Calculation of saturated coupling loss in Rutherford cables

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Outline

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2. Saturated coupling loss
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   • Network modelling

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Introduction

Development of network models for Rutherford cables:
• 70’s, Morgan et al: crossing strand coupling, weak excitation, analytical results
• 80’s, Sytnikov et al: crossing and adjacent, weak and strong, analytical
• 90’s, Verweij et al: complete numerical electromagnetic model
• 00’s – p.t.: CUDI / JackPot / THEA / THELMA multi-physics modelling tools

❖ LTS have low stability (low MQE), thus must be operated in weak excitation (coupling currents \( << \) \( I_c \)), therefore leading to small filaments and short twist-pitches.

❖ In contrast, HTS materials have high stability and could be operated in a strong excitation mode (coupling currents \( \sim \) \( I_c \)). No need for fine twisted filaments, for example Bi2223 tapes with non-twisted filaments, ReBCO monofilamentary tapes.

Large AC losses should then be properly evaluated and accounted in the design.

Impact of twisting stacks was found marginal for soldered stacks in fusion conductors, while transposing stacks (twisted or not) have a moderate effect, see https://doi.org/10.1016/j.cryogenics.2020.103118
Motivation

ASTRA conductor concept for DEMO CS coils operated up to 18 T at 5 K, $\gtrsim 0.1$ T/s transients:

- Aligned Stacks Transposed in Roebel Arrangement
- Jacket pre-compression
- Separate cooling channel (conduction cooled stacks)
- Impregnation of cable space

Strong excitation mode features interplay between coupling current loops and superconducting loops, i.e. corresponding loss contributions are no longer independent.

Furthermore, effects of saturation and screening are counteracting.

Hence, detailed study on the actual AC loss performance is needed.
Analytical calculation: weak excitation

\( R_{sc} \ll R_c \): inter-strand loss

\[
P_{\perp} = P_{R_{\perp}} + P_{R_{\parallel}} = \frac{B^2 H^2 L}{3} \left( \frac{N^2}{20R_{\perp}} + \frac{1}{NR_{\parallel}} \right) \quad [W/m]
\]

\[
H = \frac{Nw}{2}, R_a \sim N/L, R_c \sim N/L \rightarrow \quad P_c \sim N^3 L^2, \quad P_a \sim NL^2
\]

\( \rightarrow R_{sc} \) increases with \( L \), \( R_c \) decreases with \( L \):

Limit of the model applicability in terms of the cable twist-pitch?

\[
\varepsilon = \dot{B}wL_c \approx I_c R \sim \rho l_c / L_c \rightarrow L_c \sim \sqrt{\rho l_c / \dot{B}w} \quad \text{(e.g. missing geometry factor)}
\]

Geometry: slab (see Wilson’s book, section 8.3.1)

\[
L_c = \sqrt{\frac{8\rho l_c}{\dot{B}w}}
\]

Geometry: Rutherford cable

\[
L_c \approx \sqrt{\frac{96\rho l_c}{N\dot{B}w}}
\]
Analytical calculation: strong excitation

\( R_{sc} \gg R_c : \) saturated loss

\[
\varepsilon = \dot{B} x l \approx E_c l (I/I_c)^n
\]

\[
I = I_c (\dot{B} x / E_c)^{1/n}
\]

\[
P_1 / l = E \cdot I = E_c I_c (I/I_c)^{n+1}
\]

\[
= E_c I_c (\dot{B} x / E_c)^{1+1/n} \quad \text{[W/m]}
\]

Perpendicular field (2 layers):

\[
P_N / l = \sum P_1(x) / l = E_c I_c \left( \frac{\dot{B} w}{E_c} \right)^{1+\frac{1}{n}} F(N)
\]

\[
F(N) = 4 \cdot \sum_{k=0 \text{ or } 1/2}^{(N-2)/4} k^{1+1/n} \quad \text{(min } N = 4)\]

Parallel field (N/2 layers):

\[
P_N / l = E_c I_c \left( \frac{\dot{B} h}{2E_c} \right)^{1+\frac{1}{n}} N
\]

Critical-state model (\( n = \infty \)):

\[
P / l = I_c w \dot{B} N^2 / 8 \quad \text{in perp field}
\]

\[
P / l = I_c h \dot{B} N / 2 \quad \text{in parr field}
\]

or simply

\[
P / l = I_{c, total} \times \text{width} \times \dot{B} / 4
\]

\[
\dot{B}
\]

\[
\text{width} = N w / 2 \quad \text{(perp)}
\]

\[
= 2h \quad \text{(parr)}
\]

\[
I_{c, total} = N \times I_c
\]

Generally, higher loss for finite \( n \)-value and same \( I_c \):

\[
P / l \sim \dot{B}^2 N^3 \quad \text{for } n = 1 \text{ in perp field}
\]
**Network modelling: integral formulation**

<table>
<thead>
<tr>
<th>Category</th>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometry</td>
<td>$N$</td>
<td>Number of strands</td>
<td>$4 \text{ – } 40$</td>
</tr>
<tr>
<td></td>
<td>$W$</td>
<td>Strand width</td>
<td>$8 \text{ mm}$</td>
</tr>
<tr>
<td></td>
<td>$H$</td>
<td>Strand height</td>
<td>$4 \text{ mm}$</td>
</tr>
<tr>
<td></td>
<td>$L$</td>
<td>Cable twist-pitch</td>
<td>$0.1 \text{ m – inf}$</td>
</tr>
<tr>
<td>Electrical properties</td>
<td>$\rho$</td>
<td>Trans. specific resistance</td>
<td>$1 \text{ – } 100 \mu\Omega\text{.m}$</td>
</tr>
<tr>
<td></td>
<td>$I_c$</td>
<td>Critical current</td>
<td>(strand props)</td>
</tr>
<tr>
<td></td>
<td>$n$</td>
<td>Index of transition</td>
<td>$1 \text{ – } 1000$</td>
</tr>
<tr>
<td>Operating conditions</td>
<td>$\dot{B}$</td>
<td>Ramp rate of external magnetic field</td>
<td>$0.01 \text{ – } 1 \text{ T/s}$</td>
</tr>
<tr>
<td></td>
<td>$\varphi$</td>
<td>Field orientation</td>
<td>$0 \text{ – } 90 \text{ deg}$</td>
</tr>
<tr>
<td></td>
<td>$I_{op}$</td>
<td>Operating current</td>
<td>$0 \text{ to } I_c$</td>
</tr>
</tbody>
</table>

Find *current distribution* by solving:

- Kirchhoff current law for each node (yellow points)
- Kirchhoff voltage law for each elementary circuit
- Conservation of total operating current
- Specified boundary conditions:

1. Dirichlet type: fixed input/output currents (i.e. ‘insulated’ strands outside the modeling region)
2. Neumann type: fixed current derivatives (zero derivatives $\rightarrow$ equipotential ends, i.e. short-circuited strands)
3. Periodic type: account for symmetry in current distribution (suitable to model infinitely long transposed strands) $\rightarrow$

https://doi.org/10.1016/j.cryogenics.2018.10.003
Network modelling: setup and benchmark

$N = 10$: Length = 1 twist-pitch

Length = 1 twist-pitch / $N$

$\Rightarrow P = \sum I^2 R/l$ [W/m] $Q = \int P dt$ [J/m]

Note: intra-strand loss neglected

Using periodic BC, $L/N$-long segment is sufficient for fast analysis w/o accuracy loss.

The model is validated wrt analytical solutions.
Network modelling: power loss

Study on twist-pitch:

- Power loss at the critical value of twist-pitch \(\sim 50-70\%\) of the saturated loss

Study on critical twist-pitch:

- Typically, \(n\sim 10-20\) for fusion conductors, thus its impact on \(L_c\) some 20 – 30%

\[
L_c = \frac{8\rho I_c}{B_w} \quad \text{slab} \quad L_c \approx \frac{96\rho I_c}{NB_w} \quad \text{Rutherford}
\]

- \(L_c \sim N^{-0.5}\) scaling:
  - less strands -> higher \(L_c\) -> lower P inter-strand, but higher P intra-strand
**Network modelling: energy loss**

- Typical AC loss cycles studied at zero transport current.
- However, changing of both transport current and produced magnetic field should be analyzed for actual coil operation.

\[ P_{total} = P_{long} + P_{cross} + P_{adj} \]

\[ Q \sim \nu \] (weak excitation)
\[ Q \sim \nu^{1/n} \] (saturation)
\[ Q \sim \nu^{-1} \] (screening)

\[ I \ll I_c \] (Weak excitation: \( Q \sim \nu \))
\[ I \gg I_c \] (Saturation: \( Q \sim \nu^{1/n} \))

\[ \tau \sim \mu_0 L^2 / \rho \]
DEM0 CS graded coils: quick overview

- Target: 60 kA / 18 T / 5 K, pulsed operation

- R&D on HTS conductors for DEMO CS started this year: 4 concepts by SPC, ENEA and KIT; performance demonstration before 2024

- Operating cycle in pulsed tokamaks: advanced 18 T CS graded coils allow increasing burn time, thus higher plant availability by ~10-20% compared to 12 T baseline option.

- Coils operated independently, but detailed scenario not yet specified.

Assuming 15 T sweep for RU:

<table>
<thead>
<tr>
<th>Bdot</th>
<th>RU</th>
<th>Flat-top</th>
<th>RD</th>
</tr>
</thead>
<tbody>
<tr>
<td>MIRA</td>
<td>0.1 T/s</td>
<td>0.004 T/s</td>
<td>0.1 T/s</td>
</tr>
<tr>
<td>PROCESS</td>
<td>0.5 T/s</td>
<td>0.003 T/s</td>
<td>0.6 T/s</td>
</tr>
</tbody>
</table>

https://idm.euro-fusion.org/?uid=2M4P9J

https://dx.doi.org/10.5445/IR/100095873
DEMO CS graded coils: power loss estimates

### Calculation of saturated coupling loss in Rutherford cables

<table>
<thead>
<tr>
<th>CS module</th>
<th>CS3L</th>
<th>CS2L</th>
<th>CS1</th>
<th>CS2U</th>
<th>CS3U</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PREMAG: 14.66 T / 70.1°</td>
<td>EOF: 12.59 T / 70.1°</td>
<td>PREMAG: 18.14 T / 83.7°</td>
<td>EOF: 13.16 T / 70.3°</td>
<td>PREMAG: 15.03 T / 70.2°</td>
</tr>
<tr>
<td>Most demanding operating conditions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td># of 3.3 mm tapes (aligned along z-axis)</td>
<td>~220</td>
<td>~200</td>
<td>~140</td>
<td>~200</td>
<td>~220</td>
</tr>
<tr>
<td>P max for L ~ 1 m</td>
<td>≈ 2.2 W/m</td>
<td>≈ 1.7 W/m</td>
<td>≈ 1.1 W/m</td>
<td>≈ 1.9 W/m</td>
<td>≈ 2.4 W/m</td>
</tr>
<tr>
<td>P max for L &gt;&gt; 1 m</td>
<td>≈ 110 W/m</td>
<td>≈ 80 W/m</td>
<td>≈ 60 W/m</td>
<td>≈ 90 W/m</td>
<td>≈ 120 W/m</td>
</tr>
</tbody>
</table>

For non-transposed strands, ~100 W/m applied to strands may increase their temperature above 10 K, thus need for higher T_{cs}.

Strands transposition may reduce losses by about 10 times.

2.4 W/m + intra-strand loss = 10–20 W/m?

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**PROCESS cycle: CS3U**

Operating current \( \sim B \) during the cycle

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Intra-strand loss should be accounted...

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Intra-strand loss is negligible
Conclusion

- Analytical and numerical models are developed to analyze saturated coupling loss in Rutherford cables.

- Relatively low loss is obtained by keeping twist-pitch below its critical value expressed as $L_c \approx \sqrt{96\rho I_c / NBw}$.

- Transposition of strands is needed to avoid their overheating during fast transients in the DEMO CS coils.

- Network modelling approach is being used to simulate electromagnetic behavior of various cable layouts.

THANK YOU FOR YOUR ATTENTION!