Sensitivity analysis on material properties in mechanical analysis of YBCO tapes during cool-down and axial loading

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Outline

- Introduction
- 3D mechanical modeling with Sparselizard
- Validation
- Sensitivity analysis
- Conclusion

Introduction

- Developed a 3D simulation model for YBCO tape, using ***Sparselizard****
- Noticed a wide spread of material properties in literature
- Properties of super thin layers in YBCO tape ≠ properties of bulk materials
- Detailed material characterization seems necessary
- Which material should we measure?
- Properties which are critical for **YBCO layer** and **average Tape** strain and stress
- Performed a sensitivity analysis to determine impact of each property

* HALBACH, Alexandre. Sparselizard-the user friendly finite element c++ library. 2017. ***(http://sparselizard.org/)***

Elastic model

$$\begin{cases} \nabla.(\underline{\boldsymbol{\sigma}}) + \vec{f_b} = 0 \\ \underline{\boldsymbol{\sigma}} = \boldsymbol{D} : \underline{\boldsymbol{\varepsilon}} - \underline{\boldsymbol{\sigma}}_{th} \\ \underline{\boldsymbol{\varepsilon}} = \frac{1}{2}(\nabla\vec{u} + \nabla\vec{u}^T) \\ \underline{\boldsymbol{\sigma}}_{th} = \frac{E}{1-2\nu}\underline{\boldsymbol{\varepsilon}}_{th} = \frac{E}{1-2\nu}\underline{\boldsymbol{\alpha}}_{th}\Delta T \end{cases}$$

weak formulation:

$$\int_{\Omega} -\boldsymbol{D}\underline{\boldsymbol{\varepsilon}}(\vec{u})\underline{\boldsymbol{\varepsilon}}(\vec{u}')dV + \int_{\Omega} \underline{\boldsymbol{\sigma}}_{th}\underline{\boldsymbol{\varepsilon}}(\vec{u}')dV + \int_{\Omega'} \vec{f_b}\,\vec{u}'dV = 0$$

$\underline{\boldsymbol{\sigma}}$	Cauchy stress tensor
$\underline{\boldsymbol{\varepsilon}}$	Cauchy strain tensor
$\boldsymbol{D}$	Elastic tensor
$\vec{f_b}$	body force vector
$\vec{u}$	Displacement vector
E	Young modulus
ν	Poisson's ratio
ΔT	Thermal change
$\underline{\boldsymbol{\varepsilon}}_{th}$	Thermal strain tensor
$\underline{\boldsymbol{\alpha}}_{th}$	Thermal expansion coefficient tensor

Bi-linear plasticity model

Plasticity?

$$\begin{cases} f < 0 & \text{elastic} \\ f = 0 & \text{plastic} \end{cases}$$

$$f = ||\underline{\boldsymbol{s}} - \underline{\boldsymbol{\alpha}}|| - \kappa$$

$$\underline{s} = \underline{\boldsymbol{\sigma}} - \frac{1}{3}\mathrm{tr}(\underline{\boldsymbol{\sigma}})\underline{\mathbf{1}}$$

$$\dot{\underline{\boldsymbol{\varepsilon}}} = \dot{\underline{\boldsymbol{\varepsilon}}}^e + \dot{\underline{\boldsymbol{\varepsilon}}}^p$$

$$\dot{\underline{\boldsymbol{\sigma}}} = D : (\dot{\underline{\boldsymbol{\varepsilon}}} - \dot{\underline{\boldsymbol{\varepsilon}}}^p)$$

$$\dot{\underline{\boldsymbol{\varepsilon}}}^p = \dot{\lambda}\frac{\partial f}{\partial \underline{\boldsymbol{\sigma}}} = \dot{\lambda}\hat{n}$$

$$\dot{\kappa} = \beta H \dot{\lambda}\ ,\ \dot{\underline{\boldsymbol{\alpha}}} = (1-\beta) H \dot{\lambda}\hat{n}$$

$$\dot{f} = 0$$

Computational form:

$$\Delta\underline{\boldsymbol{\varepsilon}} = \Delta\underline{\boldsymbol{\varepsilon}}^e + \Delta\underline{\boldsymbol{\varepsilon}}^p$$

$$\underline{\boldsymbol{\sigma}}_{n+1} = \underline{\boldsymbol{\sigma}}^{tr}_{n+1} - 2\mu\Delta\lambda\hat{n}_{n+1}$$

$$\Delta\underline{\boldsymbol{\varepsilon}}^p \approx \Delta\lambda\hat{n}_{n+1}$$

$$\kappa_{n+1} = \kappa_n + \beta H \Delta\lambda$$

$$\underline{\boldsymbol{\alpha}}_{n+1} = \underline{\boldsymbol{\alpha}}_n + (1-\beta) H \Delta\lambda\hat{n}_{n+1}$$

$$f_{n+1} = 0$$

f	Yield function
$\underline{\boldsymbol{s}}$	Deviatoric stress tensor
κ	Isotropic hardening
$\underline{\alpha}$	Kinematic hardening
$\underline{\boldsymbol{\varepsilon}}^e$	Elastic strain tensor
$\underline{\boldsymbol{\varepsilon}}^p$	Plastic strain tensor
λ	Plasticity multiplier (norm)
$\hat{n}$	Unit direction vector
H	Generalized plastic modulus

Numerical elasto-plastic model for axial loading

1) Elastic prediction:

$$\begin{cases} \int_{\Omega} -\boldsymbol{D}\Delta\underline{\varepsilon}(\vec{u})\Delta\underline{\varepsilon}(\vec{u}')dV + \int_{\Omega'} \vec{f}_b\,\vec{u}'\,dV = 0 \\ \underline{\varepsilon}_{n+1} = \underline{\varepsilon}_n + \Delta\underline{\varepsilon} \\ \underline{\boldsymbol{\sigma}}^{\mathrm{tr}}_{n+1} = \underline{\boldsymbol{\sigma}}_n + \boldsymbol{D} : \Delta\underline{\varepsilon} \end{cases}$$

2) Plasticity?

$$\begin{cases} p = \frac{1}{3}\mathrm{tr}(\underline{\boldsymbol{\sigma}}^{tr}_{n+1}) \\ \underline{\boldsymbol{s}}^{tr}_{n+1} = \underline{\boldsymbol{\sigma}}^{tr}_{n+1} - p\mathbf{1} \\ f = ||\underline{\boldsymbol{s}}^{tr}_{n+1} - \underline{\boldsymbol{\alpha}}_n|| - \kappa_n \end{cases}$$

3) Plasticity correction:

$$\begin{cases} \hat{n}_{n+1} = \frac{\underline{\boldsymbol{\sigma}}^{tr}_{n+1}}{||\underline{\boldsymbol{\sigma}}^{tr}_{n+1}||} \\ \Delta\lambda = \frac{||\underline{\boldsymbol{s}}^{tr}_{n+1} - \underline{\boldsymbol{\alpha}}_n|| - \kappa_n}{2\mu + H} \\ \underline{\varepsilon}^p_{n+1} = \underline{\varepsilon}^p_n + \Delta\lambda\hat{n}_{n+1} \\ \underline{\boldsymbol{\sigma}}_{n+1} = \underline{\boldsymbol{\sigma}}^{tr}_{n+1} - 2\mu\Delta\lambda\hat{n}_{n+1} \\ \kappa_{n+1} = \kappa_n + \beta H\Delta\lambda \\ \underline{\boldsymbol{\alpha}}_{n+1} = \underline{\boldsymbol{\alpha}}_n + (1-\beta)H\Delta\lambda\hat{n}_{n+1} \end{cases}$$

Numerical elasto-plastic thermal model for cool-down

1) Initial elastic prediction:

$$
\begin{cases}
\int_\Omega -\boldsymbol{D}\Delta\underline{\varepsilon}(\vec{u})\Delta\underline{\varepsilon}(\vec{u}')dV + \int_\Omega \Delta\underline{\boldsymbol{\sigma}}_{th}\Delta\underline{\varepsilon}(\vec{u}')dV = 0 \\
\Delta\underline{\boldsymbol{\sigma}}_{th} = \dfrac{E}{1-2\nu}\,\underline{\boldsymbol{\alpha}}_{th}\,\Delta T \\
\underline{\boldsymbol{\varepsilon}}_{n+1} = \underline{\boldsymbol{\varepsilon}}_n + \Delta\underline{\boldsymbol{\varepsilon}} \\
\underline{\boldsymbol{\sigma}}^{\mathrm{tr}}_{n+1} = \underline{\boldsymbol{\sigma}}_n + (\boldsymbol{D} : \Delta\underline{\boldsymbol{\varepsilon}} - \Delta\underline{\boldsymbol{\sigma}}_{th})
\end{cases}
$$

2) Plasticity?

$$
\begin{cases}
p = \dfrac{1}{3}\mathrm{tr}(\underline{\boldsymbol{\sigma}}^{tr}_{n+1}) \\
\underline{\boldsymbol{s}}^{tr}_{n+1} = \underline{\boldsymbol{\sigma}}^{tr}_{n+1} - p\mathbf{1} \\
f = ||\underline{\boldsymbol{s}}^{tr}_{n+1} - \underline{\boldsymbol{\alpha}}_n|| - \kappa_n
\end{cases}
$$

3) Initial Plasticity correction:

$$
\begin{cases}
\hat{n}_{n+1} = \dfrac{\underline{\boldsymbol{\sigma}}^{tr}_{n+1}}{||\underline{\boldsymbol{\sigma}}^{tr}_{n+1}||} \\
\Delta\lambda = \dfrac{||\underline{\boldsymbol{s}}^{tr}_{n+1} - \underline{\boldsymbol{\alpha}}_n|| - \kappa_n}{2\mu + H} \\
\Delta\underline{\boldsymbol{\sigma}}_c = 2\mu\Delta\hat{n}_{n+1}
\end{cases}
$$

Numerical elasto-plastic thermal model for cool-down

4) Updated elastic prediction:

$$
\begin{cases}
\int_{\Omega} -\boldsymbol{D}\Delta\underline{\varepsilon}(\vec{u})\Delta\underline{\varepsilon}(\vec{u}')dV + \int_{\Omega} \left(\Delta\underline{\boldsymbol{\sigma}}_{th} + \Delta\underline{\boldsymbol{\sigma}}_{c}\right)\Delta\underline{\varepsilon}(\vec{u}')dV = 0 \\
\underline{\varepsilon}_{n+1} = \underline{\varepsilon}_n + \Delta\underline{\varepsilon} \\
\underline{\boldsymbol{\sigma}}^{\mathrm{tr}}_{n+1} = \underline{\boldsymbol{\sigma}}_n + (\boldsymbol{D} : \Delta\underline{\varepsilon} - \Delta\underline{\boldsymbol{\sigma}}_{th})
\end{cases}
$$

5) Plasticity?

$$
\begin{cases}
p = \frac{1}{3}\mathrm{tr}(\underline{\boldsymbol{\sigma}}^{tr}_{n+1}) \\
\underline{\boldsymbol{s}}^{tr}_{n+1} = \underline{\boldsymbol{\sigma}}^{tr}_{n+1} - p\mathbf{1} \\
f = ||\underline{\boldsymbol{s}}^{tr}_{n+1} - \underline{\boldsymbol{\alpha}}_n|| - \kappa_n
\end{cases}
$$

6) Plasticity correction:

$$
\begin{cases}
\hat{n}_{n+1} = \dfrac{\underline{\boldsymbol{\sigma}}^{tr}_{n+1}}{||\underline{\boldsymbol{\sigma}}^{tr}_{n+1}||} \\
\Delta\lambda = \dfrac{||\underline{\boldsymbol{s}}^{tr}_{n+1} - \underline{\boldsymbol{\alpha}}_n|| - \kappa_n}{2\mu + H} \\
\underline{\boldsymbol{\varepsilon}}^p_{n+1} = \underline{\boldsymbol{\varepsilon}}^p_n + \Delta\lambda\hat{n}_{n+1} \\
\underline{\boldsymbol{\sigma}}_{n+1} = \underline{\boldsymbol{\sigma}}^{tr}_{n+1} - 2\mu\Delta\lambda\hat{n}_{n+1} \\
\kappa_{n+1} = \kappa_n + \beta H\Delta\lambda \\
\underline{\boldsymbol{\alpha}}_{n+1} = \underline{\boldsymbol{\alpha}}_n + (1-\beta)H\Delta\lambda\hat{n}_{n+1}
\end{cases}
$$

YBCO tape and the constituent layers

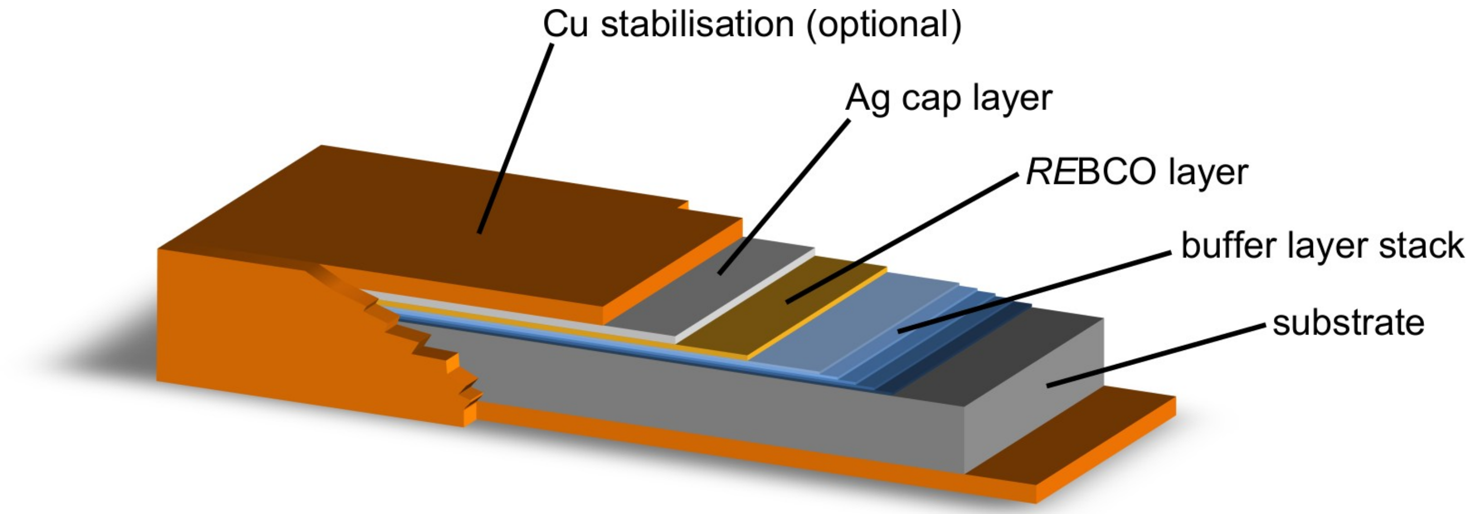


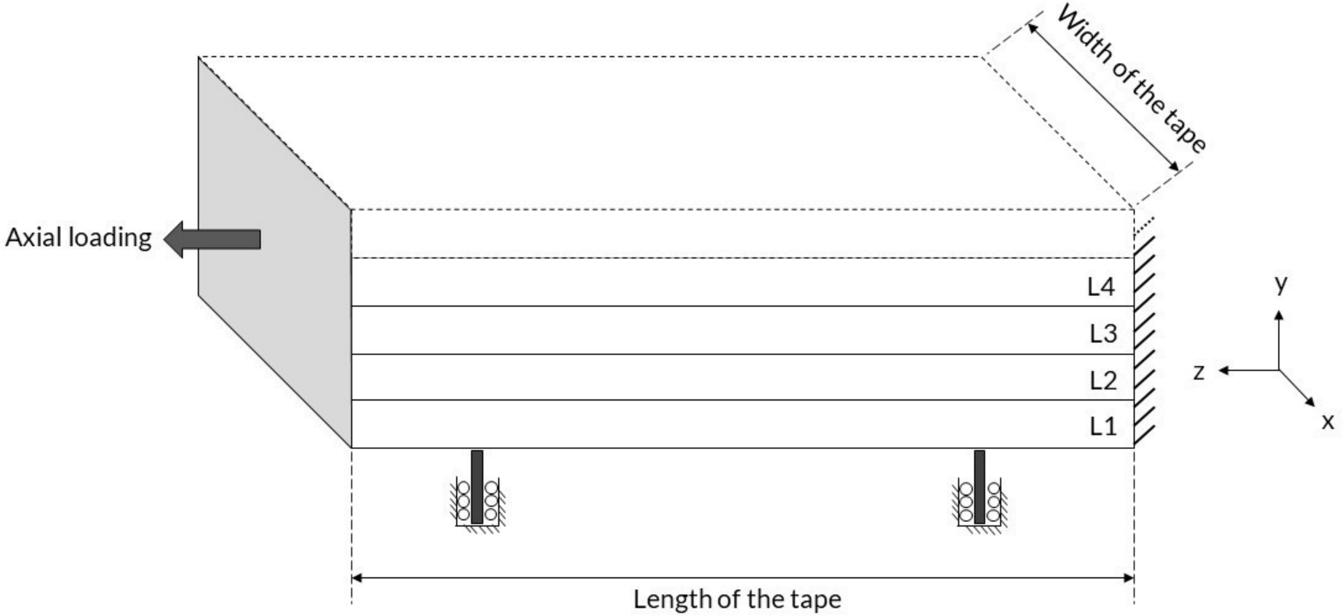
Figure from: Barth, Christian. *High temperature superconductor cable concepts for fusion magnets*. Vol. 7. KIT Scientific Publishing, 2013.

	Tape 1	Tape 2	Tape 3	Tape 4
	Demo Tape Sensitivity analysis	validation with COMSOL	validation with Tufts measurements*	
Manufacturer			SuperPower	SuNAM
Number of layers	7	4	4	4
Materials	Copper Silver Hastelloy Buffer YBCO	Copper Hastelloy YBCO	Copper Hastelloy YBCO	Copper Sainless steel YBCO
length of tape	8 cm	8 cm	5 cm	5 cm
tape width	5.3 mm	5.3 mm	4.027 mm	4.062 mm
substrate thickness	100 μm	100 μm	50 μm	100 μm
cool-down	✓	✓	✗	✗
axial -loading	✓	✓	✓	✓

* Allen, N. C., L. Chiesa, and M. Takayasu. "Structural modeling of HTS tapes and cables." *Cryogenics* 80 (2016): 405-418.

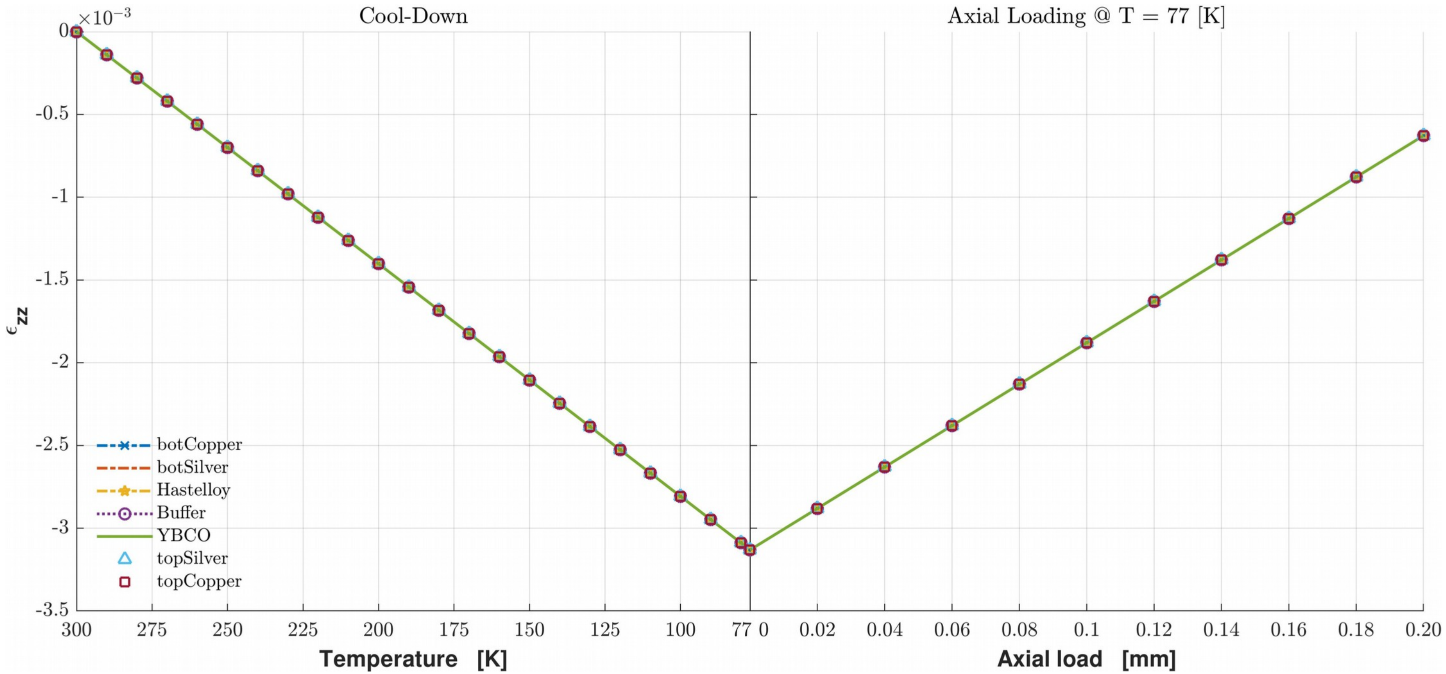
Mechanical boundary conditions for Demo tape

- Fixed clamping at one end
- Partially constrained at the bottom



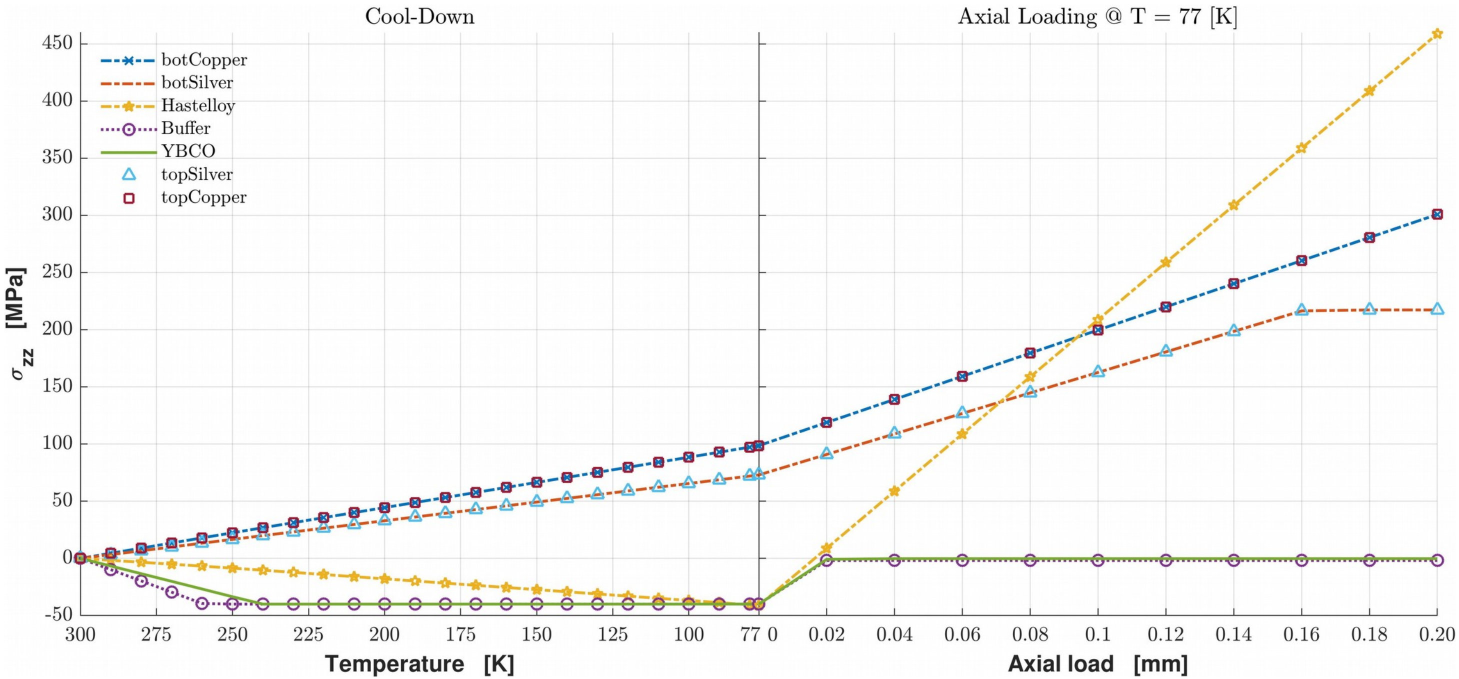
	dimension [m]
L1 : Copper	20×10^{-6}
L2 : Silver	2×10^{-6}
L3 : Hastelloy	100×10^{-6}
L4 : Buffer	1×10^{-6}
L5 : YBCO	2×10^{-6}
L6 : Silver	2×10^{-6}
L7 : Copper	20×10^{-6}
width	5.3×10^{-3}
length	8×10^{-2}

Demo tape (model tape 1) mechanical behavior – strain results



H. Milanchian, et al., "Novel 3D-simulation model for REBCO tape mechanical behaviour", *to be submitted to IEEE Trans. On appl. Supercond.*

Demo tape (model tape 1) mechanical behavior – stress results



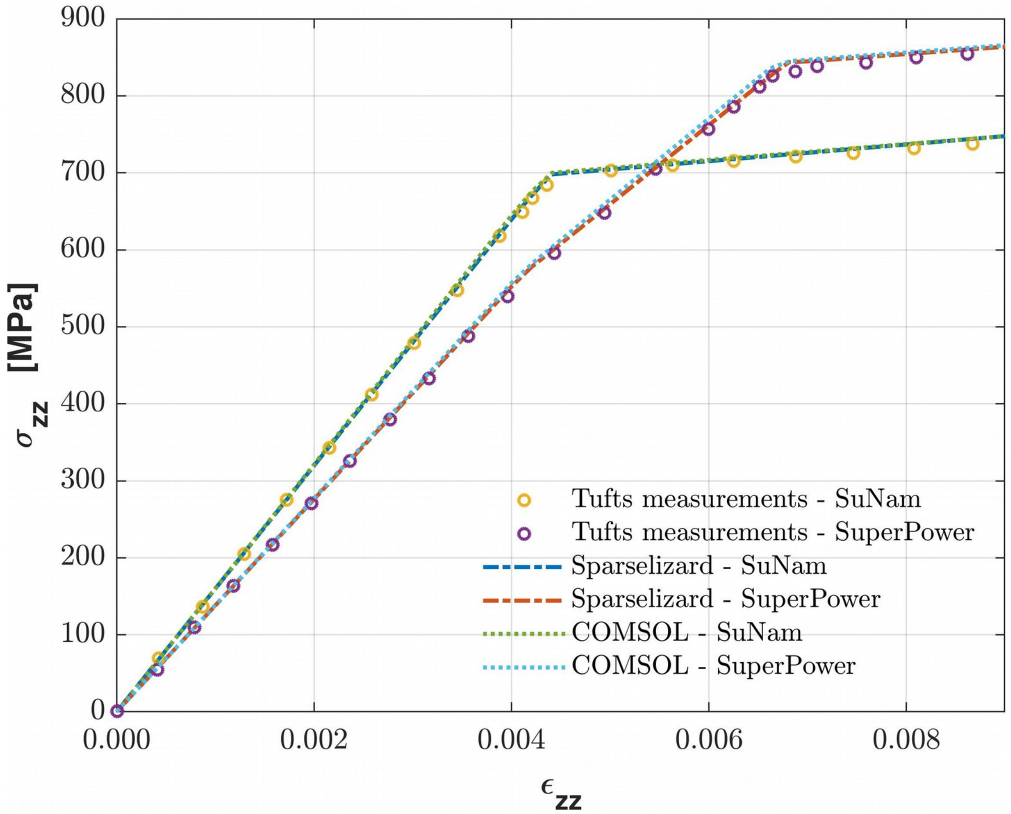
H. Milanchian, et al., ”Novel 3D-simulation model for REBCO tape mechanical behaviour”, *to be submitted to IEEE Trans. On appl. Supercond.*

Validation with COMSOL – model tape 2

			botCopper	Hastelloy	YBCO	topCopper
Relative Error (%)	cool-down	ε_{zz}	0.0946	0.0946	0.0946	0.0946
		σ_{zz}	0.3240	0.2005	0.0226	0.3124
	axial loading	ε_{zz}	0.0756	0.0756	0.0756	0.0756
		σ_{zz}	0.0031	0.0030	0.0734	0.0031

H. Milanchian, et al., ”Novel 3D-simulation model for REBCO tape mechanical behaviour”, *to be submitted to IEEE Trans. On appl. Supercond.*

Validation with Tufts measurements – model tapes 3 & 4

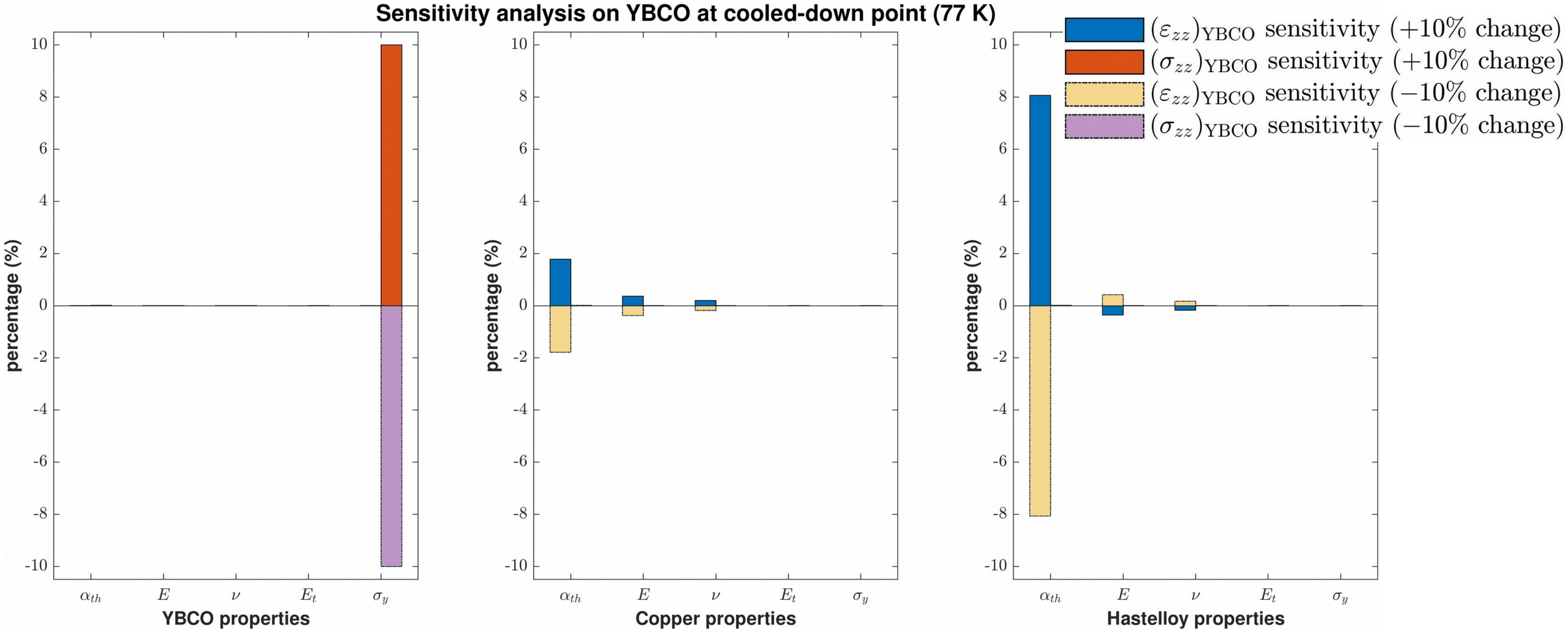


Tufts measurement data from: Allen, N. C., L. Chiesa, and M. Takayasu. "Structural modeling of HTS tapes and cables." *Cryogenics* 80 (2016): 405-418.
Comparison in: H. Milanchian, et al., "Novel 3D-simulation model for REBCO tape mechanical behaviour", *to be submitted to IEEE Trans. On appl. Supercond.*

Sensitivity analysis

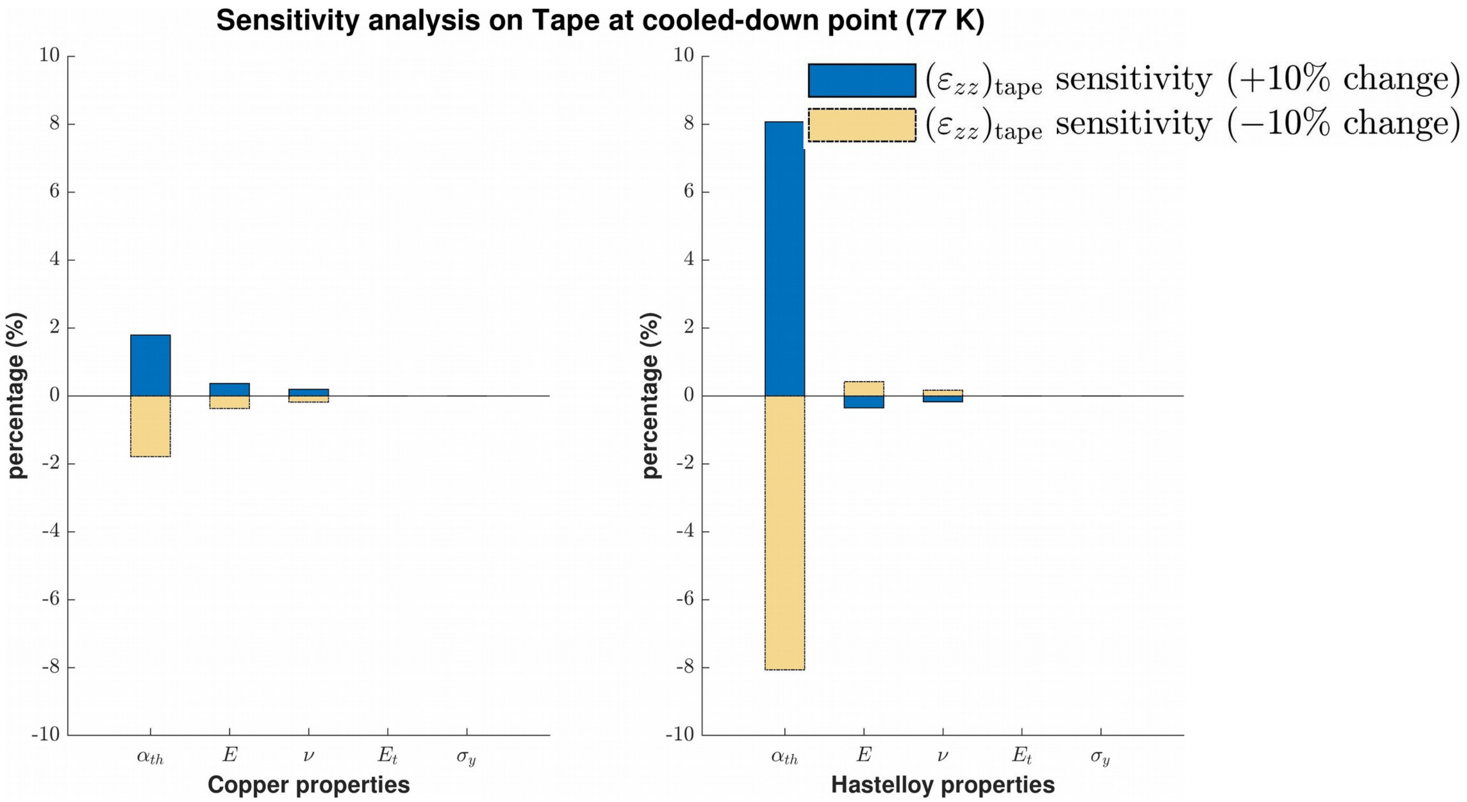
- Demo tape (model tape 1)
- Investigate the changes in ***YBCO layer*** and ***average Tape*** strain and stress
- 5 materials: Copper, Silver, Hastelloy, Buffer, YBCO
- 5 material properties: $\underline{\boldsymbol{\alpha}}_{th},\ E,\ \nu,\ E_T,\ \sigma_y$
- +10% and -10% change (one property at a time and then compare with reference case)
- 50 different cases
- Sensitivity analysis in Cooled-down point (77 K) and axial loading at 77 K

Sensitivity analysis for cool-down stage



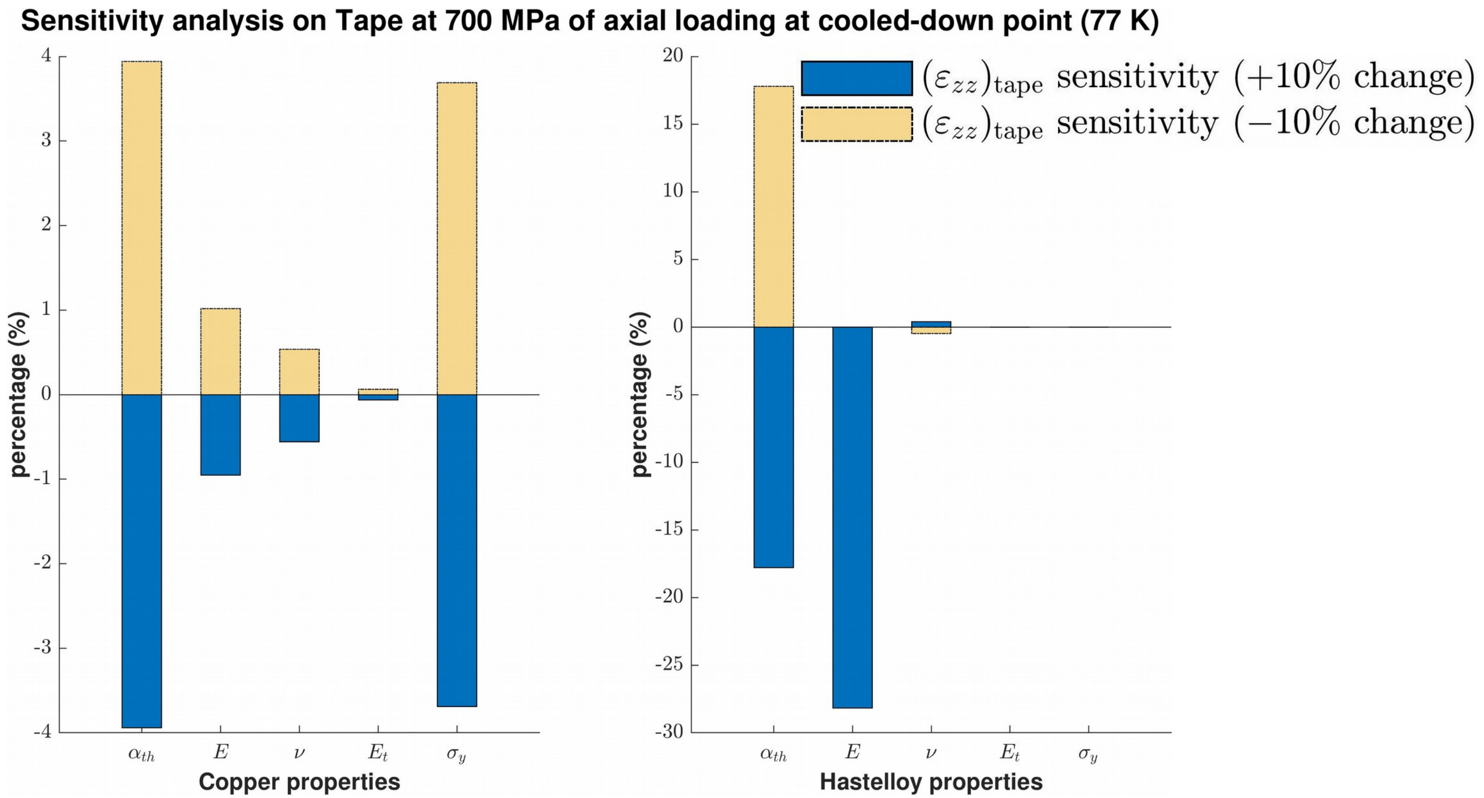
► Silver and Buffer properties had below 1% effect in all cases

Sensitivity analysis for cool-down stage



► YBCO layer properties had below 1% effect in average Tape cases

Sensitivity analysis for Axial loading



► Silver, Buffer, YBCO properties (below 1% effect)

YBCO

- $\underline{\alpha}_{th}$ of Hastelloy and Copper (8% and 2%)
- σ_y of YBCO (10% on stress)

Tape

- $\underline{\alpha}_{th}$ of Hastelloy and Copper (18% and 4%)
- E of Hastelloy (28%)
- σ_y of Copper (4%)

- Thickness sensitivity analysis
- Elastoplastic behavior of YBCO and Copper

Conclusion

Demo tape material properties

	$\alpha_{\rm th}$ [$\times 10^{-6}$]	E [Gpa]	ν	σ_y [Mpa]	E_T [Gpa]
Copper	17.7	80	0.338	275	4
Silver	17	70	0.367	140	0
Hastelloy	13.4	200	0.307	980	8.5
Buffer	9.5	170	0.226	40	0
YBCO	11	157	0.3	40	0

COMSOL validation tape material properties

	α_{th} [$\times 10^{-6}$]	E [Gpa]	ν	σ_y [Mpa]	E_T [Gpa]
Copper	17.7	80	0.338	275	4
Hastelloy	13.4	200	0.307	980	8.5
YBCO	11	157	0.300	40	0

Tufts tape material properties

	E [Gpa]	ν	σ_y [Mpa]	E_T [Gpa]
Copper	85	0.355	350	4
Substrate (SuperPower)	180	0.29	1225	7.5
Substrate (SuNam)	190	0.29	840	10
YBCO[1]	150	0.3	-	-

1 assumed elastic