GAM MHD spectroscopy in spherical tokamaks

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in collaboration with

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18th Technical Meeting on EPs in Magnetic Confinement Systems
Seville University, March, 2025

Supported in part by DoE contracts DE-AC02-09CH11466 and CRADA NFE-19-07769

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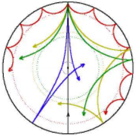
MHD spectroscopy has roots in Helioseismology (Goedbloed'93, Keppens'02)

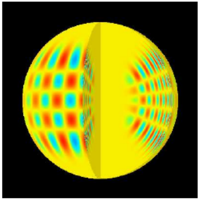
- Spectral properties of waves in solar conditions connect important plasma characteristics ⟶ $\Rightarrow$ Solar Model $T(r), \rho(r), \ldots \to$ p-, g-, and f-modes

  Spectral problem leads to:

  calculate waves, instabilities
  deduce equilibrium structures

- Goedbloed et al., PPCF'93 introduced (and termed) "MHD spectroscopy" the application of the TAE spectroscopy technique for tokamaks.





$\Rightarrow$ p sound waves, g gravitational oscillations, f surface wave

Applications to new devices are important when their diagnostics are not matured.

MHD spectroscopy in tokamaks is working

- Earlier works: Goedbloed, PPCF'93, Fasoli, PPCF'02, Sharapov, PoP'02, PRL'04, Classen,PPCF'11, Garcia-Munoz,NF11, Fredrickson, PoP'07 **Van Zeeland**, NF'06, **NF'16** ⟶
- Most often used are the rational values of q vs time,

$$q_{mn}(t) = m/n.$$

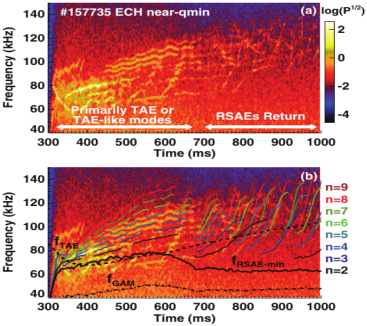
but full sweep up frequency evolution is needed.

- Typically a simplified *ad hoc* frequency dispersion is used:

$$f = nf_{\varphi rot} + mf_{\theta rot} + af_{GAM} + bf_{\nabla} + cf_{RSAE,TAE}.$$

This is due to $f_{GAM}, f_{\nabla}, f_{RSAE,TAE}$ parametric dependence complexities.

(RSAE ≡ Alfven Cascades)



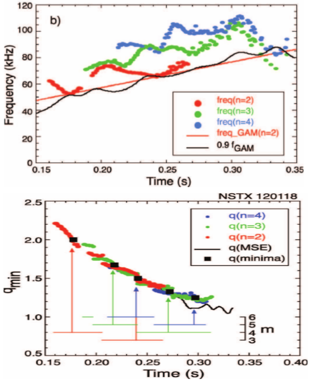
MHD spectroscopy in tokamaks is working in STs

- Fredrickson, PoP'07 inferred $\sigma_s = 0.9$ for GAM upshift in NSTX
- Selfconsistent theory prescribes (ignoring $f_{\theta rot}$):

$$f = nf_{\varphi rot} + \sigma_s\,(f,r)\,\sqrt{f^2_{GAM} + f^2_{RSAE,TAE}}$$

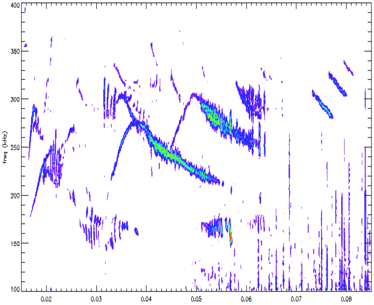
[Breizman et al., PoP'05].

- Full upswing RSAE cycle is needed.



- *Can we extend MHD spectroscopy to infer more information? Ask for less restrictions?*

RSAEs in ST40 contain information on $q_{min}(t,r)$ and EP pressure at q_{min} location



Sweeping up RSAEs observed in ST40.
Preliminaries for #11523:

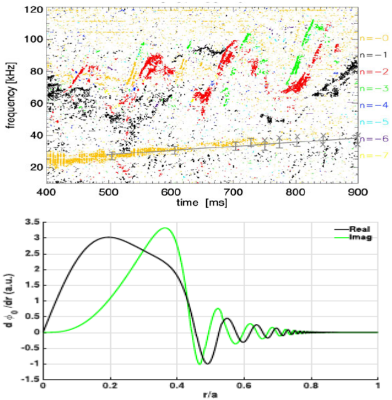
- multiple unstable n RSAEs help to infer $f_{\varphi rot}$
- iterations of $q_{min}(t)$ is useful to construct equilibrium
- find ideal MHD upshifts of Aa continuum
- EGAM frequency due to EPs correction at q_{min} is possible. Adjust gyrokinetic formulation for adiabatic index γ from its MHD value 5/3.

*We propose a **G**AM **M**HD **S**pectroscopy **(GMS)** in fusion devices, basing on rigorous theory (Fu et al.,PoP'06, Gorelenkov et al. PPCF'06)*

$$f = nf_{\varphi rot} + \sigma_s(f,r)\sqrt{f^2_{GAM} + f^2_{\nabla} + f^2_{RSAE}}.$$

Can be used for verification of plasma and EP profiles consistency.
(GMS) is expected to work even if RSAEs are seen as sweeping up only partially.

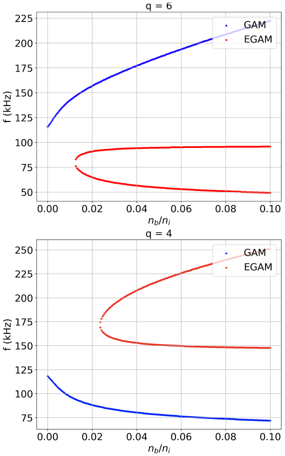
EGAMs in DIII-D agree with gyrokinetic predictions $nf_{\varphi rot}+\sigma_s(f,r)\sqrt{\underbrace{f_{GAM}^2}_{\uparrow}+f_{\nabla}^2+f_{RSAE}^2}$



F. Camilo de Souza et al., JPP'22

- Make use of earlier studies Fu, PRL'08, Qu, Hole, PPCF'10,'17. Qiu, Zonca, Chen, PST'11 & Camilo de Souza et al., JPP'22.
- apply NOVA, ideal MHD
  - $\sigma_s\left(f_{GAM}, r_{q_{min}}\right) \sim 0.7$
  - due to poloidal harmonics $m \pm 2$ and higher
- profiles are taken from DIII-D/TRANSP
- f_{GAM} is an essential function of EP distribution
- can we use it to diagnose EP component?

In ST40 what should we use for GAM upshift $nf_{\varphi rot} + \sigma_s(f,r)\sqrt{\underbrace{f_{GAM}^2}_{\uparrow} + f_\nabla^2 + f_{RSAE}^2}$?



- Winsor's PoFl'68 GAM is modified

  $\omega_{GAM,MHD} = \sqrt{\frac{\gamma p}{\rho R^2}\left(2 + \frac{1}{q^2}\right)}$.
  - include EP parameters
- GAM upshift depends on EP $n_b(\rho)/n_i$ *(Fu, PRL'08)*.
- In devices with relatively new diagnostics this technique is expected to help to infer EP n_b at q_{min}.
- The GAM/EGAM eqn:

$$\omega^2 = \sigma_s^2\omega_{GAM,mhd}^2 + \sigma_s^2\frac{n_b}{n_i}\frac{(2-\lambda_0)\,\omega^2 q^2}{(1-\lambda_0)^2}\left[16\left((2-\lambda_0)\frac{\omega^7}{\omega_{tr}^7} + 4\frac{\omega^5}{\omega_{tr}^5}\right)Z\left(\frac{\omega}{\omega_{tr}}\right),\right.$$
$$\left. -8(2-\lambda_0)\frac{\omega^6}{\omega_{tr}^6} + 4(6+\lambda_0)\frac{\omega^4}{\omega_{tr}^4} + 2(2+3\lambda_0)\frac{\omega^2}{\omega_{tr}^2} + 3(2-5\lambda_0)\right]$$

where $\omega_{tr} = v_{tb}\sqrt{1-\lambda_0}/R_0 q$, $\lambda_0 = v_\perp^2/v^2$, Z is the plasma disp. function.

$\Rightarrow$ Gyrokinetic theory is expected to work for LFM/BAAE/GAM-BAE frequency modes.

Positive upshift from pressure gradient $nf_{\varphi rot}+\sigma_s(f,r)\sqrt{f_{GAM}^2+\underbrace{f_\nabla^2}_{\uparrow}+f_{RSAE}^2}$.

and from eigenfrequency $nf_{\varphi rot}+\sigma_s(f,r)\sqrt{f_{GAM}^2+f_\nabla^2+\underbrace{f_{RSAE}^2}_{\uparrow}}$.

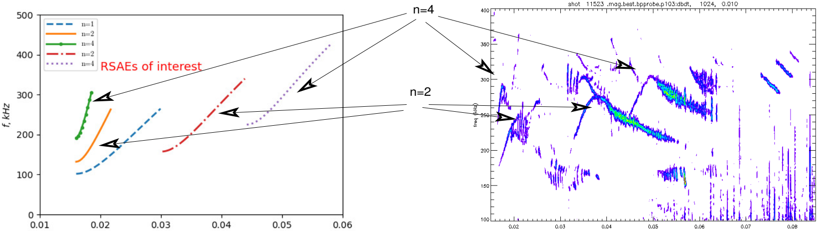
RSAE theory upshift, $\alpha=-q^2R\beta'$:

- RSAE dispersion includes *(G.-Y. Fu & H. Berk, PoP'06, N.N. Gorelenkov et al., PPCF'06)*:
  - $\sqrt{S_\omega}=\sqrt{3/4+Q_p}-1$, where $S_w=4\pi f_\nabla\left(mq_{min}^2/r_{q_{min}}^2q_{min}''\right)$, $Q_p\simeq\varepsilon\left(1-q_{min}^{-2}\right)m\alpha/r_{q_{min}}^2q_{min}''k_\parallel R\Rightarrow$ threshold condition for RSAEs to exist is $S_\omega=0$ and $Q_p=1/4$.
  - f_∇ is positive and finite in ST40.

- In a plasma frame:
  - $f_{RSAE}=\left(k_\parallel v_{A|q_{min}}+S'\right)/2\pi$,
  - S' is small.
  - Sideband's "downshift" effect is reproduced by MHD.

GAM MHD spectroscopy gives a good initial guess for ST40 plasma and EP profiles

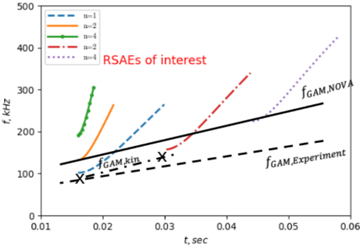
Inferred toroidal mode numbers are consistent with MHD spectroscopy!

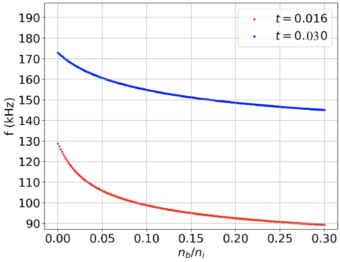


Also consistent with TRANSP simulations, but not perfect.

- Further improvements are needed:
  - The model for GAM is analytic and limited.
  - EP density $\sim 10\%$ of plasma density
- EP effects on sidebands & other branches are not included here.

GMS is sensitive to ST40 EP parameters: compare $t = 16msec$ and $t = 30msec$





q_{min} decreases as EP beta builds up.

- Further improvements are needed:
  - $t = 16msec\ q = 2,\ T_i = 0.64keV,\ T_e = 0.58keV$
  - $t = 30msec\ q = 1.5,\ T_i = 1.0keV,\ T_e = 0.9keV$
- RSAE frequency upshift is a lot due to EP effect on GAM frequency.

Summary

- Various *n* RSAEs observed in ST40 justify GAM MHD spectroscopy (GMS) applications.
  - they improve *q*-profile reconstruction
- GMS technique helps to improve the integrity of whole device simulations.
  - needs accurate EP distribution function parametrization
- GMS is expected to be useful for new devices
- We can apply GMS to earlier NSTX and forthcoming new NSTX-U experiments