Runaway Acceleration Prevention by External Whistler Waves

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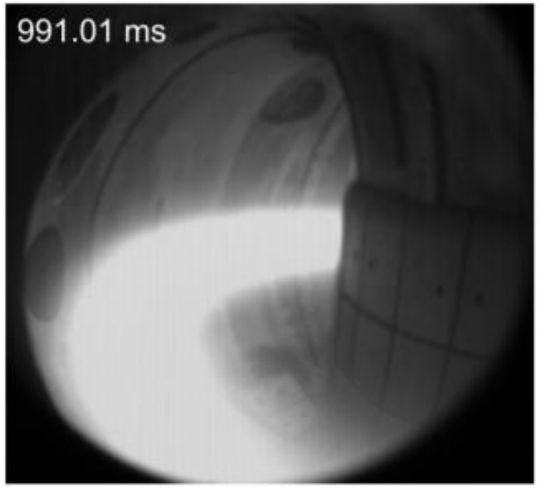
CONTROL INTEGRATED WITH DIAGNOSTICS

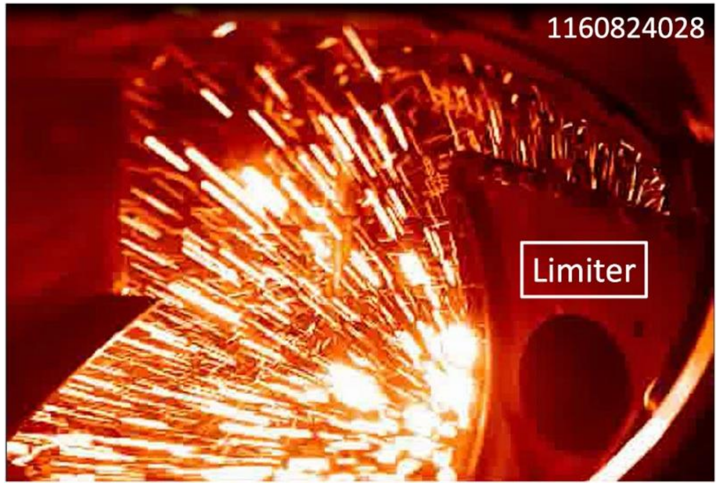
Runaway electron (RE) mitigation by resonant wave injection

RE beam (COMPASS)



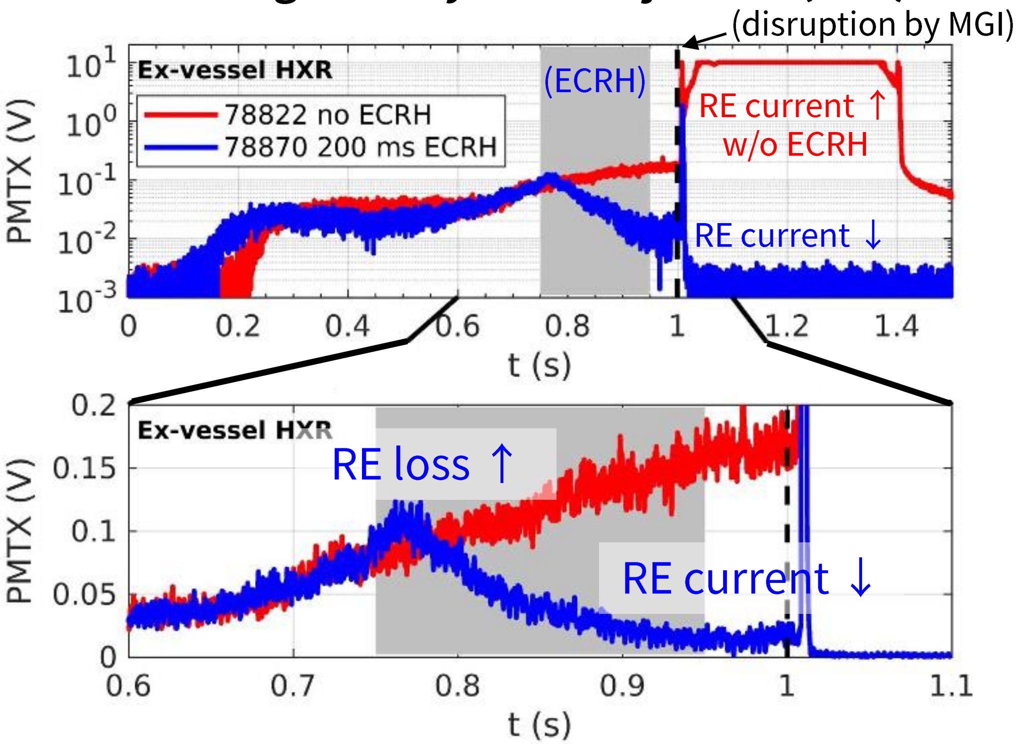
M. Vlainic et al., *J. Plasma Phys.* (2015)

Impact on tokamak by RE (Alcator C-Mod)



R. A. Tinguely et al., *Nucl. Fusion* (2018)

RE mitigation by ECRH injection (TCV)



J. Decker et al., *Nucl. Fusion* (2024)

- Mitigation of runaway electron (RE) beam at disruption is necessary for the safe operation of tokamaks.
- One of the strategies for RE mitigation is the resonant wave injection, scattering pitch angle and limiting parallel momentum of the RE. [Z. Guo et al., *PoP* (2018)] [J. Decker et al., *NF* (2024)]

Mechanism of RE mitigation by resonant wave injection – pitch angle scattering of RE

- There are many studies on particle-wave interaction and pitch angle scattering, and most studies treat broadband wave interactions and neglect single-particle dynamics.

[G. Pokol et al., *PPCF* (2008)] [A. Stahl et al., *PRL* (2015)] [C. Liu et al., *PRL* (2018)]

- However, single-particle analyses reveal that it is also a significant mechanism of the pitch angle scattering.
  - According to this analysis, electron motion in an optimal coherent whistler wave can result in instantaneous scattering of the RE within a microsecond.

[P.Bellan, *PoP* (2013)] [Y. D. Yoon et al., *PoP* (2021)]

[P.Bellan, *PoP* (2013)] [Y. D. Yoon et al., *PoP* (2021)]

Single particle motion in B_0 with wave perturbation:

$$\mathbf{B} = B_0\hat{x} + B_1\left[\left(\hat{y}\sin\left(kx-\omega t\right) + \hat{z}\cos\left(kx-\omega t\right)\right)\right],$$
$$\mathbf{E} = -\frac{\omega}{k}\hat{x}\times\mathbf{B}$$

E.o.m. in perturbation wave frame:

$$\frac{\mathrm{d}}{\mathrm{d}t'}\left(\gamma'\boldsymbol{\beta}'\right) = -\frac{e}{m_\mathrm{e}}\boldsymbol{\beta}'\times\mathbf{B}'$$

E.o.m. of pseudo-particle in pseudo-potential:

$$\frac{1}{\omega'_\mathrm{ce}}\frac{\mathrm{d}^2\xi}{\mathrm{d}^2t'} = -\frac{\partial\psi\left(\xi\right)}{\partial\xi}$$

(Introducing⋯)

Frequency mismatch parameter ξ:

$$\xi = -\frac{\gamma}{|\omega_{\mathrm{ce},0}|}\left(\omega - kv_x - \frac{|\omega_{\mathrm{ce},0}|}{\gamma}\right)$$

Pseudo-potential ψ:

$$\psi\left(\xi\right) = \frac{1}{8}\xi^4 + \frac{a_2}{2}\xi^2 - \frac{|B_1|}{|B_0|}\sqrt{1-\frac{\omega^2}{c^2k^2}}\,\xi,$$

(determined by initial conditions)

Review of single-particle analysis

E.o.m. of pseudo-particle in pseudo-potential:

$$\frac{1}{\omega'_{\mathrm{ce}}}\frac{\mathrm{d}^2\xi}{\mathrm{d}^2t'} = -\frac{\partial\psi(\xi)}{\partial\xi},$$

Frequency mismatch parameter ξ :

$$\xi = -\frac{\gamma}{|\omega_{\mathrm{ce},0}|}\left(\omega - k\, v_x - \frac{|\omega_{\mathrm{ce},0}|}{\gamma}\right)$$

(fixed parameter)

Pseudo-potential ψ :

$$\psi(\xi) = \frac{1}{8}\xi^4 + \frac{a_2}{2}\xi^2 - \frac{|B_1|}{|B_0|}\sqrt{1-\frac{\omega^2}{c^2k^2}}\,\xi,$$

(determined by initial conditions)

⇨ varying ξ ⬌ varying v_x, v_y, v_z ⬌ pitch angle scattering

Review of single-particle analysis

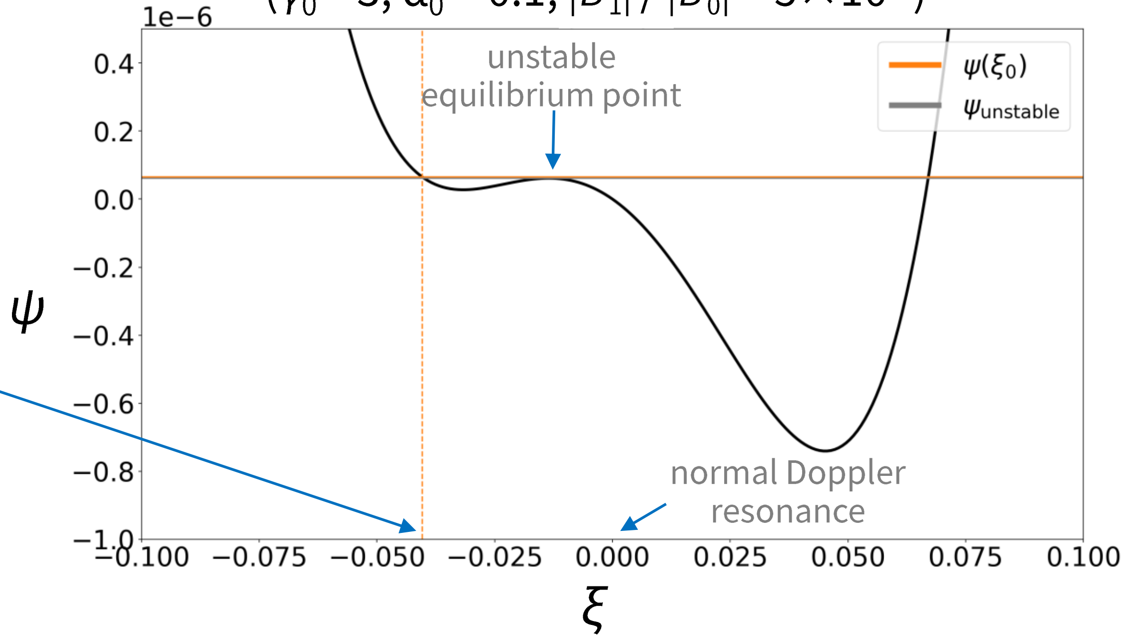
- If ψ (ξ) is a two-valley function and the total pseudo-energy matches the unstable equilibrium point, ξ will vary sensitive to its initial condition.

"deterministic pitch angle scattering"

Optimal scattering condition ξ_s:

$$\xi_{\mathrm{s}} = -\left[\left(4+\sqrt{8}\right)\frac{|B_1|^2}{|B_0|^2}\left(1-\frac{\omega^2}{c^2k^2}\right)\right]^{1/3}$$

$(\gamma_0 = 3,\ \alpha_0 = 0.1,\ |B_1|/|B_0| = 5\times10^{-4})$



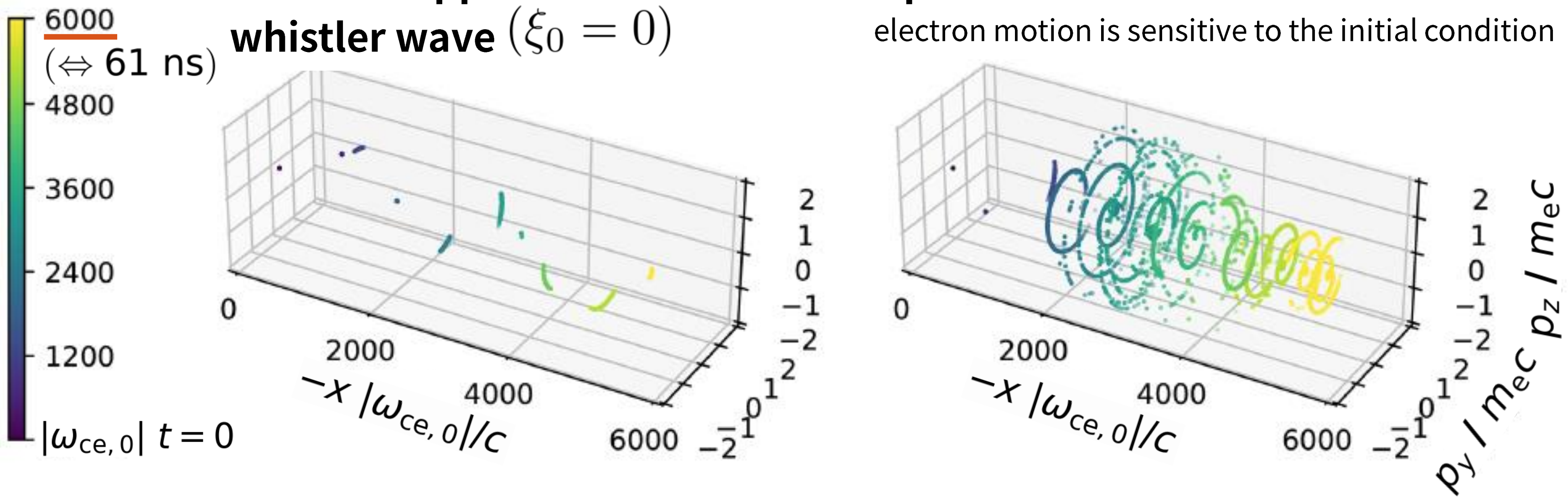
- Combining optimal scattering condition with the whistler wave dispersion, we can obtain the optimal wave condition.

Review of single-particle analysis

$$* \ \xi = -\frac{\gamma}{|\omega_{\mathrm{ce},0}|}\left(\omega - kv_x - \frac{|\omega_{\mathrm{ce},0}|}{\gamma}\right)$$

Normal Doppler resonant whistler wave $(\xi_0 = 0)$

Optimal whistler wave: $(\xi_0 = \xi_{\mathrm{s}})$
electron motion is sensitive to the initial condition



Y. D. Yoon et al., *Phys. Plasmas* (2021)

- Single-particle simulation verified that an optimal coherent whistler wave induces instantaneous scattering on the order of nanoseconds.

“Firewall” of momentum acceleration and reproduction by particle-in-cell simulation

(Idea from single-particle analysis)

Acceleration

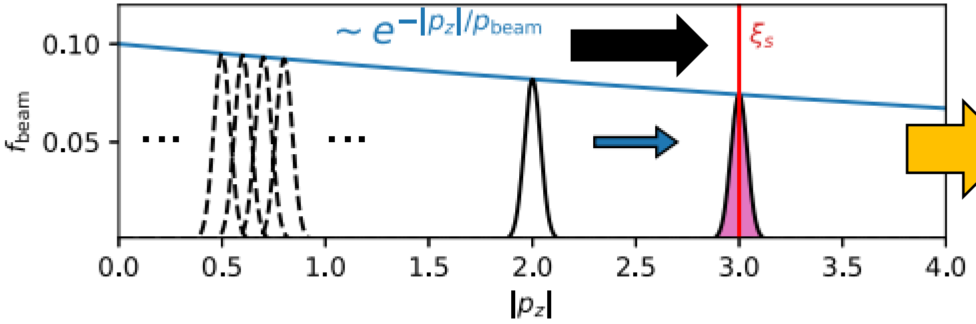


FIG. 6. An example of the proposed scattering process. A hypothetical runaway distribution (blue curve; $p_{\mathrm{beam}} = 10$) may be expressed as a sum of partial segments (dashed curves) that accelerate (black solid curve and blue arrow). Each segment is scattered when its mean $|p_z|$ satisfies $\xi_0 = \xi_s$ (magenta color).

Instantaneous pitch angle scattering

Prevent further acceleration

Y. D. Yoon et al., *Phys. Plasmas* (2021)

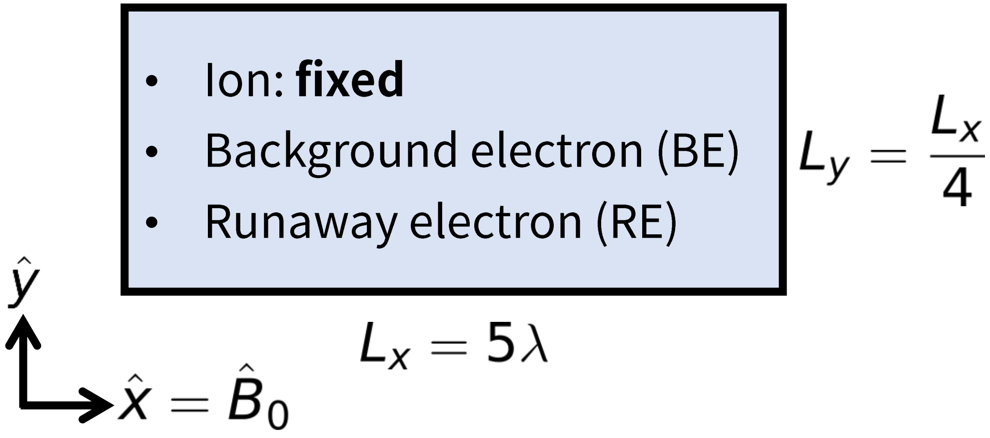
Reproduction by PIC simulation

- This study suggest that whistler waves can serve as a "firewall" in momentum space.
- Here, we construct self-consistent particle-in-cell (PIC) simulations for a beam of electrons interacting with external waves in plasma background relevant to tokamak.

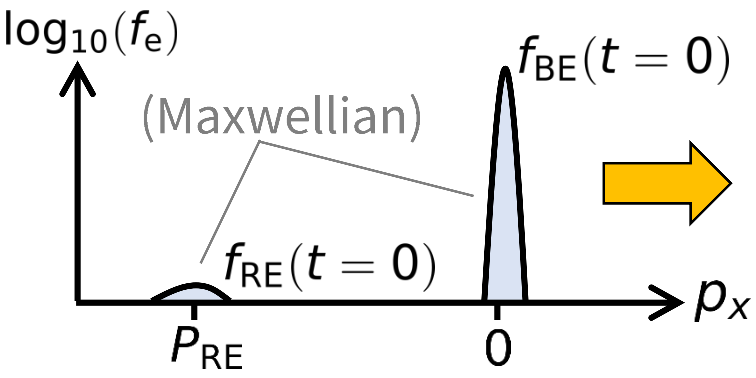
Simulation setup

J. Derouillat et al., *Comput. Phys. Commun.* (2018) : Smilei)

Simulation domain (Slab geometry)



Parallel momentum distribution



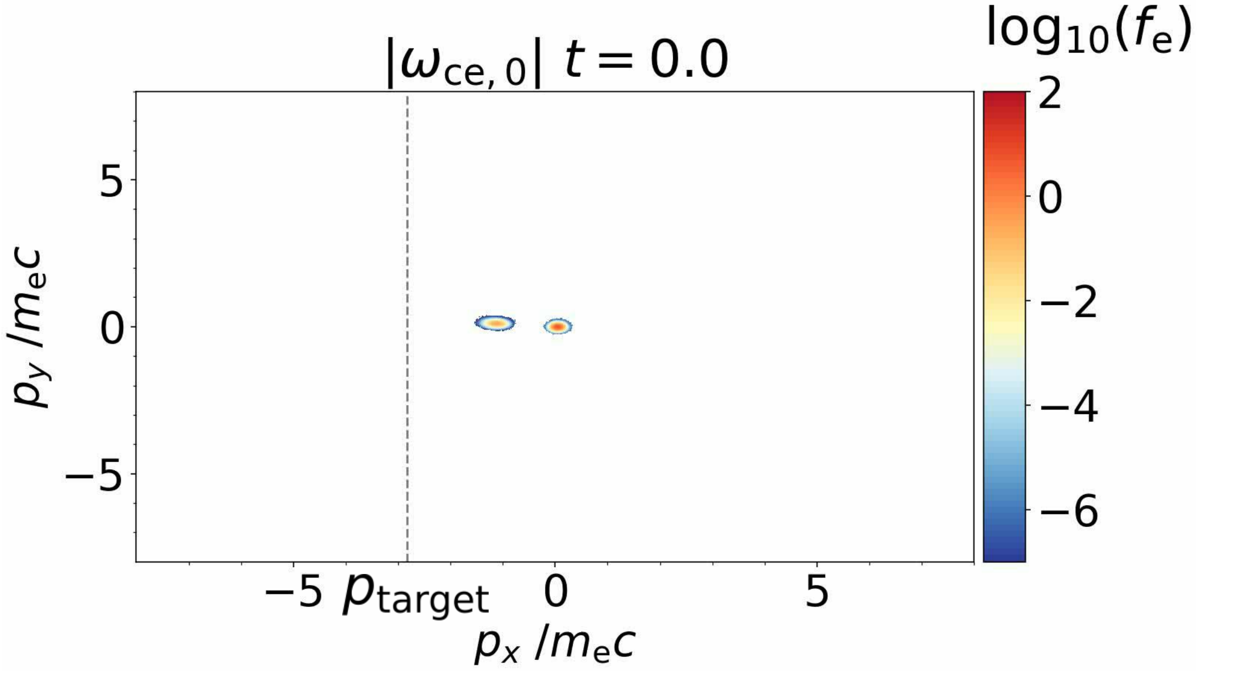
bump-on-tail distribution

Input parameters

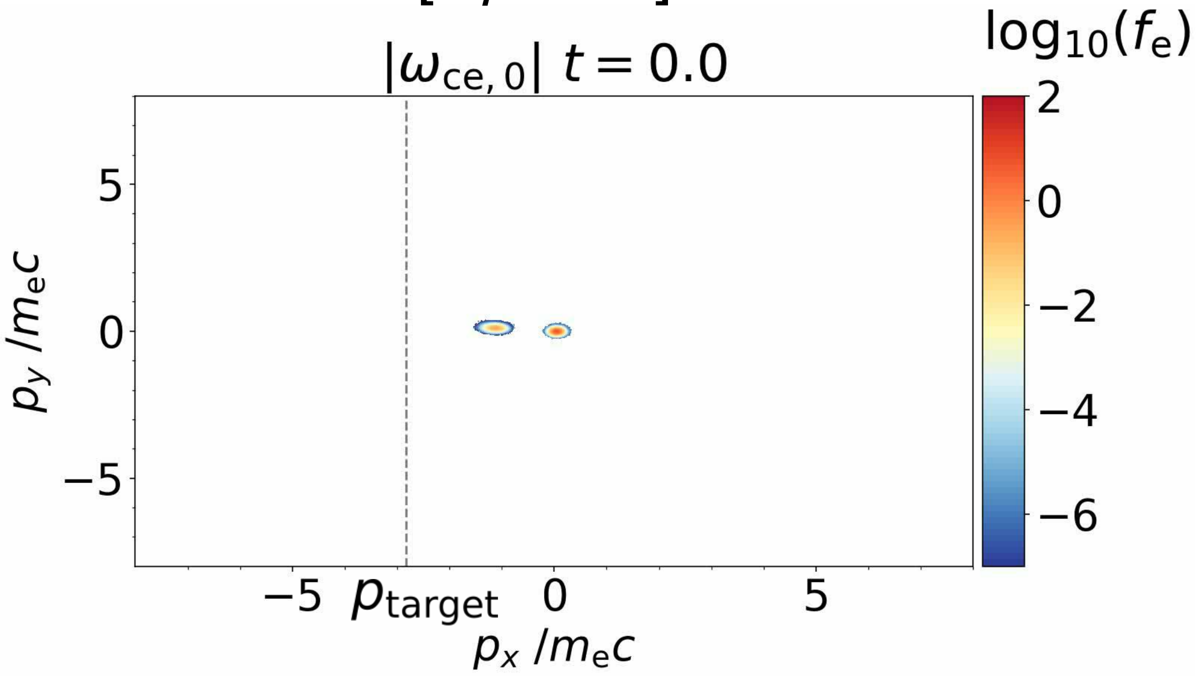
Simulation time	$t_{sim} = 5000\ /\lvert\omega_{ce,0}\rvert = 51\ \ [\mathrm{ns}]$
$\omega_{pe}\ /\lvert\omega_{ce,0}\rvert$	$\omega_{pe}\ /\lvert\omega_{ce,0}\rvert \simeq 0.29$
Boundary condition	Periodic in all direction
Initial pitch angle	$\alpha_{RE}(t = 0) = \ 0.1\ \ [\mathrm{rad}]$
Lorentz factor of RE	$\gamma_{RE}(t = 0) = 1.5$
Temperature	$T_{BE} = T_{RE} = 1\ [\mathrm{keV}]$
Population of RE	$n_{RE,0}\ /n_{e,0} = 1/16$
Wave amplitude	$B_1\ /B_0 = 5 \times 10^{-4}$
Wave frequency	$\omega \simeq 0.15\ /\ \lvert\omega_{ce,0}\rvert$
Wave number	$k = k_x \simeq 0.19\ \lvert\omega_{ce,0}\rvert/c$

$t_{\mathrm{sim}} = 5000\ /|\omega_{\mathrm{ce},0}| = 51\ \ [\mathrm{ns}]$

[w/o wave]

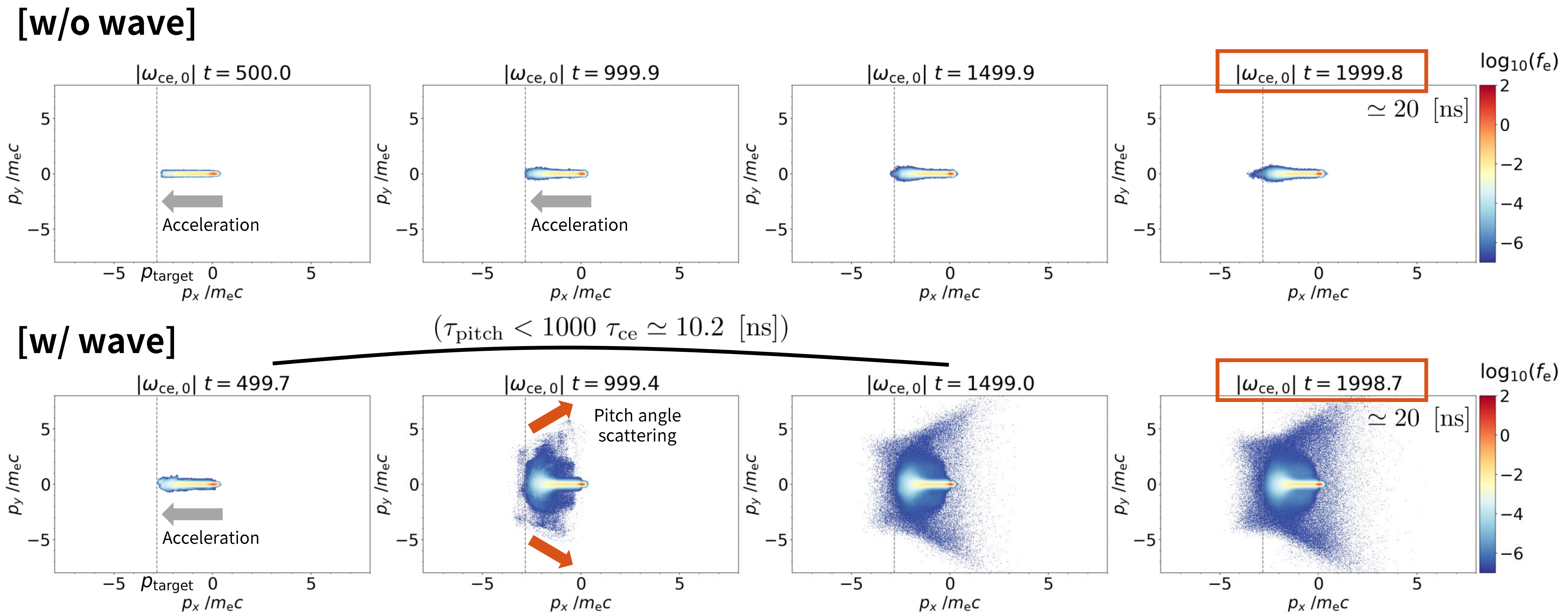


[w/ wave]



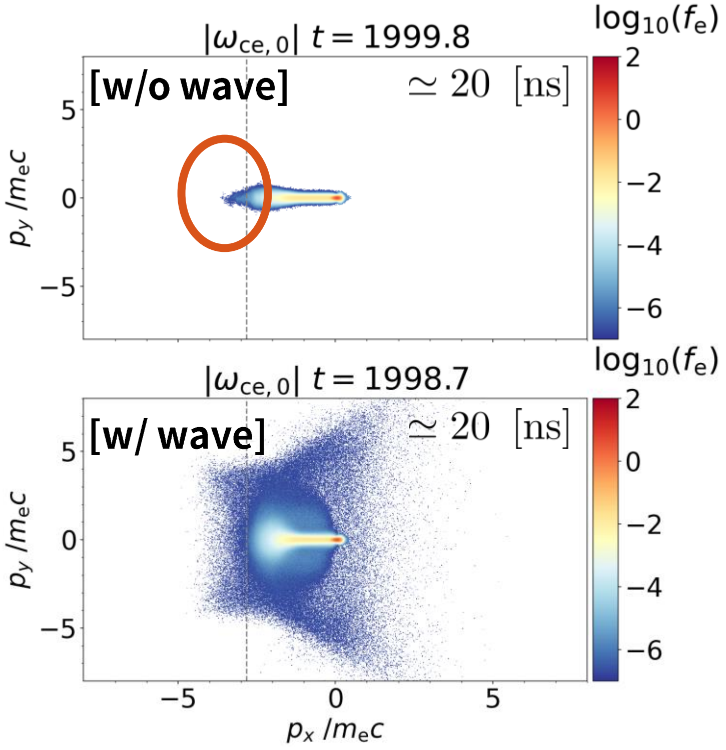
- The simulation results show that an optimal whistler wave can prevent parallel acceleration of the electrons by prompt pitch angle scattering.

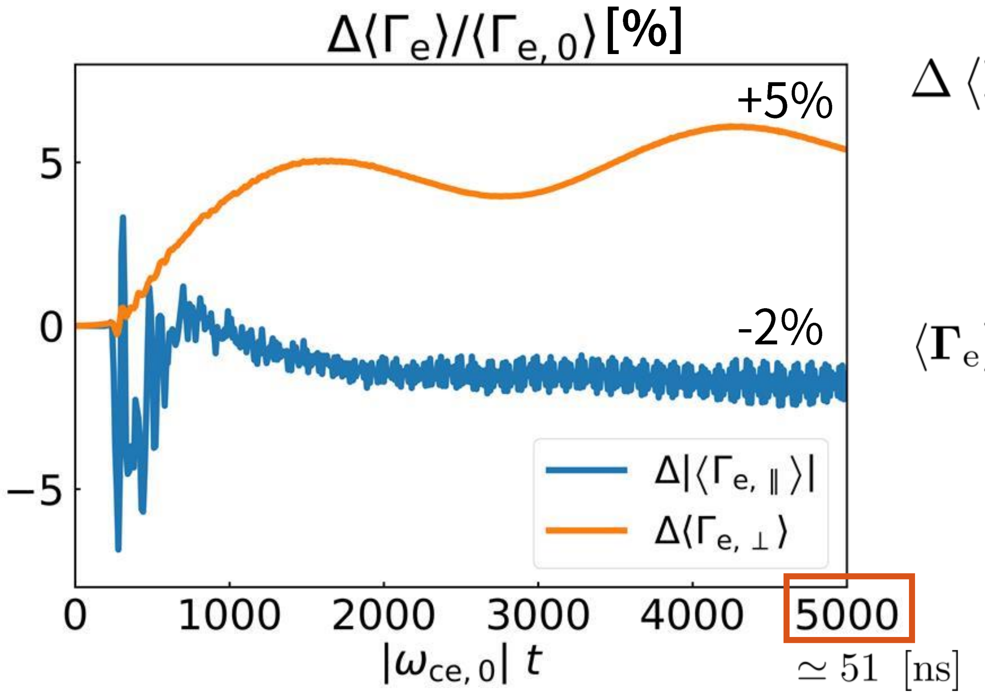
Results – Time evolution of momentum distribution function



- The simulation results show that an optimal whistler wave can prevent parallel acceleration of the electrons by prompt pitch angle scattering.

Results – (kinetic energy flux density change)





$$\Delta \langle \boldsymbol{\Gamma}_{\mathrm{e}} \rangle = \langle \boldsymbol{\Gamma}_{\mathrm{e}} \rangle \, (\mathrm{w/\ wave}) - \langle \boldsymbol{\Gamma}_{\mathrm{e}} \rangle \, (\mathrm{w/o\ wave}) \, ,$$

$$\langle \boldsymbol{\Gamma}_{\mathrm{e}} \rangle = \int m_{\mathrm{e}} c^2 \, (\gamma - 1) \, \mathbf{v} \, f_{\mathrm{e}} \mathrm{d}^3 p$$

- We compared the average kinetic energy flux of the RE with wave and without wave case.
  - The external wave decreases parallel energy flux about 2%, but increases perpendicular energy flux of the RE about 5%.
  - Note that the population accelerated over target momentum is small.
  - Electron acceleration method is under development.

Discussion – characteristic timescale estimation

$* \ \tau_{\mathrm{coll}} = 4\pi\varepsilon_0^2 m_{\mathrm{e}}^2 c^3 / n_{\mathrm{e},0} e^4 \ln\Lambda$

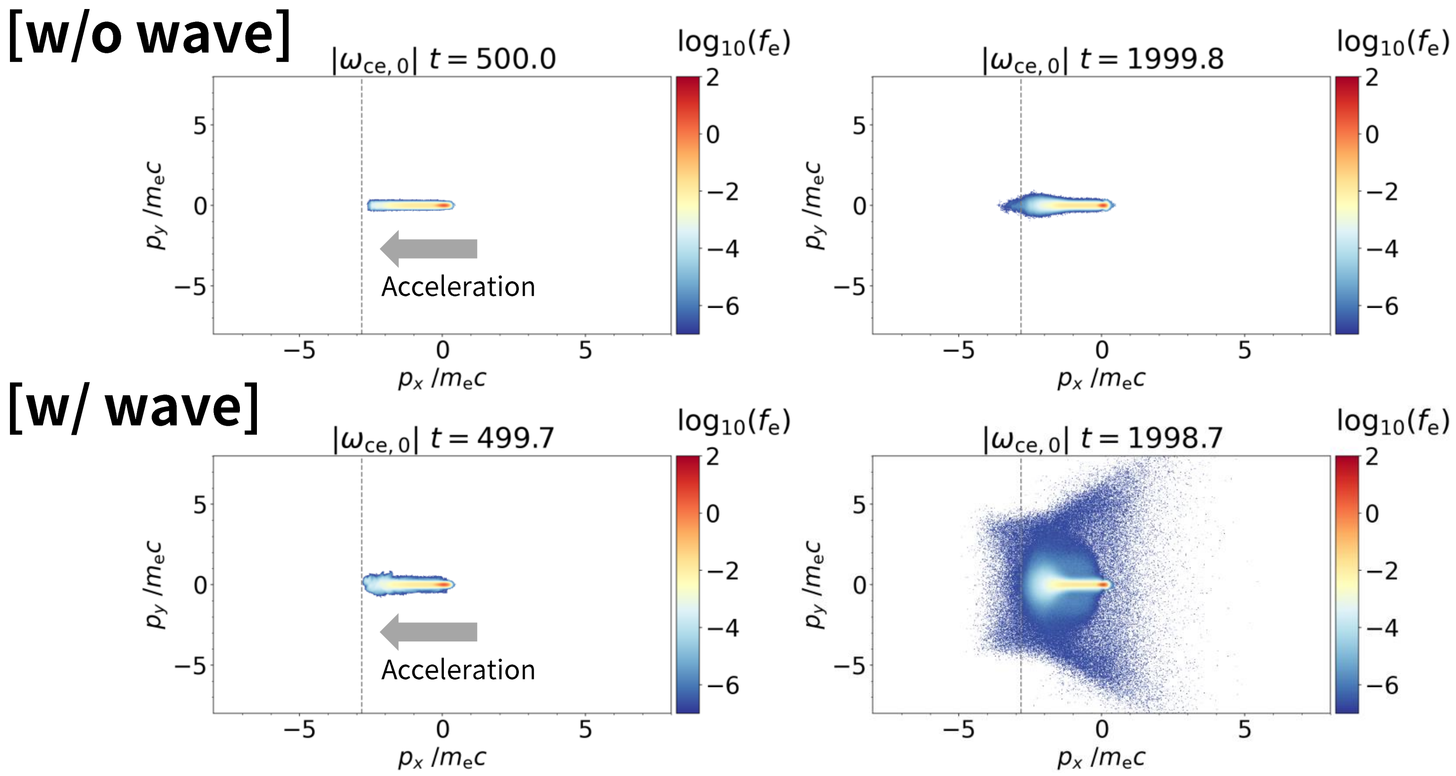
- For $n_{\mathrm{e},0} = 1 \times 10^{19}\ [\mathrm{m}^{-3}],\ T_{\mathrm{BE}} = 1\ \ [\mathrm{keV}],\ B_0 = 3.5\,[\mathrm{T}],$

Electron gyro period	$\tau_{\mathrm{ce}} = 2\pi / \lvert\omega_{\mathrm{ce},0}\rvert \simeq 10.2\ \ [\mathrm{ps}]$
Pitch angle scattering timescale	$\tau_{\mathrm{pitch}} < 1000\ \tau_{\mathrm{ce}} \simeq 10.2\ \ [\mathrm{ns}]$
Collision time of RE	$\tau_{\mathrm{coll}} \simeq 201\ \ [\mathrm{ms}]$

- The timescale of the pitch angle scattering from the simulation results is few hundreds of electron cyclotron period, or sub-microsecond.
- Such fast phenomena is new observation unexpected from the previous studies.

Summary

- We reviewed the single-particle analyses revealing that an electron in an optimal coherent whistler wave can be scattered instantaneously within a microsecond.
- Using PIC simulation, we verified the single-particle analysis and showed that the external whistler wave works as a “firewall” of the RE acceleration.



Thank you for your attention.

※ This work was supported by the NRF of Korea under grant no. RS-2022-00154676, RS-2023-00281272, and RS-2024-00409564.
※ The computational resources for the simulations are provided by the **KAIROS** supercomputing system at **KFE**.

Backup Slides

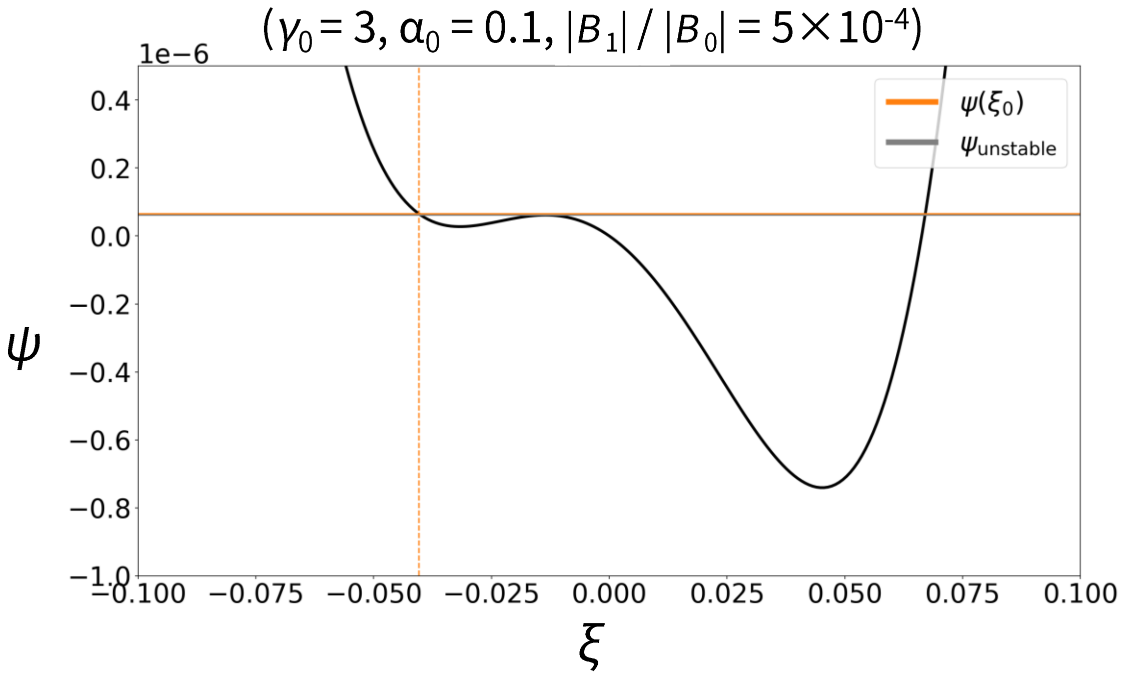
Assumptions for the two valley pseudo-potential

- The initial pitch angle of the particle is small:

$$|\beta_{x,0}| \gg |\beta_{\perp,0}|.$$

- The wave is "perturbation" :

$$|B_1| \ll |B_0|.$$



Input parameters

Simulation domain	$L_x = 5 \times 2\pi/k = 0.49\ \ [\mathrm{m}]\,,$ $L_y = L_x/4 = 0.12\ \ [\mathrm{m}]$
Simulation time	$t_{\mathrm{sim}} = 5000\ /\lvert\omega_{\mathrm{ce},0}\rvert = 51\ \ [\mathrm{ns}]$
$\omega_{\mathrm{pe}}/\lvert\omega_{\mathrm{ce},0}\rvert$	$\omega_{\mathrm{pe}}\ /\lvert\omega_{\mathrm{ce},0}\rvert \simeq 0.29$
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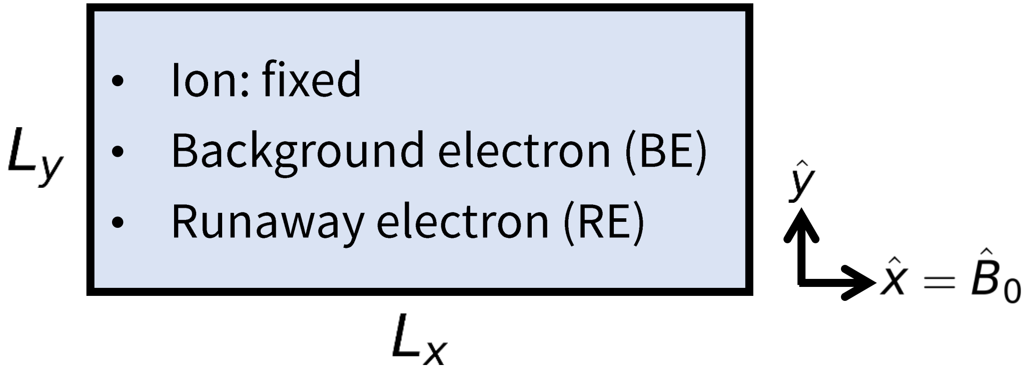
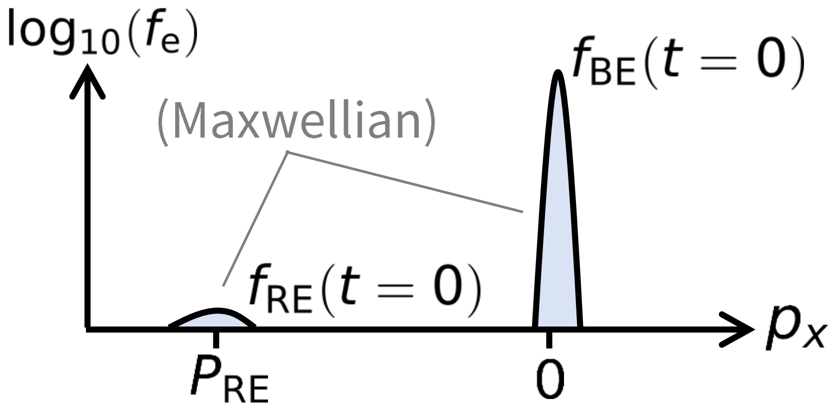
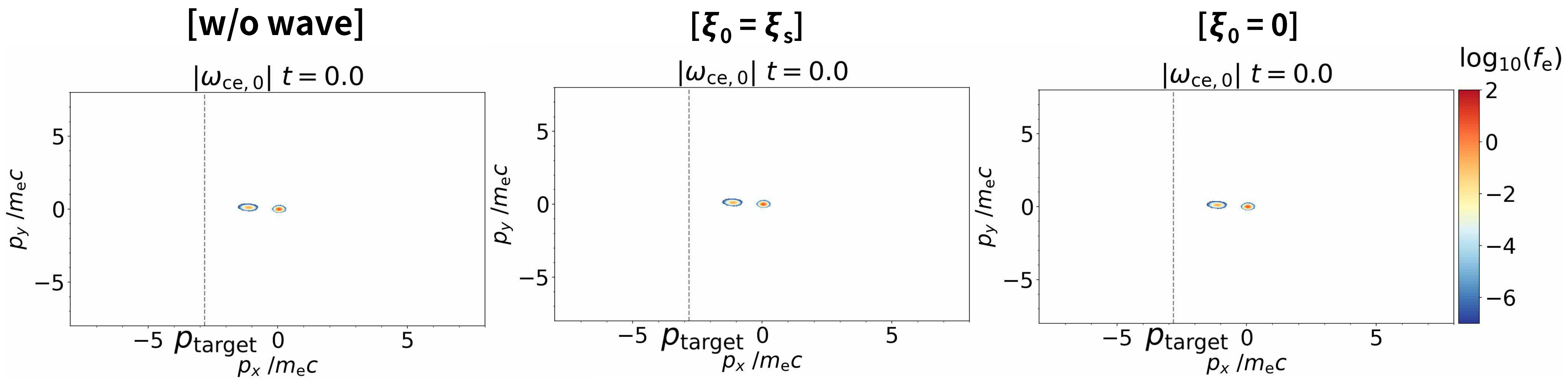
Simulation domain



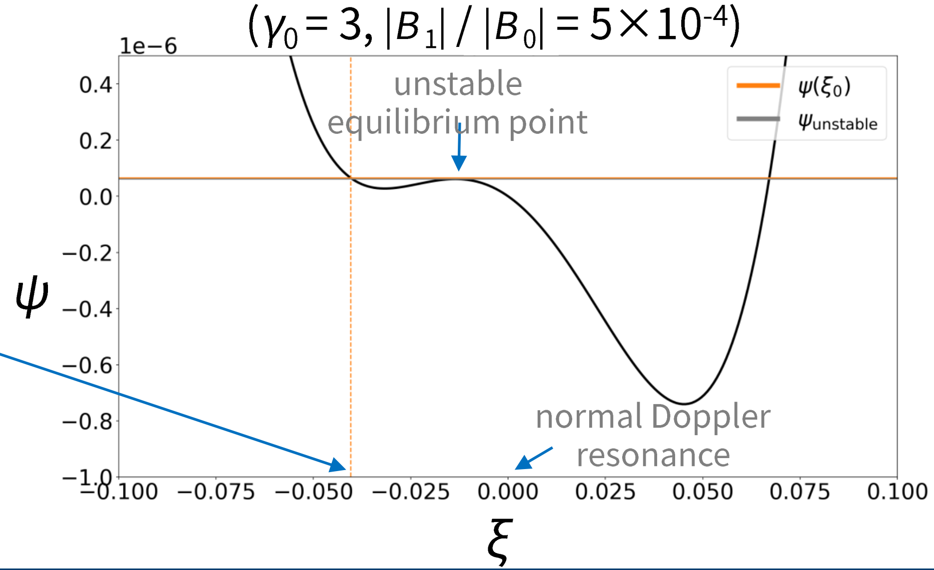
Parallel momentum distribution



Comparison – optimal case ($\xi_0 = \xi_s$) v.s. normal Doppler resonance case ($\xi_0 = 0$)



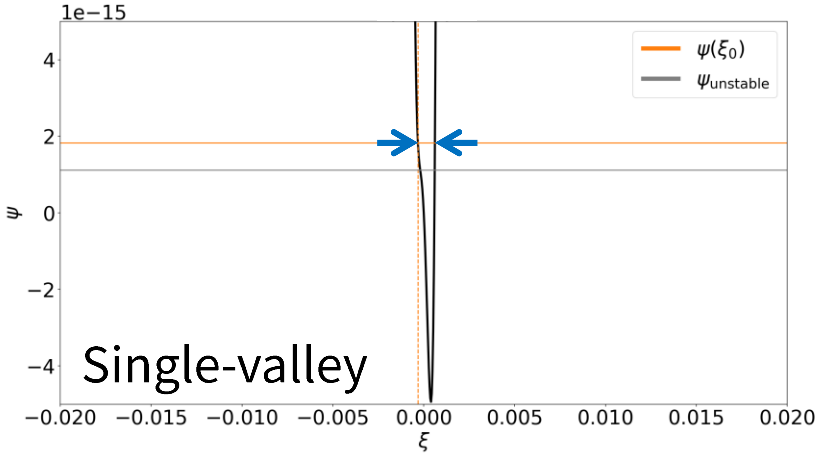
$$\xi_s = -\left[\left(4+\sqrt{8}\right)\frac{|B_1|^2}{|B_0|^2}\left(1-\frac{\omega^2}{c^2k^2}\right)\right]^{1/3}$$



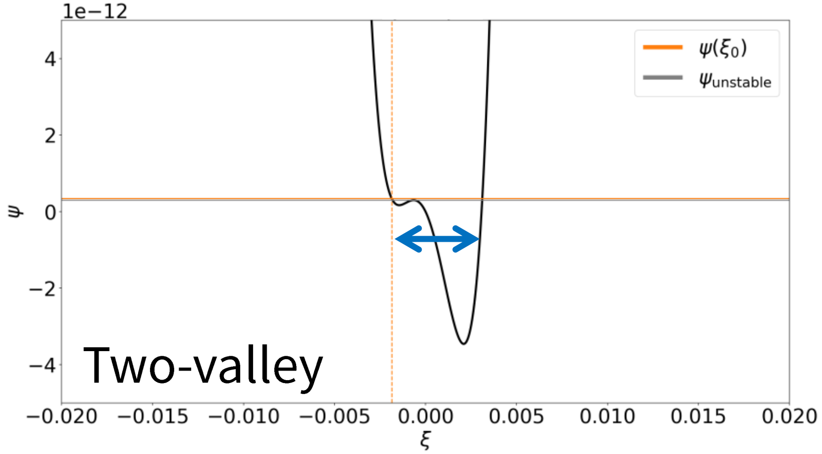
Comparison – various perturbation amplitudes

[$|B_1|/|B_0|$ =0]

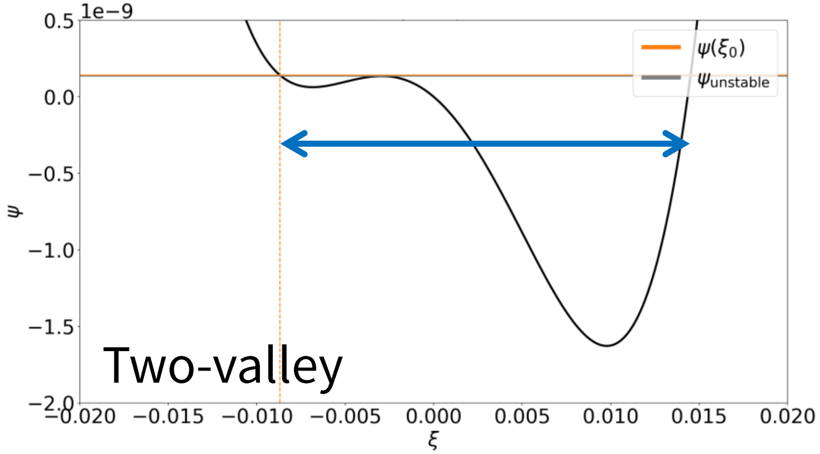
[$|B_1|/|B_0| = 5\times10^{-6}$]



[$|B_1|/|B_0| = 5\times10^{-5}$]

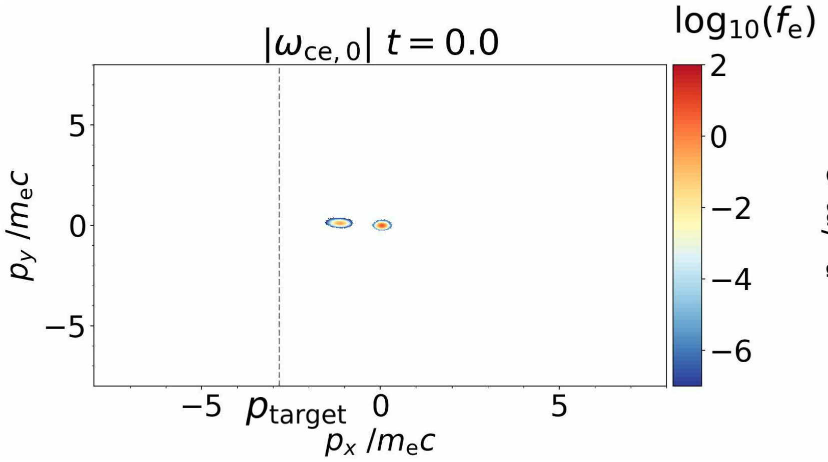


[$|B_1|/|B_0| = 5\times10^{-4}$]

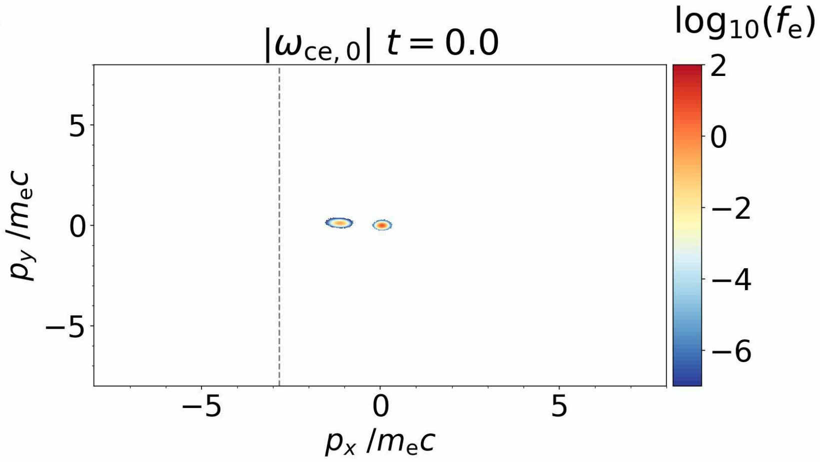


Comparison – various perturbation amplitudes

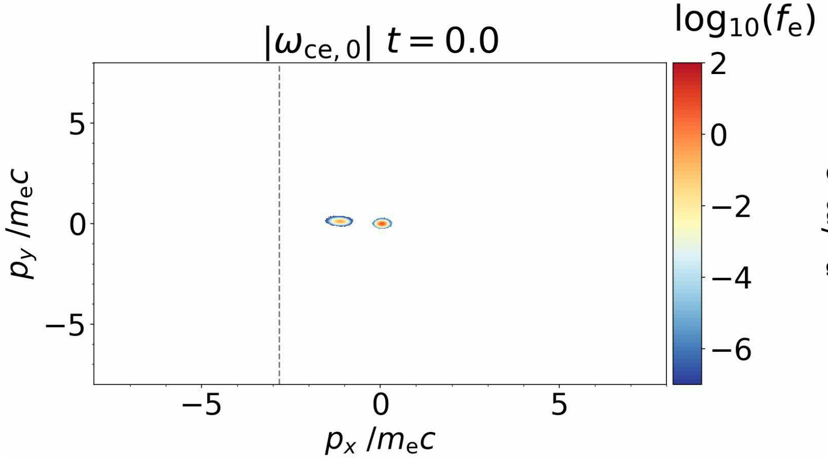
$[|B_1|/|B_0| = 0]$



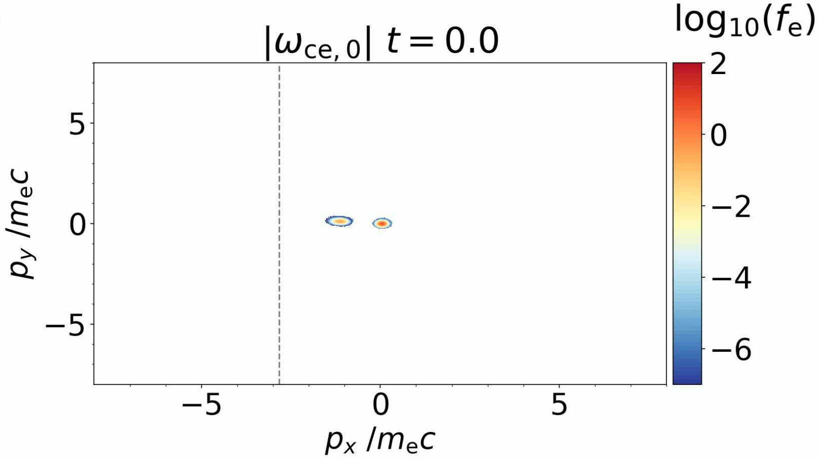
$[|B_1|/|B_0| = 5\times10^{-6}]$



$[|B_1|/|B_0| = 5\times10^{-5}]$



$[|B_1|/|B_0| = 5\times10^{-4}]$



Discussion – characteristic timescale estimation

$* \ \tau_{\mathrm{coll}} = \dfrac{4\pi\varepsilon_0^2 m_{\mathrm{e}}^2 c^3}{n_{\mathrm{e},0} e^4 \ln\Lambda}, \ \tau_{\mathrm{rad}} = \dfrac{6\pi\varepsilon_0 (m_{\mathrm{e}} c)^3}{e^4 B_0^2}, \ \tau_{\mathrm{accel}} = \dfrac{m_{\mathrm{e}} c}{e E_{\mathrm{ext}}}$

- For $n_{\mathrm{e},0} = 1 \times 10^{19} \left[\mathrm{m}^{-3}\right], \ T_{\mathrm{BE}} = 1 \ [\mathrm{keV}], \ B_0 = 3.5\,[\mathrm{T}], \ E_0 = 20\,[\mathrm{V/m}],$

Electron plasma oscillation period	$\tau_{\mathrm{pe}} = 2\pi/\omega_{\mathrm{pe}} \simeq 35.2\,[\mathrm{ps}]$
Electron gyro period	$\tau_{\mathrm{ce}} = 2\pi/\lvert\omega_{\mathrm{ce},0}\rvert \simeq 10.2 \ [\mathrm{ps}]$
Deuterium gyro period	$\tau_{\mathrm{cD}} = 2\pi/\lvert\omega_{\mathrm{cD},0}\rvert \simeq 37.5 \ [\mathrm{ns}]$
Acceleration time by external E-field	$\tau_{\mathrm{accel}} \simeq 85.2 \ [\mu\mathrm{s}]$
Collision time of RE	$\tau_{\mathrm{coll}} \simeq 201 \ [\mathrm{ms}]$
Synchrotron radiation reaction time	$\tau_{\mathrm{rad}} \simeq 421 \ [\mathrm{ms}]$