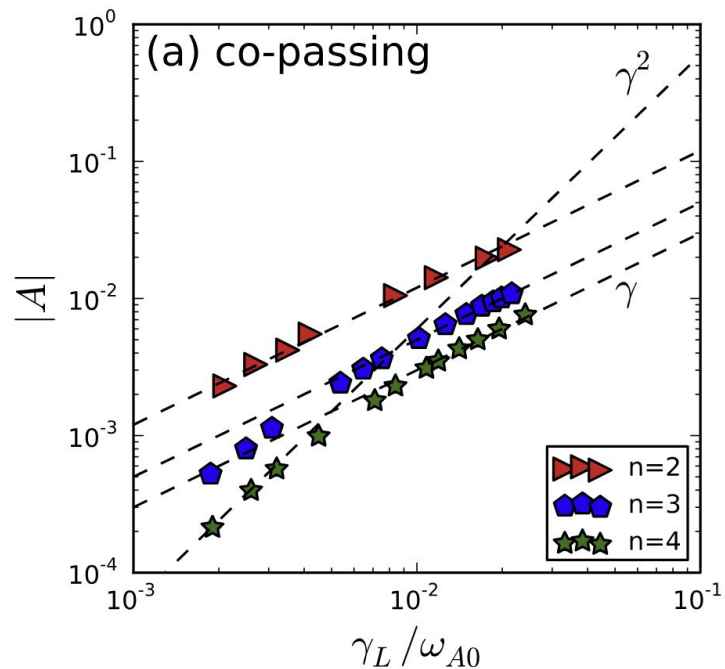


X Wang and S Briguglio 2016 *New J. Phys.* 18 085009

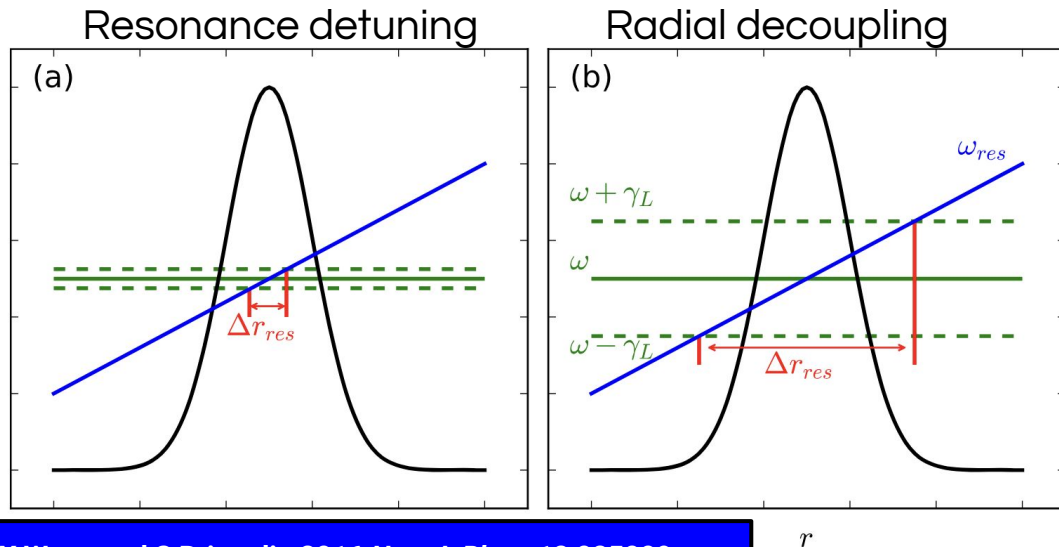
Discussion: On the possible origin of the fixed constant, a mixing length constant in the radial decoupling AE saturation regime



X Wang and S Briguglio 2016 *New J. Phys.* 18 085009

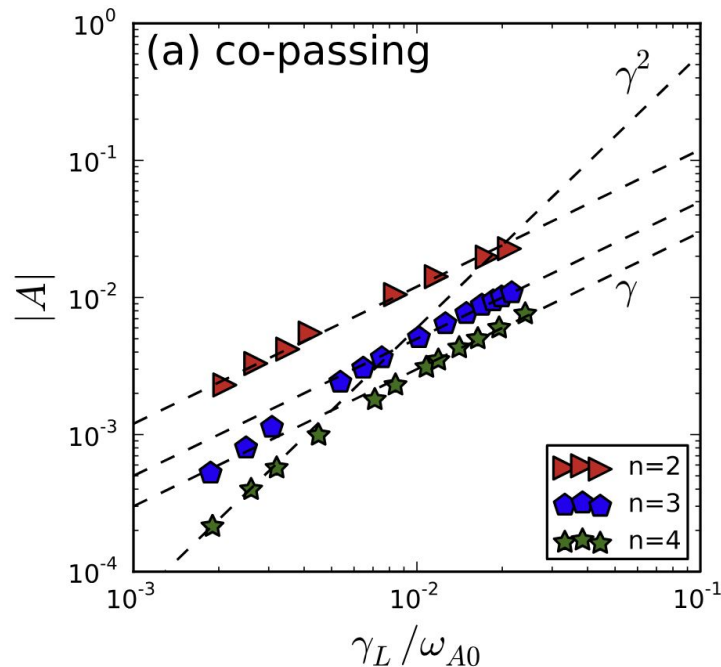


Discussion: On the possible origin of the fixed constant, a mixing length constant in the radial decoupling AE saturation regime

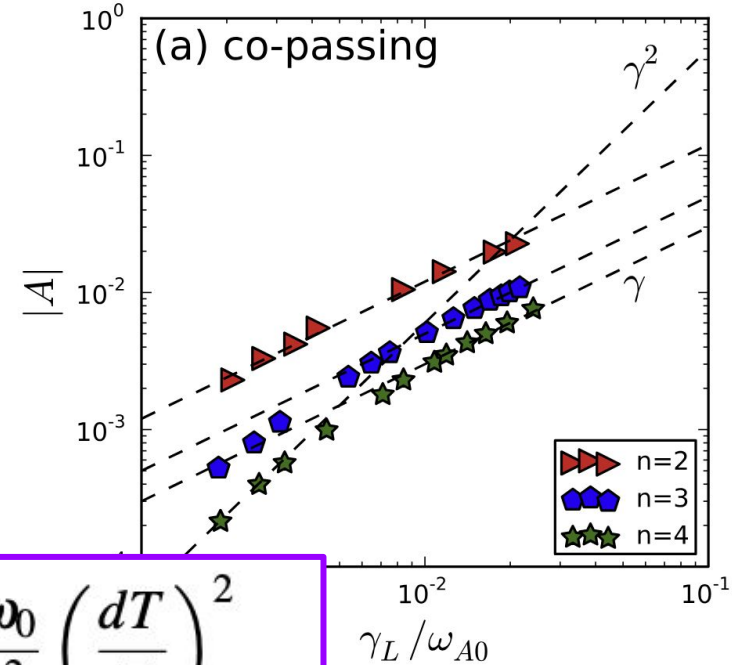
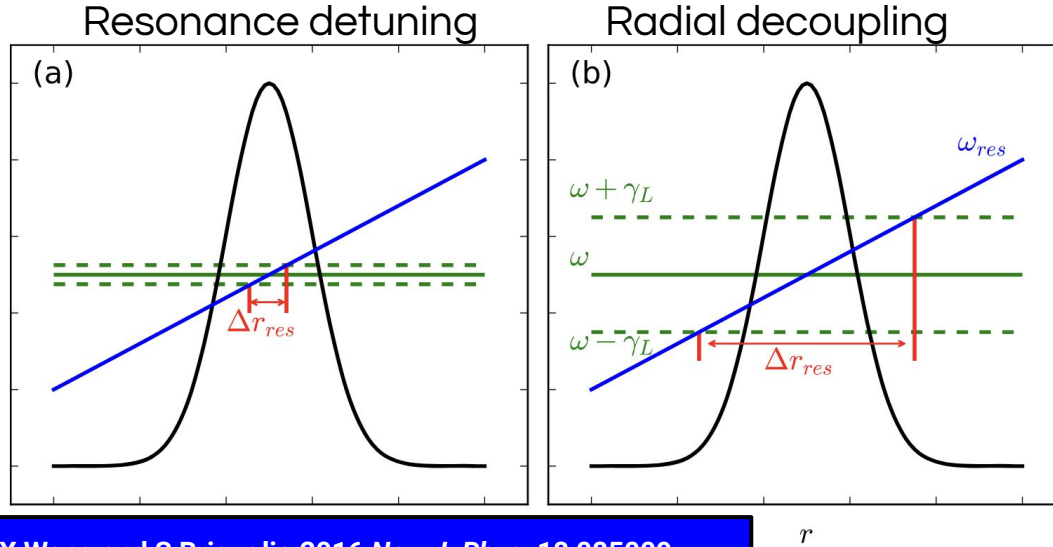


X Wang and S Briguglio 2016 *New J. Phys.* 18 085009

$$D_{ML} \approx \gamma_{linear} / k_{\perp}^2$$



Discussion: On the possible origin of the fixed constant, a mixing length constant in the radial decoupling AE saturation regime



$$D_{ML} \approx \gamma_{linear} / k_{\perp}^2$$

$$D_{resonance} \approx \frac{\omega_0}{k_{\perp}^2} \left(\frac{dT}{T} \right)^2$$

$$D_{radial} \approx \frac{\omega_0}{k_{\perp}^2} \frac{dT}{T}$$

Discussion: Mixing length estimates of Alfvén Eigenmode transport from a single unstable mode

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Discussion: Mixing length estimates of Alfvén Eigenmode transport from a single unstable mode

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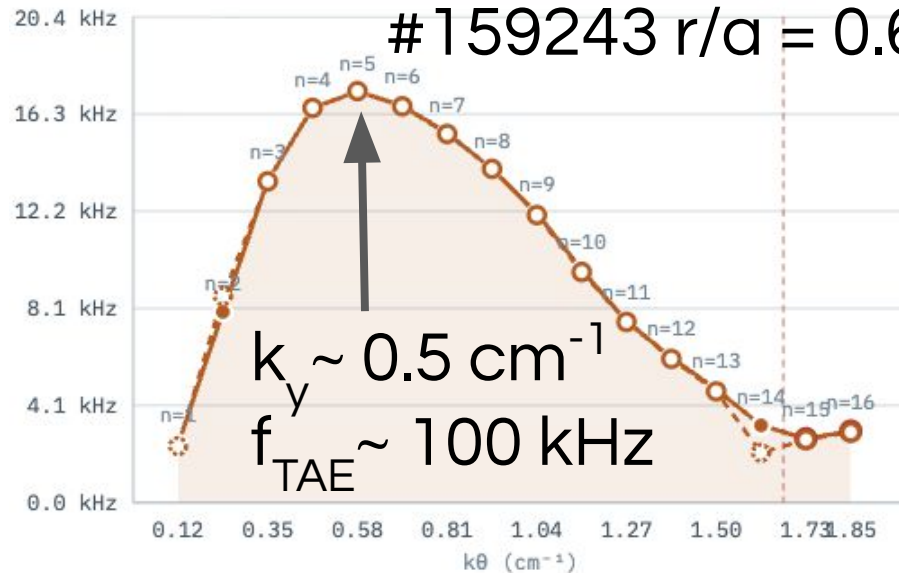
$$D_{radial} \approx \frac{\omega_0}{k_{\perp}^2} \frac{dT}{T}$$

Growth rate γ

vs. k_{θ} · kHz vs. cm^{-1}

Linear CGYRO

#159243 $r/a = 0.6$



41

Discussion: Mixing length estimates of Alfvén Eigenmode transport from a single unstable mode

~~$$D_{\text{resonance}} \approx \frac{\omega_0}{k_{\perp}^2} \left(\frac{dT}{T} \right)^2$$~~

$$D_{\text{radial}} \approx \frac{\omega_0}{k_{\perp}^2} \frac{dT}{T}$$

$$\omega_0/k_{\perp}^2 \approx 2.5 \times 10^6 \text{ cm}^2/\text{s}$$

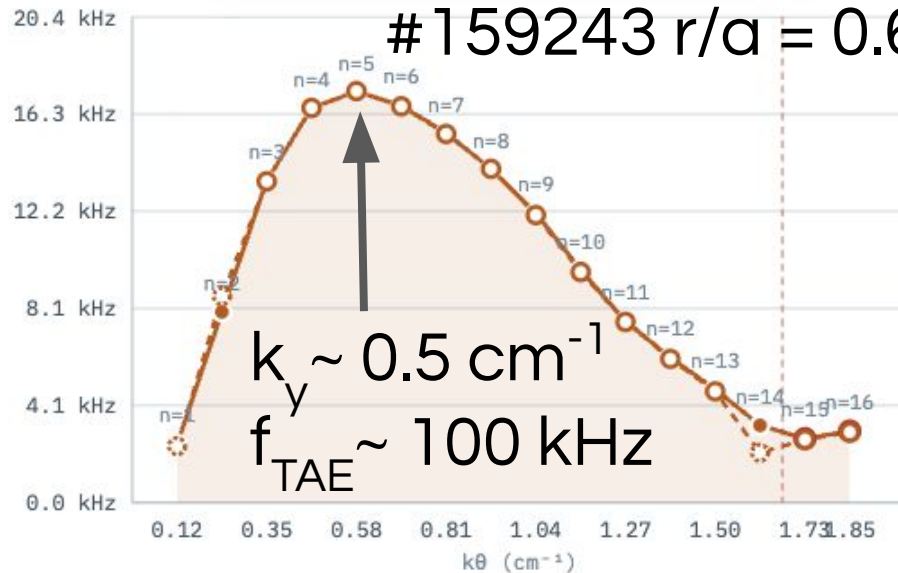
$$D_0 = 2.66 \times 10^6 \text{ cm}^2/\text{s}$$

Growth rate γ

vs. k_{θ} · kHz vs. cm^{-1}

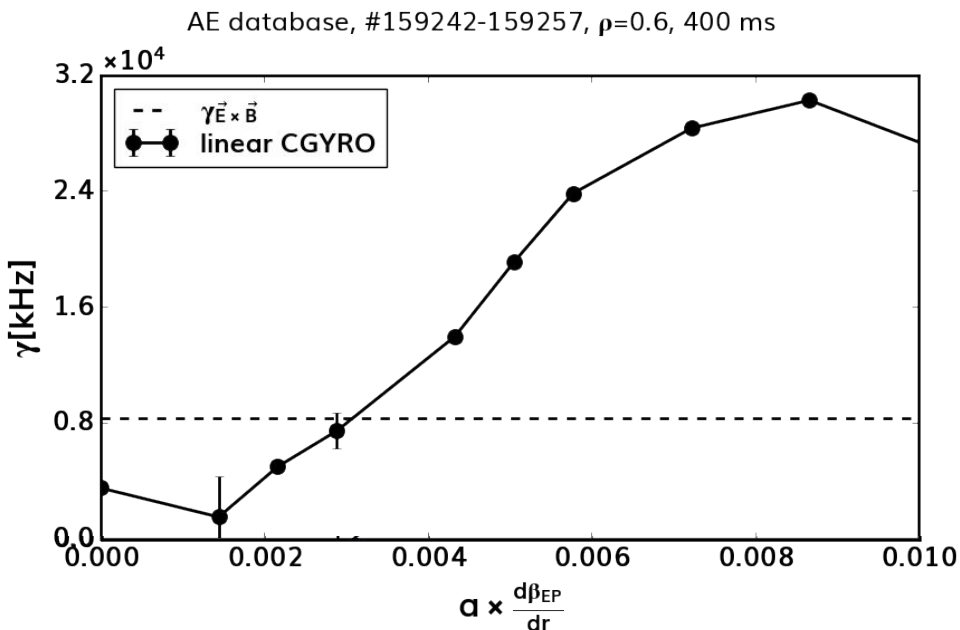
Linear CGYRO

#159243 $r/a = 0.6$



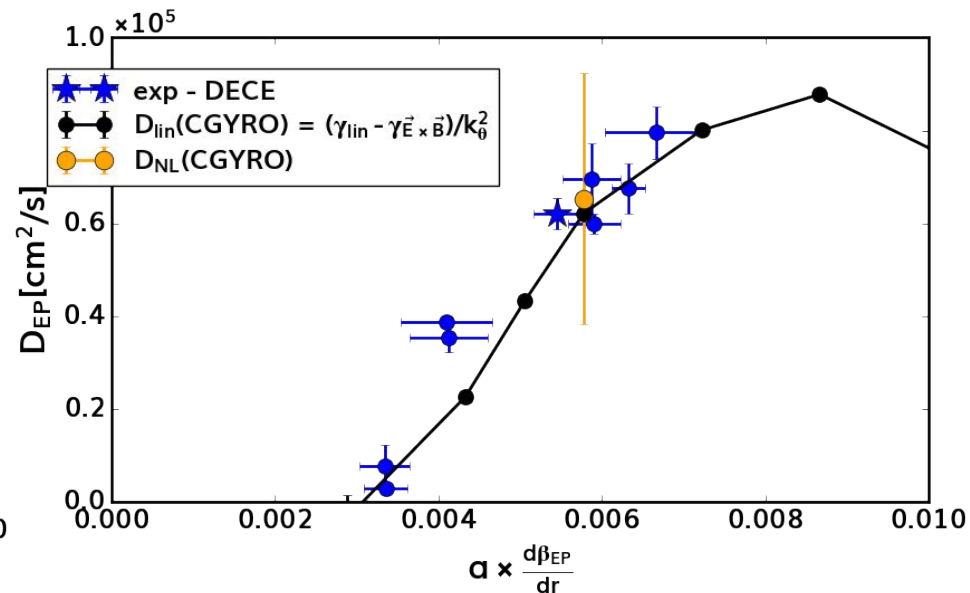
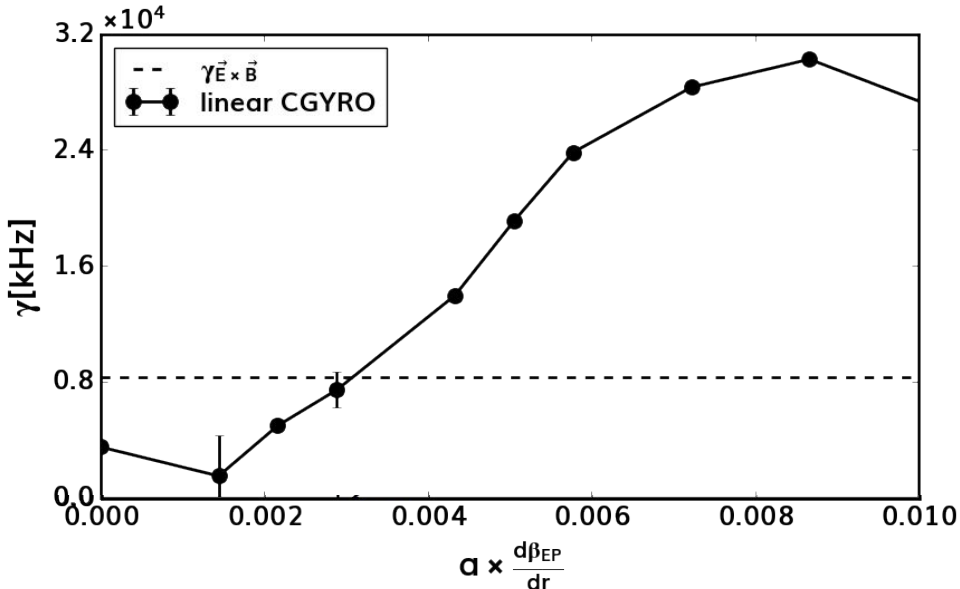
42

Discussion: D_{ECE} analysis of existing discharges demonstrates critical gradient behavior reasonably consistent with gyrokinetic estimates



Discussion: D_{ECE} analysis of existing discharges demonstrates critical gradient behavior reasonably consistent with gyrokinetic estimates

AE database, #159242-159257, $\rho=0.6$, 400 ms

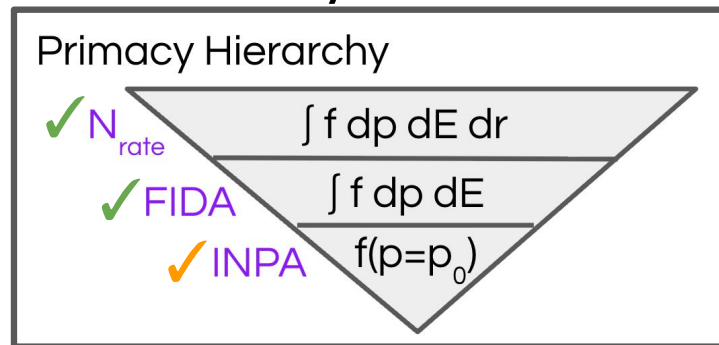


E.M.Bass-Self saturation of Alfvén eigenmodes with a reactor-like low rho-star (10/01 11:55 am)

Conclusions and ongoing work

Conclusions: A new, computationally efficient method for determining fast ion transport in tokamak experiments is now available, which is validated by

- 1) Primacy hierarchy
- 2) A database of +40 DIII-D AE discharges



RESEARCH ARTICLE | SEPTEMBER 29 2026

A diffusive empirical fast ion transport model using electron temperature fluctuation data 🛒

K. J. Callahan ; M. A. Van Zeeland ; W. W. Heidbrink ; M. E. Austin ; L. Liu ; X. D. Du ;
D. Liu ; J. Rueda-Rueda ; J. B. Lestz

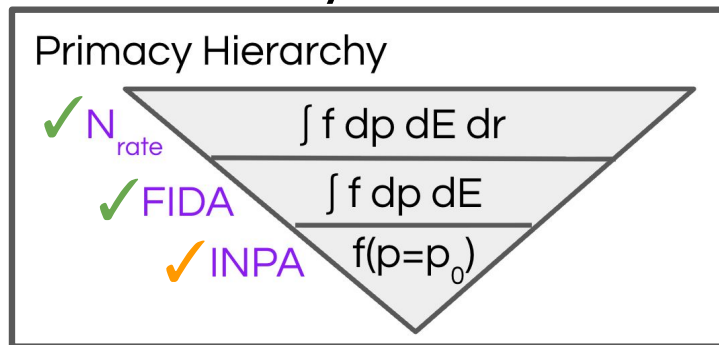
Conclusions and ongoing work

Conclusions: A new, computationally efficient method for determining fast ion transport in tokamak experiments is now available, which is validated by

- 1) Primacy hierarchy
- 2) A database of +40 DIII-D AE discharges

Ongoing work:

- 1) Add an energy dependence to this model (back-up slides)
- 2) Extend workflow to other devices (JET, JT-60SA)
- 3) Use to help verify and validate reduced fast ion transport models like TGLF-EP and nonlinear CGYRO

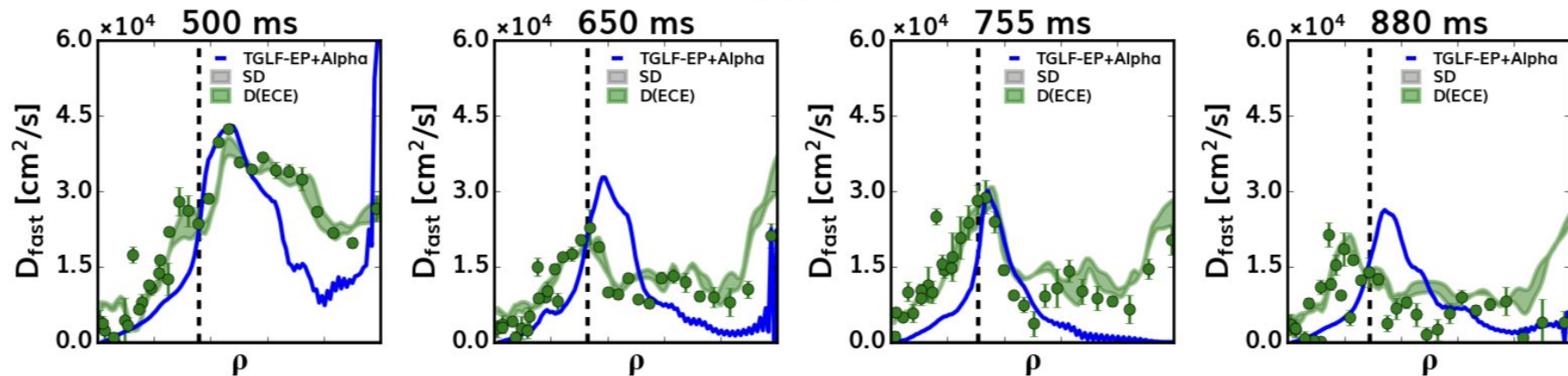


Backup slides and acknowledgements

Acknowledgements: This work was supported by the US Department of Energy under Awards No. DE-FC02- 04ER54698, DE-SC0020337, DE-AC02-09CH11466, DE-SC0020287, DE-SC0024399, DE-SC0014664, and DE-FG02-08ER54999.

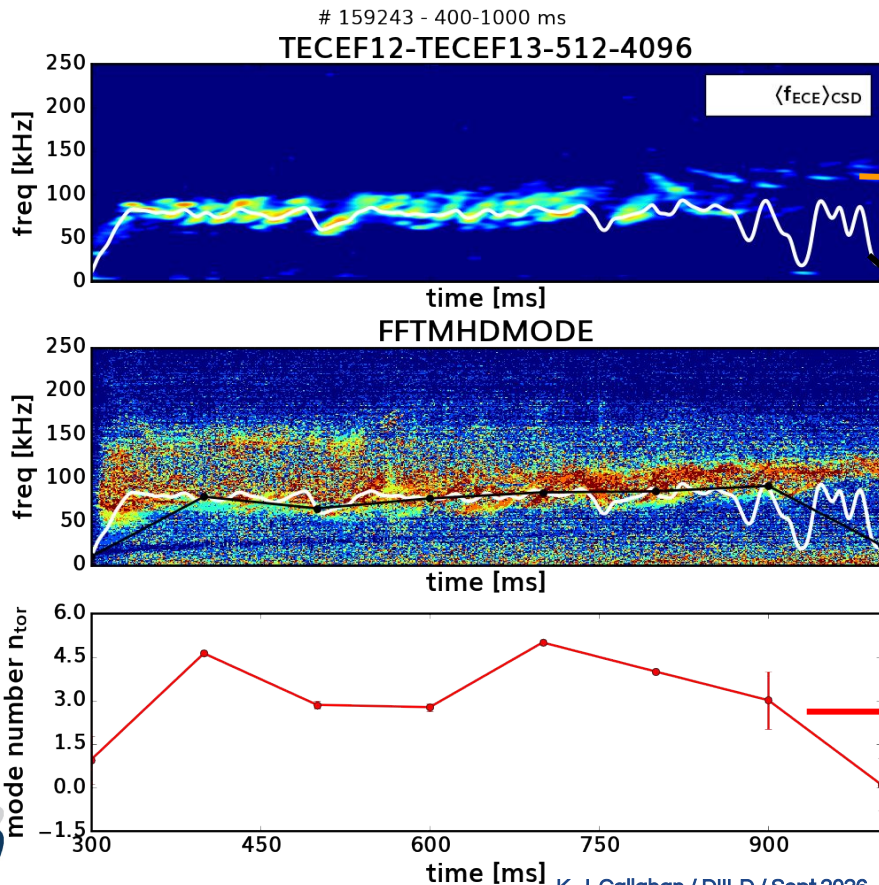
Back-up slides: TGLF-EP vs DECE cross-comparison

#202017



$$D_j = D_{AE} (a/p_{tot}) \left[(-\partial p_{eff}/\partial r) - (-\partial p_{eff}/\partial r)_{crit} \right]_{>0} + D_j^{ITG/TEM}.$$

TGLF-EP+Alpha comparison to experimental data: Electron Cyclotron Emission Data



$$D_{ECE}(\rho, \tau) = D_0 \frac{dT}{T}(\rho, \tau)$$

$$\langle f_{ECE}(t) \rangle_{\text{CSD}}$$

$n_{\text{tor}}(f, t)$ spectrogram
from magnetics

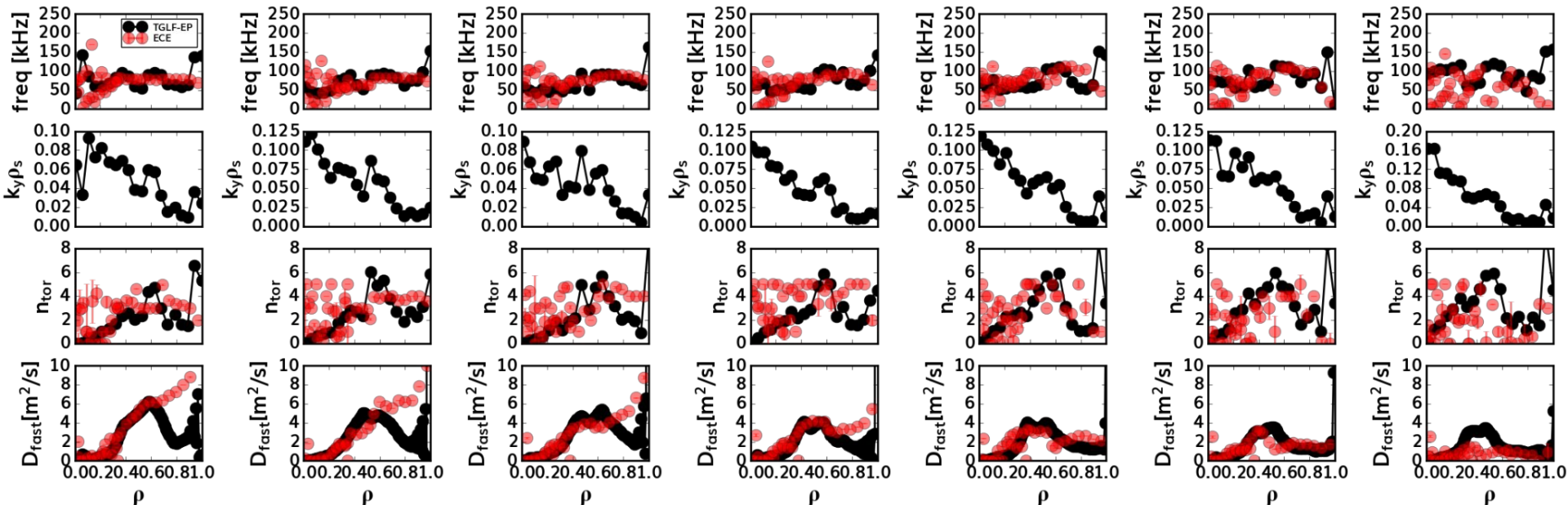
$$n_{\text{tor}}(\langle f_{ECE} \rangle, t)$$

TGLF-EP+Alpha comparison vs 159243

experiment :

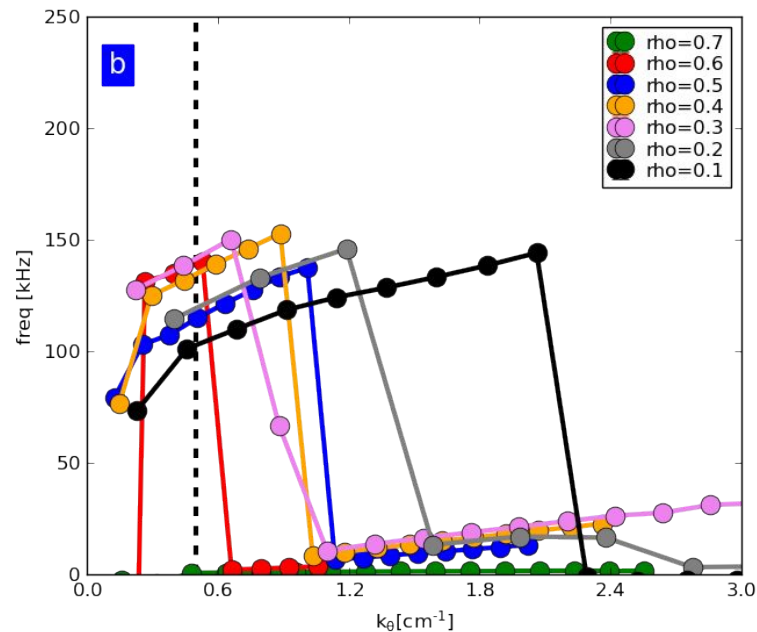
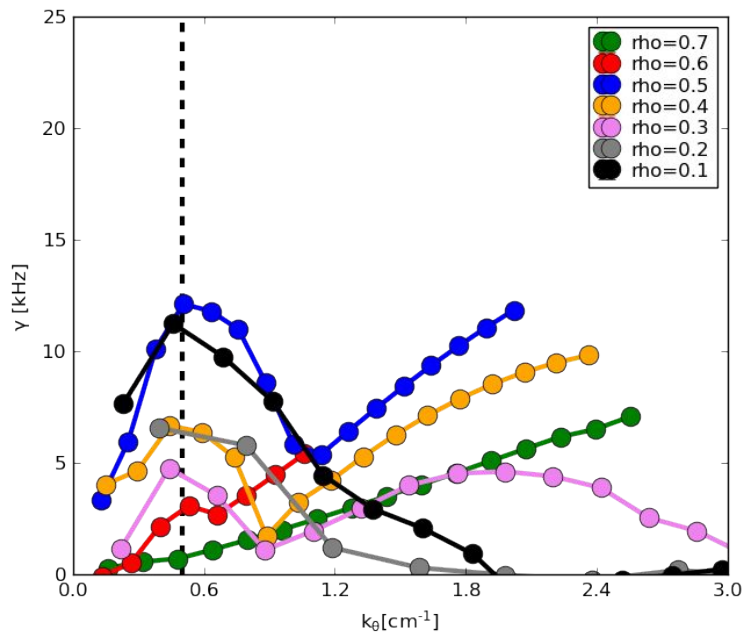
400 ms

1000 ms



Back-up slides: Peak growth rate perpendicular wave-number appears pinned to 0.5 cm^{-1} for all radial locations with AE activity

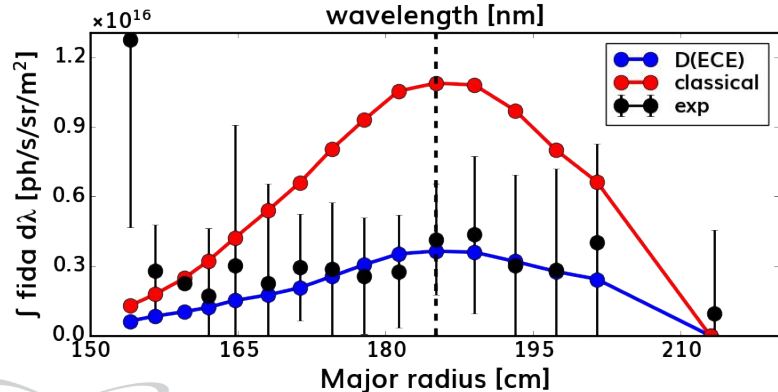
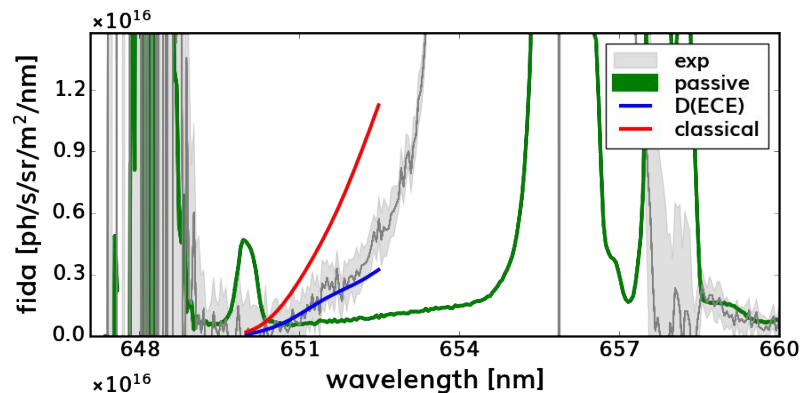
#204232 at 550 ms, linear CGYRO



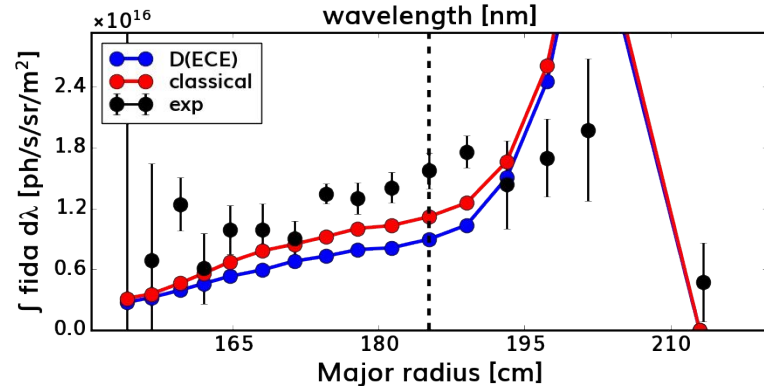
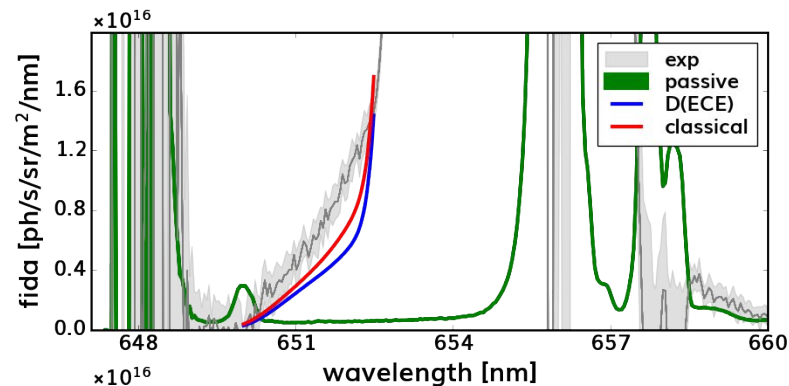
$$\omega_0/k_{\perp}^2 \approx 2.5 \times 10^6 \text{ cm}^2/\text{s}$$

Back-up slides: extra FIDA comparisons at other times

FIDA-channel f05, #202014, 390-410 ms, 650.0-652.5 nm

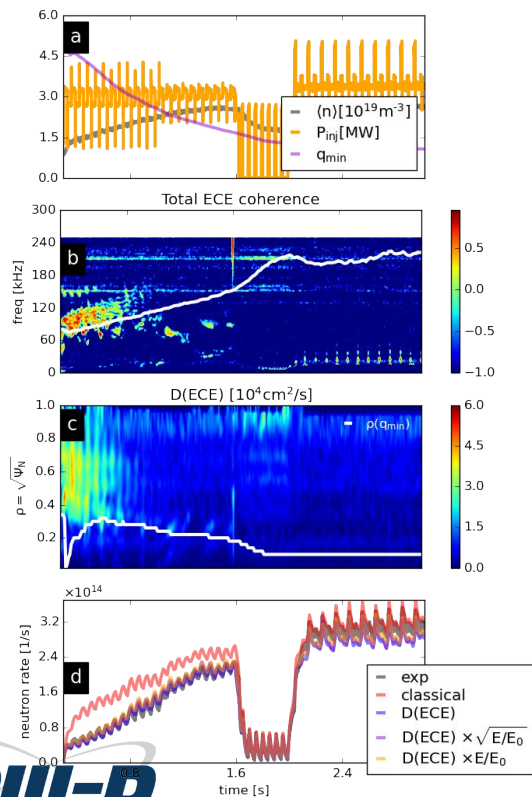


FIDA-channel f05, #202014, 2200-2220 ms, 650.0-652.5 nm

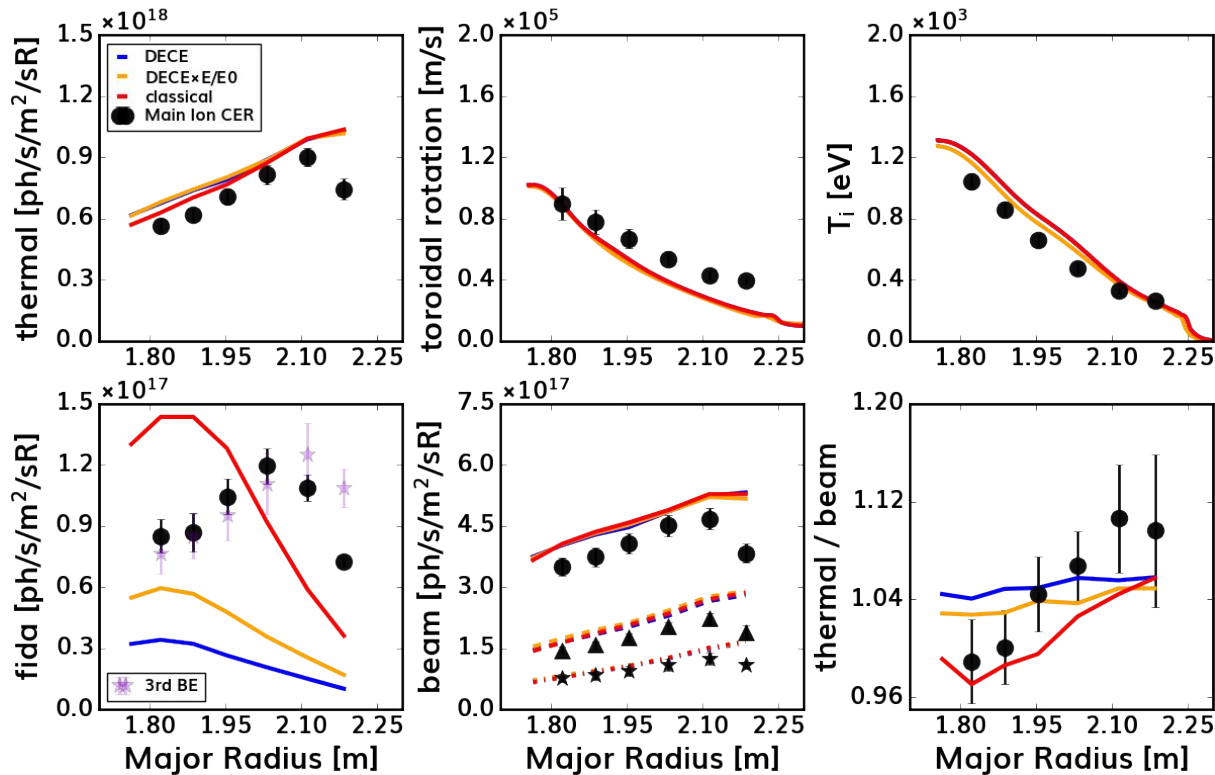


Back-up slides: main ion CER comparisons

DIII-D # 202014

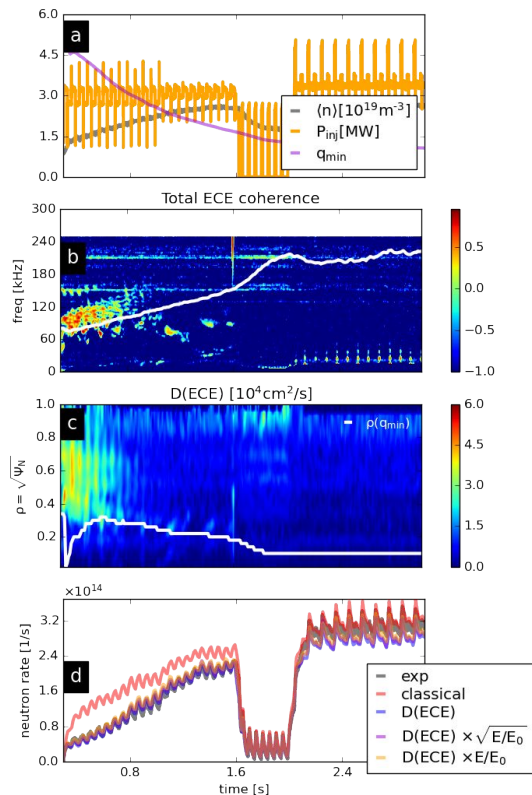


#202014 - 430 ms

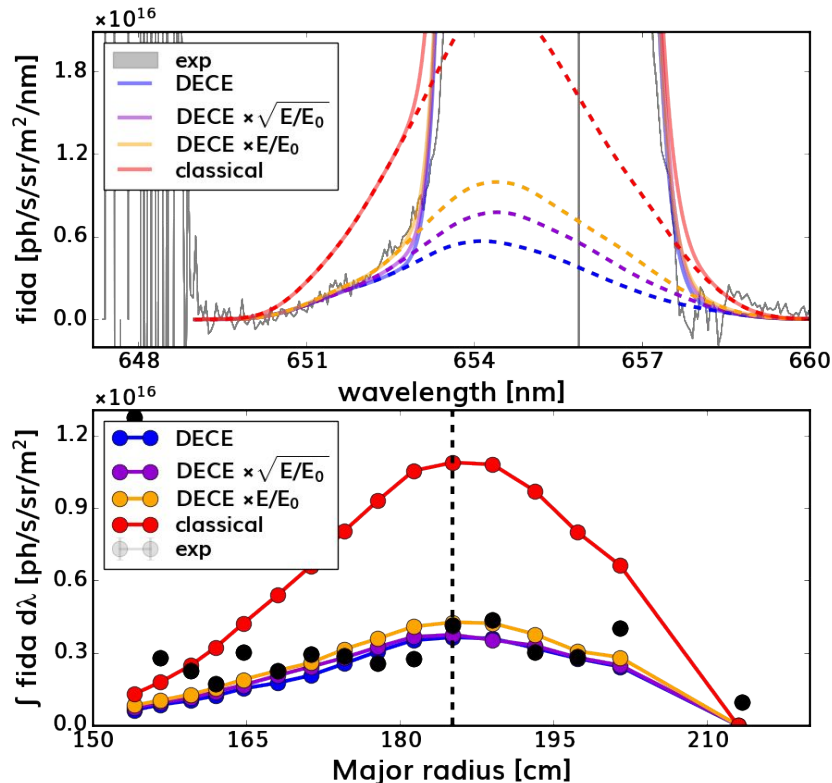


Back-up slides: Adding in an energy dependence normalized to injection energy at 80 keV

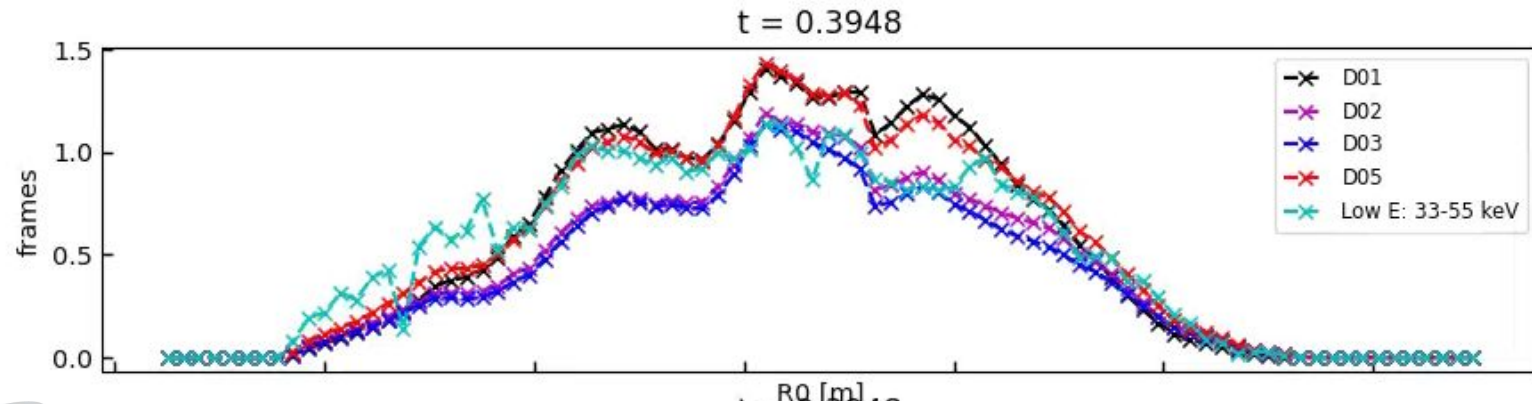
DI-II-D # 202014



FIDA-channel f05, #202014, 390-410 ms, 650.0-652.5 nm



Back-up slides: Adding in an energy dependence normalized to injection energy at 80 keV for INPA data



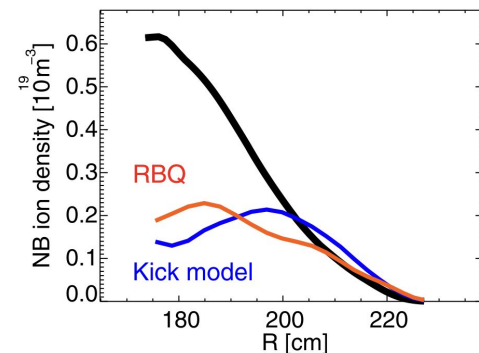
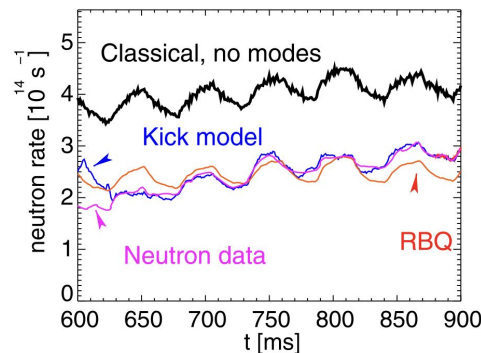
Back-up slide: Comparison of empirical AE diffusion modeling to historical transport predictions demonstrates strong agreement

Diffusion modeling using ECE amplitudes strongly consistent with RBQ1D reduced modeling

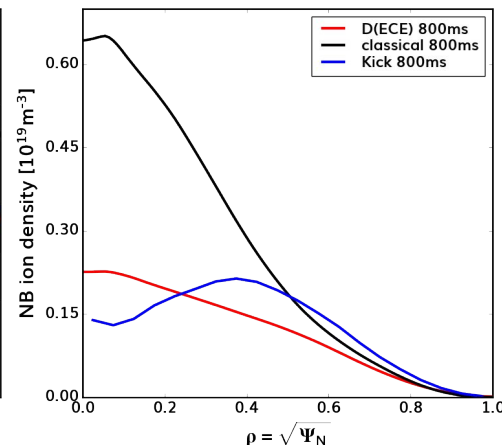
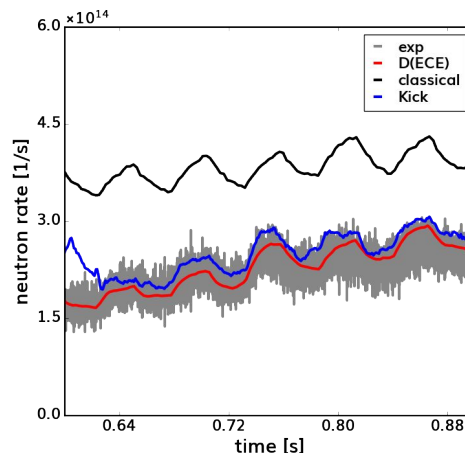
Collins, C. S., et al. *Nuclear Fusion* 57.8 (2017): 086005.

Podesta, Mario, Marina Gorelenkova, and Roscoe B. White. *PPCF* 56.5 (2014): 055003.

Gorelenkov, N. N., et al. *Physics of Plasmas* 26.7 (2019)



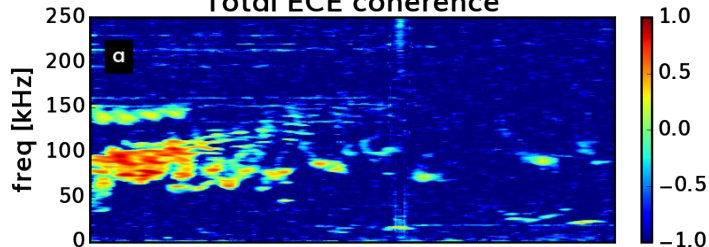
DIII-D # 159243



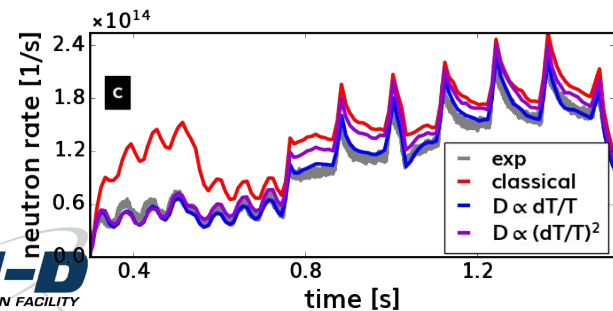
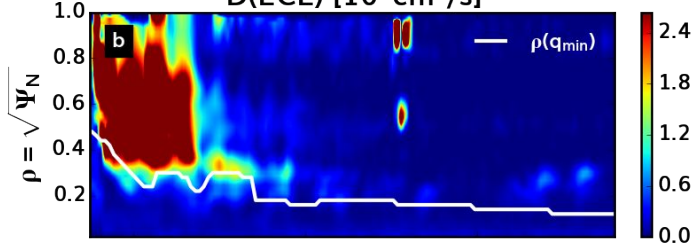
Back-up slide: Test on alternative D^{ECE} scalings with dT/T show linear dependence characteristics specifically

DIII-D # 202017

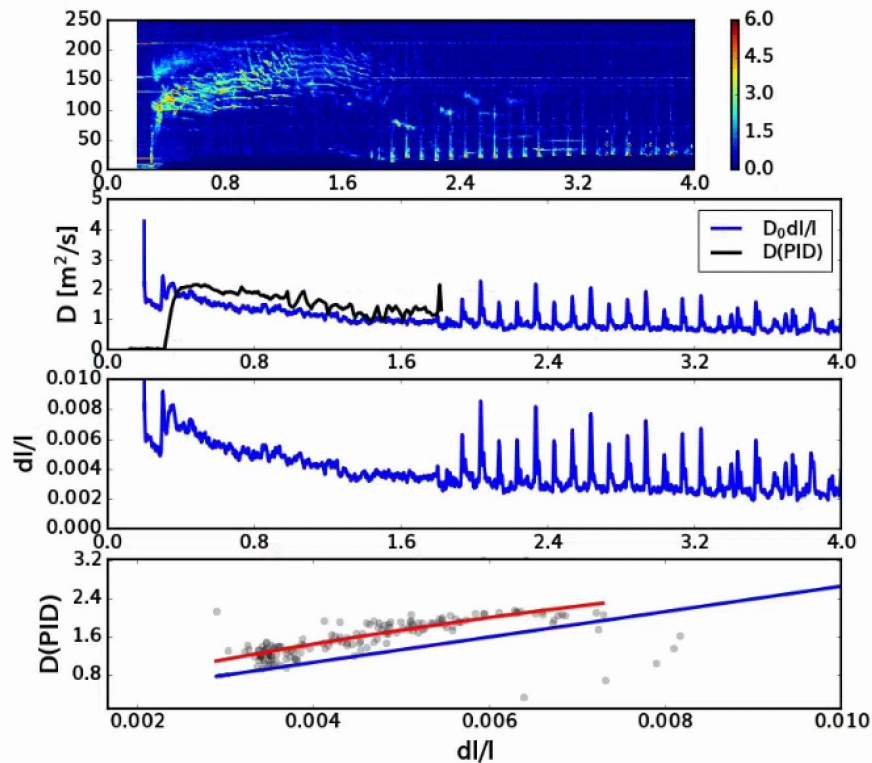
Total ECE coherence



D(ECE) [$10^4 \text{cm}^2/\text{s}$]

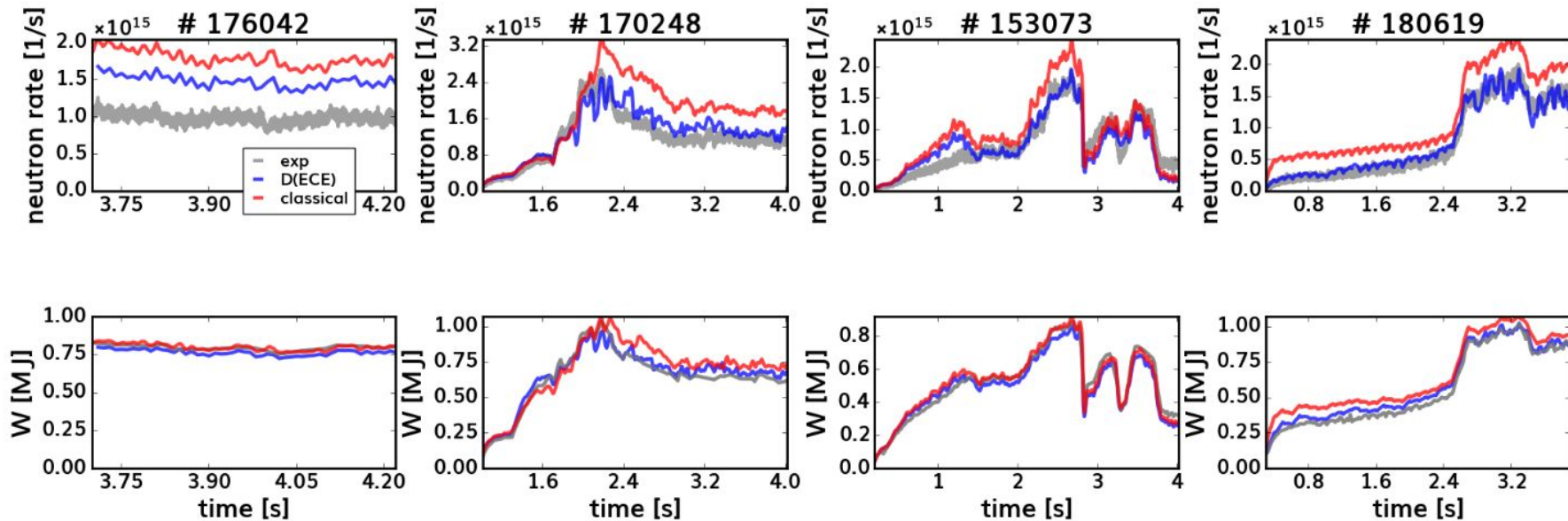


DIII-D # 204232



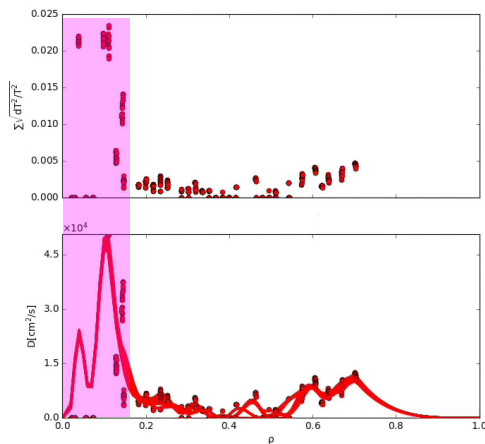
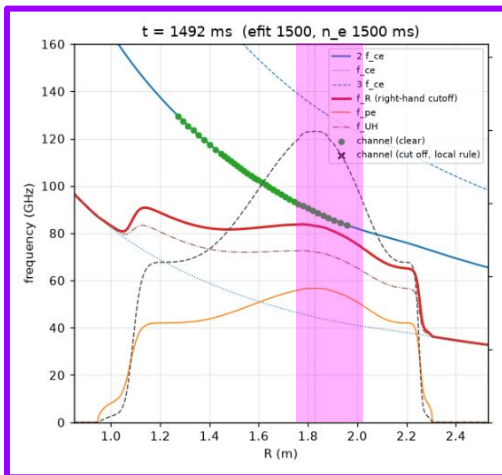
$D[\text{m}^2/\text{s}] = 266.700 \text{ dl}/l$
 Fitted Equation: $D = -13919.610 * dl/l^2 + 418.582 * dl/l$

Discharges w/ stronger n = 1 activity observe stronger disagreement between D_{ECE} and exp



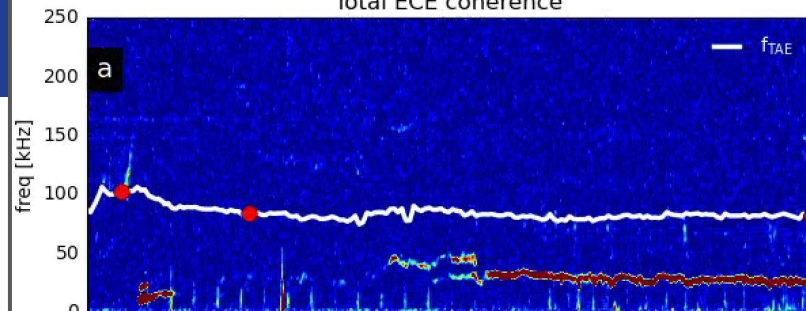
$$\frac{dT}{T}(\tau) = \sqrt{\int_{f_{tor,core}}^{f_{Nyquist}} \frac{|\tilde{T}_x(f, \tau) \tilde{T}_y^*(f, \tau)|}{\bar{T}_x(\tau) \bar{T}_y(\tau)} df}$$

Discharges w/ low $B_{\perp}$ can have incomplete spatial coverage & density cut-off



DIII-D # 153072

Total ECE coherence



D(ECE) [$10^4 \text{cm}^2/\text{s}$]

