Invited talk

21st Joint Workshop on Electron Cyclotron Emission and Electron Cyclotron Resonance Heating
June 20-24, 2022, ITER Organization HQ, France

Propagation properties of electron cyclotron wave with helical wavefront in magnetized plasma

Toru I. Tsujimura
National Institute for Fusion Science, Japan

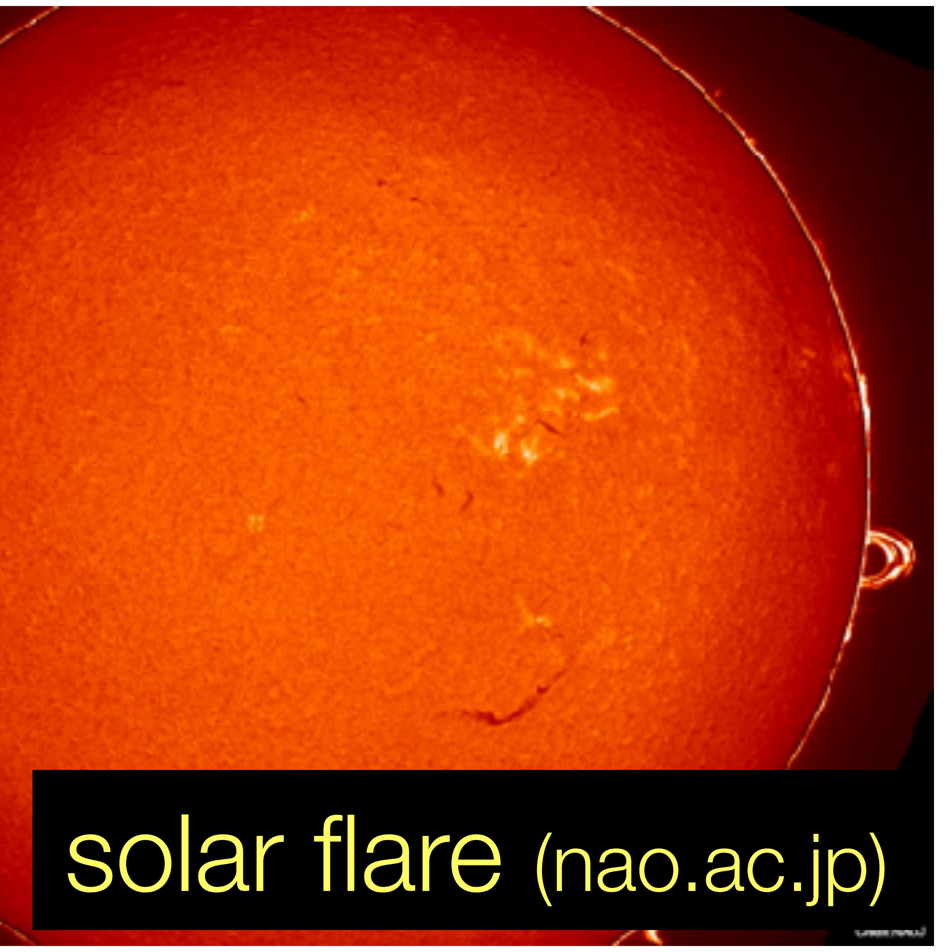
Shin Kubo
Chubu University, Japan

Reference: T. I. Tsujimura and S. Kubo, Phys. Plasmas **28**, 012502 (2021)

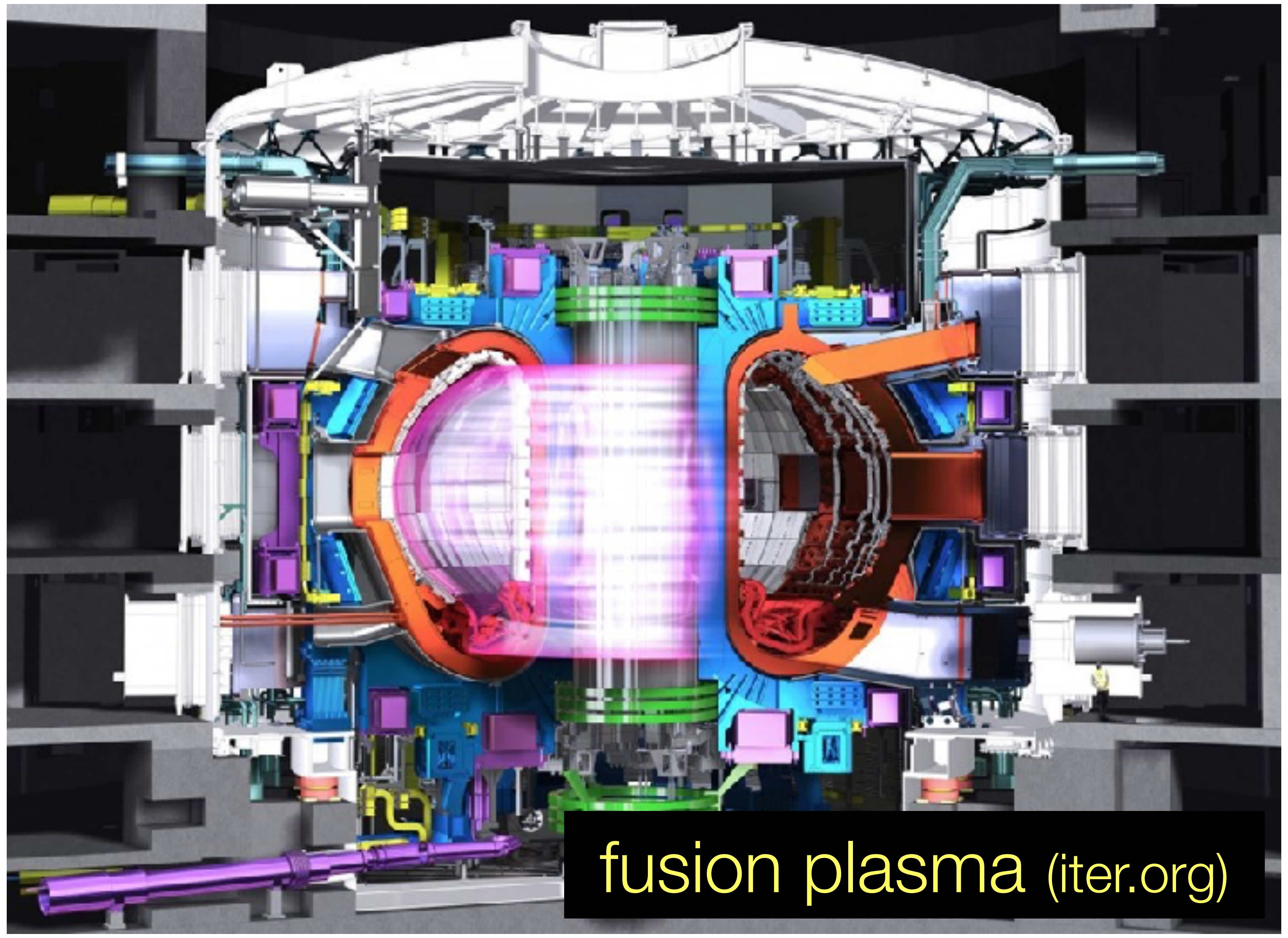
This work was supported by the NINS program for cross-disciplinary study (Grant Numbers 01311802 and 01311903), JSPS KAKENHI Grant Number JP19K14687, two grants from The Murata Science Foundation, and NIFS grants ULRR703, ULRR036, and UFEX106.

Contents

Waves in magnetized plasmas


solar flare (nao.ac.jp)


aurora (nasa.gov)


fusion plasma (iter.org)

- Various waves emitted from magnetized plasmas
  - Cyclotron waves or RF (radiofrequency) waves for heating and diagnostics in fusion plasma
- Knowledge of the propagation properties
  - Plane wave
  - **Phase**:

$$\boldsymbol{k} \cdot \boldsymbol{r} - \omega t$$

wave vector

position vector

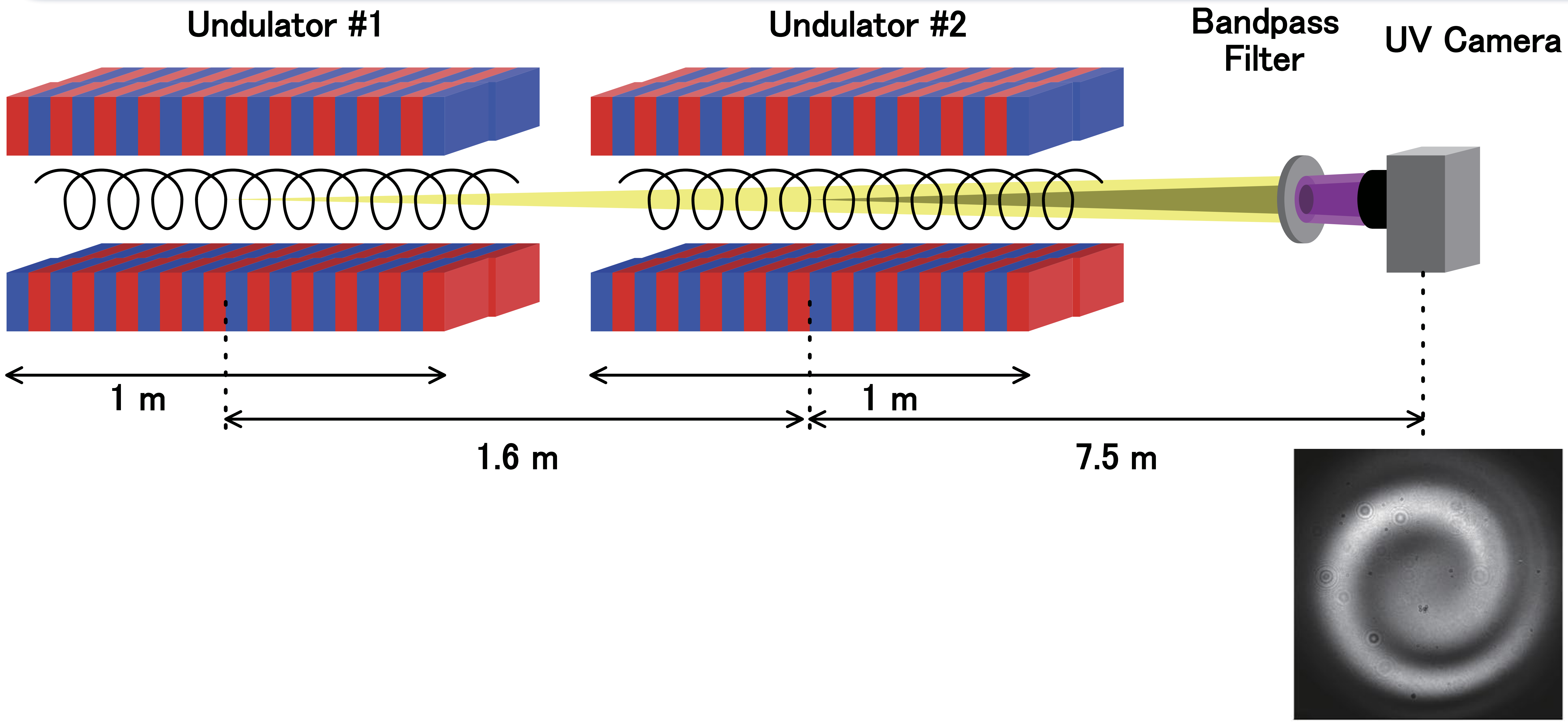
angular frequency

time

  - Advanced methods for the description of wave beams*

*I. Y. Dodin *et al*., Phys. Plasmas **26**, 072110 (2019), K. Yanagihara *et al*., Phys. Plasmas **26**, 072111 (2019).

Cyclotron motion of electrons emits twisted photons (high-harmonic optical vortices)



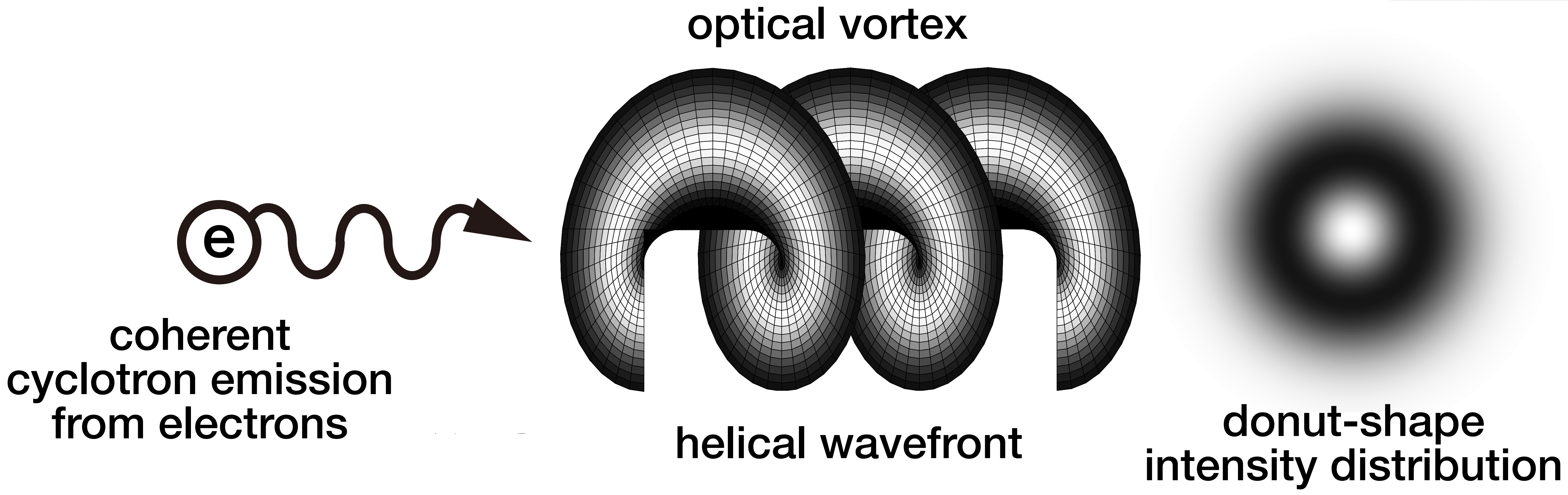
UltraViolet Synchrotron Orbital Radiation Facility

UVSOR Synchrotron Facility

at Institute for Molecular Science, Japan

Higher-harmonic synchrotron radiation from undulators in a UV range has helical wavefront.

M. Katoh *et al.*, Sci. Rep. **7**, 6130 (2017)



Numerical simulation shows coherent cyclotron emission from electrons has helical wavefront.

Y. Goto, S. Kubo, and T. I. Tsujimura, New J. Phys. **23**, 063021 (2021)

induced VORTex Electron Cyclotron Emission Device

New **iVORTECE** device is under development at NIFS.

Cyclotron motion of electrons emits twisted photons

Radiation field intensity
from an electron

Theory shows that a single free electron in circular motion emits twisted photons carrying orbital angular momentum (OAM) in addition to spin angular momentum.*

*M. Katoh *et al.*, Phy. Rev. Lett. **118**, 094801 (2017); Sci. Rep. **7**, 6130 (2017)

- Ubiquitous in nature
- **Phase**:

$$l\varphi + k_z z - \omega t$$

topological charge

azimuthal angle around the optical axis z

How an optical vortex propagates in magnetized plasma?
Beneficial for heating or diagnostics in fusion plasma?

Contents

Maxwell equations in magnetized plasma

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t},$$

$$\nabla \times \boldsymbol{B} = \mu_0 \left(\boldsymbol{j} + \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} \right)$$

Assuming a monochromatic wave in time: $\mathrm{e}^{\mp \mathrm{i}\omega t}$

Using the dielectric tensor: $\boldsymbol{\varepsilon}_r$

$$\nabla \times (\nabla \times \boldsymbol{E}) - \underset{\substack{\uparrow \\ \text{wavenumber in vacuum}}}{k_0^2} \boldsymbol{\varepsilon}_r \cdot \boldsymbol{E} = \boldsymbol{0}$$

Electric field of an optical vortex

Start with a sufficiently general ansatz for the wavefield of an optical vortex

$$\boldsymbol{E}(r,\varphi,z)=\frac{1}{2}\left\{\tilde{\boldsymbol{E}}(r,\varphi,z)\alpha r^{|l|}\exp\left[\mathrm{i}(l\varphi+\psi(r,\varphi,z)-\omega t)\right]+\mathrm{c.c.}\right\}$$

complex-valued phase function: $\psi(r,\varphi,z)=\int_0^z k_z(r,\varphi,z')\mathrm{d}z'$

z component of the local wave vector: $k_z=\partial_z\psi$

When $\tilde{E}$ and k_z are constant on space, this simple form of the optical vortex satisfies the Maxwell equations in the vacuum without any approximation.

The wavefield can have a parallel component to the propagation direction *z* even in the vacuum although a plane wave is a transverse wave without a parallel component.*

*T. Takahashi, Kogaku (Jpn. J. Opt.) **47**, 30 (2018)

T. I. Tsujimura & S. Kubo, Phys. Plasmas **28**, 012502 (2021)

Complex eikonal approximation

① Short wavelength condition

$$\epsilon = \frac{\lambda_0}{L_0} \ll 1$$

scale length

② Weakly varying amplitude

$$\frac{|\nabla \tilde{E}_\sigma|}{|\tilde{\boldsymbol{E}}|} \sim \frac{1}{L_0}, \quad \frac{|\nabla \times (\nabla \times \tilde{\boldsymbol{E}})|}{|\tilde{\boldsymbol{E}}|} \sim \frac{1}{L_0^2}$$

$$\nabla E_\sigma \approx \frac{1}{2}\left\{ \mathrm{i}\left(-\mathrm{i}\frac{|l|}{r}\nabla r + l\nabla\varphi + k_z\nabla z \right)\tilde{E}_\sigma s + \mathrm{c.c.} \right\}$$

$$s = \alpha r^{|l|} \exp\left[\mathrm{i}(l\varphi + \psi - \omega t)\right]$$

This formula suggests the **"wave vector"** of the optical vortex.

$$\begin{aligned} \boldsymbol{k} &= -\mathrm{i}\frac{|l|}{r}\nabla r + l\nabla\varphi + k_z(r,\varphi,z)\nabla z \\ &= -\mathrm{i}\frac{|l|}{r}\boldsymbol{e}_r + \frac{l}{r}\boldsymbol{e}_\varphi + k_z\boldsymbol{e}_z \end{aligned}$$

T. I. Tsujimura & S. Kubo, Phys. Plasmas **28**, 012502 (2021)

Simple approach to exclude the phase singularity in the ordering assumptions

$$r \geq r_0 > 0$$

A natural approach would be look for a solution such that

$$① \; |\boldsymbol{k}| \sim |k_z| \sim k_0 = \frac{2\pi}{\lambda_0}, \quad ② \; |\nabla k_\sigma| \sim \frac{k_0}{L_0}$$

① $|\boldsymbol{k}|^2 = \frac{2l^2}{r^2} + |k_z|^2 \leq \frac{2l^2}{r_0^2} + |k_z|^2$

$|k_z| \sim k_0 = \frac{2\pi}{\lambda_0}$ and $\frac{l^2}{r_0^2} \leq \frac{4\pi^2}{\lambda_0^2}$

$$\therefore \quad r_0 \geq \frac{|l|}{2\pi}\lambda_0$$

② $\nabla\boldsymbol{k} = \frac{\partial k_z}{\partial r}\nabla r \otimes \nabla z + \frac{\partial k_z}{\partial \varphi}\nabla\varphi \otimes \nabla z + \frac{\partial k_z}{\partial z}\nabla z \otimes \nabla z$
$+ i\frac{|l|}{r^2}\nabla r \otimes \nabla r + l\nabla\nabla\varphi - i\frac{|l|}{r}\nabla\nabla r$

$\nabla\nabla r \sim 1/r, \; \nabla\nabla\varphi \sim 1/r^2$

$\otimes$ dyadic operator

$\left|\frac{\partial k_z}{\partial r}\right| \sim \frac{k_0}{L_0}, \quad \left|\frac{\partial k_z}{\partial \varphi}\right| \sim \frac{k_0 r_0}{L_0}, \quad \left|\frac{\partial k_z}{\partial z}\right| \sim \frac{k_0}{L_0}$ and $\frac{|l|}{r_0^2} \leq \frac{k_0}{L_0}$

$$r_0^2 \geq \frac{|l|}{2\pi}\lambda_0 L_0$$

$$\boxed{r_0 = \max\left\{\frac{|l|}{2\pi}\lambda_0, \sqrt{\frac{|l|}{2\pi}\lambda_0 L_0}\right\}}$$

T. I. Tsujimura & S. Kubo, Phys. Plasmas **28**, 012502 (2021)

Limited propagation distance

$$\nabla s = i\left[\boldsymbol{k} + \underline{\int_0^z \left(\frac{\partial k_z(r,\varphi,z')}{\partial r}\nabla r + \frac{\partial k_z(r,\varphi,z')}{\partial \varphi}\nabla\varphi\right)\mathrm{d}z'}\right]s$$

$$\delta\boldsymbol{k} \sim k_0\frac{|z|}{L_0}$$

$$s = \alpha r^{|l|}\exp\left[\mathrm{i}(l\varphi + \psi - \omega t)\right]$$

$\delta\boldsymbol{k}$ can be neglected as compared to $\boldsymbol{k}$ only for a small propagation distance.

$$\boxed{|z| \ll L_0}$$

This is an *ad hoc* assumption to reduce the problem to an algebraic equation rather than a partial differential equation on the phase function.

T. I. Tsujimura & S. Kubo, Phys. Plasmas **28**, 012502 (2021)

Wave electric field and helical wavefront structure

$$\nabla \times \boldsymbol{E} = \frac{1}{2}[\nabla s \times \tilde{\boldsymbol{E}} + s\nabla \times \tilde{\boldsymbol{E}} + \text{c.c.}] \qquad s = \alpha r^{|l|} \exp[\mathrm{i}(l\varphi + \psi - \omega t)]$$

$$\boxed{\begin{aligned}\nabla \times (\nabla \times \boldsymbol{E}) = \frac{1}{2}\Bigg[&\{\boldsymbol{k} \otimes \boldsymbol{k} - (\boldsymbol{k}\cdot\boldsymbol{k})\boldsymbol{I}\}\tilde{\boldsymbol{E}}s \\ &+ O(k_0^2\epsilon) + O\left(k_0^2 \frac{|z|}{L_0}\right) + \text{c.c.}\Bigg]\end{aligned}}$$

Propagation direction “as a beam”

$$\boldsymbol{k}_z = \bar{\boldsymbol{k}} \equiv \frac{1}{2\pi}\int_0^{2\pi} \boldsymbol{k}\,\mathrm{d}\varphi = \left(\frac{1}{2\pi}\int_0^{2\pi} k_z\,\mathrm{d}\varphi\right)\boldsymbol{e}_z \equiv \bar{k}_z \boldsymbol{e}_z$$

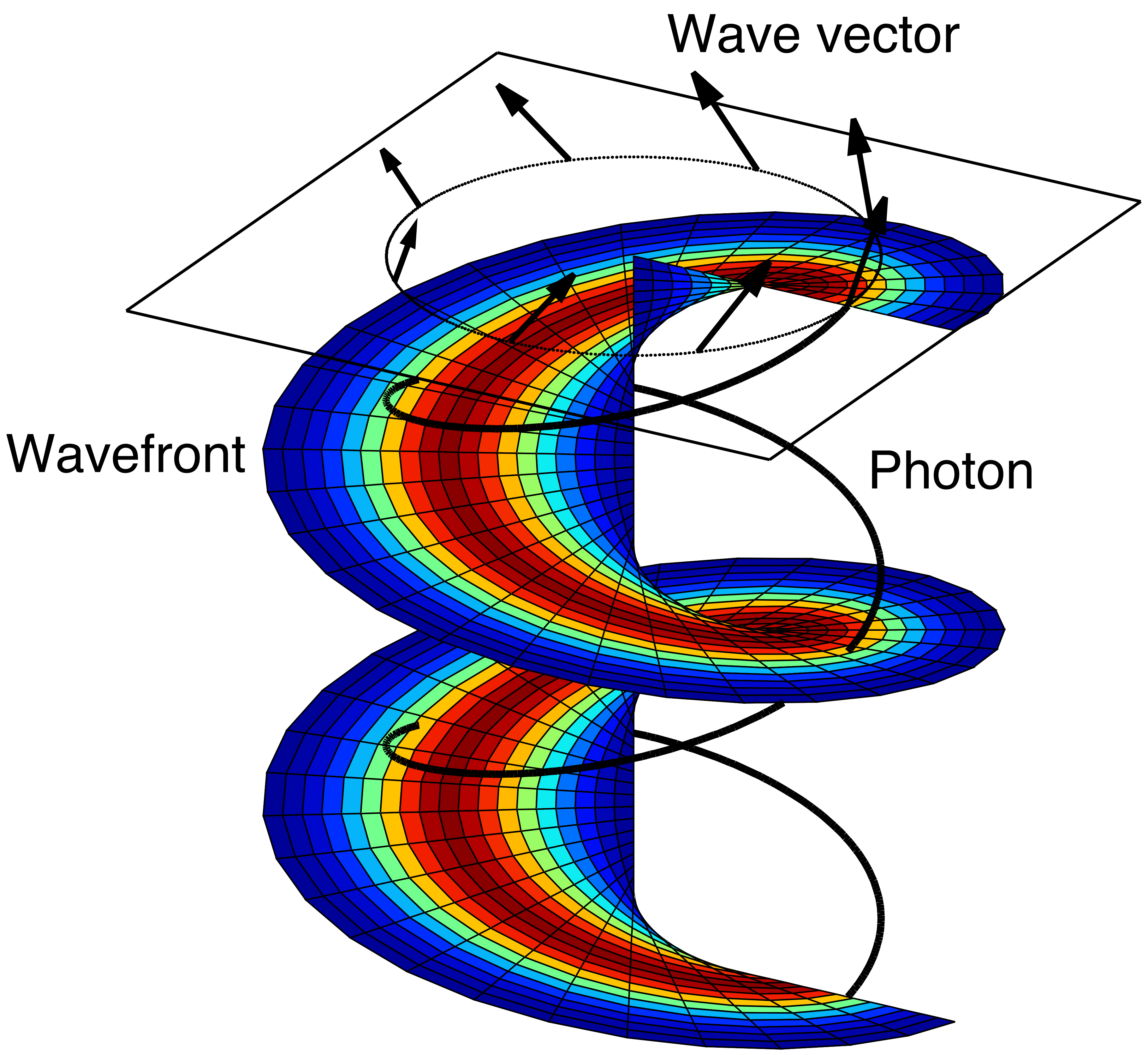
Wavefront structure in the eikonal approximation

$E_\sigma \propto \exp[\mathrm{i}S(r,\varphi,z)]$

$$S(\boldsymbol{r}) = -\mathrm{i}|l|\log r + l\varphi + \psi,$$

$$\nabla S(\boldsymbol{r}) \approx \boldsymbol{k} = -\mathrm{i}\frac{|l|}{r}\nabla r + l\nabla\varphi + k_z\nabla z,$$

$$\exp[\mathrm{i}S(\boldsymbol{r})] = r^{|l|}\exp[\mathrm{i}(l\varphi + \psi)].$$



Schematic of propagation of the optical vortex

T. I. Tsujimura & S. Kubo, Phys. Plasmas **28**, 012502 (2021)

Contents

Wave electric field redefined in the coordinate system: $\boldsymbol{B}_0 = B_0 \boldsymbol{e}_z$

Uniform and homogeneous plasma in both space and time

$$\boldsymbol{E} = \frac{1}{2}\left\{\tilde{E}\alpha(r')^{|l|}\exp\left[\mathrm{i}(l\varphi' + \psi' - \omega t)\right] + \text{c.c.}\right\}$$

$$r' = \sqrt{(x')^2 + (y')^2}, \quad \varphi' = \tan^{-1}\frac{y'}{x'},$$

$$\psi' = \int_0^{z'} k_{z'}(r', \varphi', z'')\mathrm{d}z'',$$

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\boldsymbol{k} = \boldsymbol{k}_{r'} + \boldsymbol{k}_{\varphi'} + \boldsymbol{k}_{z'},$$

$$\boldsymbol{k}_{r'} = -\mathrm{i}\frac{|l|}{r'}(\cos\varphi'\cos\theta,\ \sin\varphi',\ -\cos\varphi'\sin\theta),$$

$$\boldsymbol{k}_{\varphi'} = \frac{l}{r'}(-\sin\varphi'\cos\theta,\ \cos\varphi',\ \sin\varphi'\sin\theta),$$

$$\boldsymbol{k}_{z'} = (k_{z'}\sin\theta,\ 0,\ k_{z'}\cos\theta),$$

$$\bar{k}_{z'} = \frac{1}{2\pi}\int_0^{2\pi} k_{z'}\mathrm{d}\varphi'.$$

$$\boldsymbol{\varepsilon}_r(\omega) = \begin{pmatrix} S(\omega) & -\mathrm{i}D(\omega) & 0 \\ \mathrm{i}D(\omega) & S(\omega) & 0 \\ 0 & 0 & P(\omega) \end{pmatrix}$$

$$\boldsymbol{\varepsilon}_r^*(-\omega) = \boldsymbol{\varepsilon}_r(\omega)$$

Stix notations

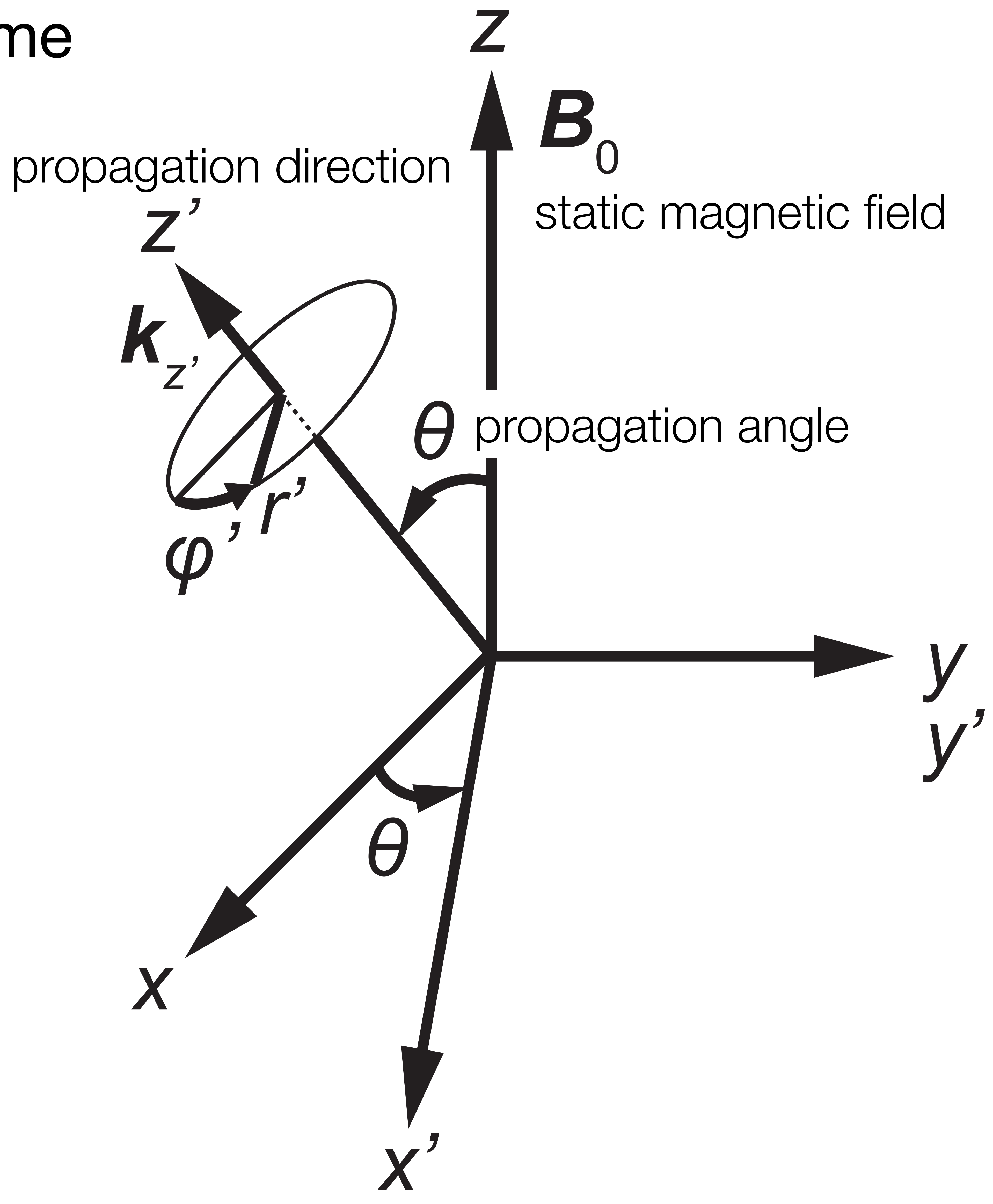
$$P(\omega) = 1 - \frac{\omega_{\mathrm{pe}}^2}{\omega^2}$$

$$R(\omega) = 1 - \frac{\omega_{\mathrm{pe}}^2}{\omega(\omega - \omega_{\mathrm{ce}})}$$

$$L(\omega) = 1 - \frac{\omega_{\mathrm{pe}}^2}{\omega(\omega + \omega_{\mathrm{ce}})}$$

$$S(\omega) = \frac{1}{2}(R + L) = 1 - \frac{\omega_{\mathrm{pe}}^2}{\omega^2 - \omega_{\mathrm{ce}}^2}$$

$$D(\omega) = \frac{1}{2}(R - L) = -\frac{\omega_{\mathrm{ce}}\omega_{\mathrm{pe}}^2}{\omega(\omega^2 - \omega_{\mathrm{ce}}^2)}$$



Wave (telegraphic) equation

$$\boxed{\frac{1}{2}\left[\boldsymbol{\Lambda}(\omega,\boldsymbol{k})\cdot\tilde{\boldsymbol{E}}s' + \boldsymbol{\Lambda}^{*}(-\omega,\boldsymbol{k})\cdot\tilde{\boldsymbol{E}}^{*}(s')^{*}\right] = \boldsymbol{0}}$$

$$s' \equiv \alpha(r')^{|l|}\exp\left[\mathrm{i}(l\varphi' + \psi' - \omega t)\right]$$

cold-plasma tensor evaluated at the complex wave vector ***k***

$$\underbrace{\boldsymbol{\Lambda}(\omega,\boldsymbol{k})}_{\text{not Hermitian}} \equiv \underbrace{\boldsymbol{k}\otimes\boldsymbol{k} - (\boldsymbol{k}\cdot\boldsymbol{k})\boldsymbol{I}}_{\text{symmetric}} + k_0^2\boldsymbol{\varepsilon}_r(\omega)$$

→ different propagation properties in comparison to a plane wave

not simply account for dispersion, but include diffraction

$$\Lambda_{mn}(\omega,\boldsymbol{k}) = \Lambda_{0,mn}(\omega,\boldsymbol{k}_R) - \frac{1}{2}(\boldsymbol{k}_I\otimes\boldsymbol{k}_I) : \frac{\partial^2\Lambda_{0,mn}(\omega,\boldsymbol{k}_R)}{\partial\boldsymbol{k}_R\partial\boldsymbol{k}_R}$$

The symbol ":" denotes the double dot product for two dyadics.

$$\boxed{+\,\mathrm{i}\boldsymbol{k}_I\cdot\frac{\partial\Lambda_{0,mn}(\omega,\boldsymbol{k}_R)}{\partial\boldsymbol{k}_R} + \mathrm{i}k_0^2\varepsilon^a_{r,mn}},$$

inhomogeneous wave

$$\boldsymbol{\Lambda}_0(\omega,\boldsymbol{k}_R) \equiv \boldsymbol{k}_R\otimes\boldsymbol{k}_R - (\boldsymbol{k}_R\cdot\boldsymbol{k}_R)\boldsymbol{I} + k_0^2\boldsymbol{\varepsilon}_r^h$$

$\boldsymbol{k}_R = \mathrm{Re}\boldsymbol{k}$

$\boldsymbol{k}_I = \mathrm{Im}\boldsymbol{k}$

T. I. Tsujimura & S. Kubo, Phys. Plasmas **28**, 012502 (2021)

Electromagnetic wave energy is conserved when propagating away from EC resonances

The Poynting vector of a monochromatic wave with complex ***n***

$$\boldsymbol{n} = (c/\omega)\boldsymbol{k}$$

$$\boldsymbol{S} = \frac{1}{\mu_0}\overline{\boldsymbol{E}\times\boldsymbol{B}}$$

the second harmonic oscillating terms are annihilated by the time average

$$\approx \frac{1}{4c\mu_0}\left\{|\tilde{\boldsymbol{E}}|^2(\boldsymbol{n}+\boldsymbol{n}^*) - (\tilde{\boldsymbol{E}}^*\cdot\boldsymbol{n})\tilde{\boldsymbol{E}} - (\tilde{\boldsymbol{E}}\cdot\boldsymbol{n}^*)\tilde{\boldsymbol{E}}^*\right\}|\alpha|^2(r')^{2|l|}\mathrm{e}^{-2\mathrm{Im}\psi'}$$

Divergence of the Poynting vector gives the source or the sink of the wave energy.

$$\nabla\cdot\boldsymbol{S} \approx -k_0^2\frac{|\alpha|^2(r')^{2|l|}\mathrm{e}^{-2\mathrm{Im}\psi'}}{2\mu_0\omega}\tilde{\boldsymbol{E}}^*\cdot\underline{\boldsymbol{\varepsilon}_r^a}\cdot\tilde{\boldsymbol{E}} = 0$$

loss-less medium

$$\boldsymbol{\varepsilon}_r^a = \boldsymbol{0}$$

The wave energy is conserved when *ε*$_r$ is Hermitian.

This energy conservation is satisfied even if ***n*** is complex due to the helical wavefront structure.

T. I. Tsujimura & S. Kubo, Phys. Plasmas **28**, 012502 (2021)

Contents

Parallel propagation: $\theta = 0$ & $k_z \,//\, B_0$

Solvability condition

$$\det[\boldsymbol{n} \otimes \boldsymbol{n} - (\boldsymbol{n} \cdot \boldsymbol{n})\boldsymbol{I} + \boldsymbol{\varepsilon}_r] = 0$$

$$\det\begin{pmatrix} S - n_z^2 - n_l^2 & -\mathrm{i}D - \mathrm{i}\,\mathrm{sgn}(l) n_l^2 & -\mathrm{i} n_l n_z \\ \mathrm{i}D - \mathrm{i}\,\mathrm{sgn}(l) n_l^2 & S - n_z^2 + n_l^2 & \mathrm{sgn}(l) n_l n_z \\ -\mathrm{i} n_l n_z & \mathrm{sgn}(l) n_l n_z & P \end{pmatrix} = 0$$

$$n_l \equiv \frac{c}{\omega} \frac{|l|}{r' \mathrm{e}^{\mathrm{i}\,\mathrm{sgn}(l)\varphi'}}, \quad r' = r = \sqrt{x^2 + y^2}, \quad \varphi' = \varphi = \tan^{-1}\frac{y}{x}$$

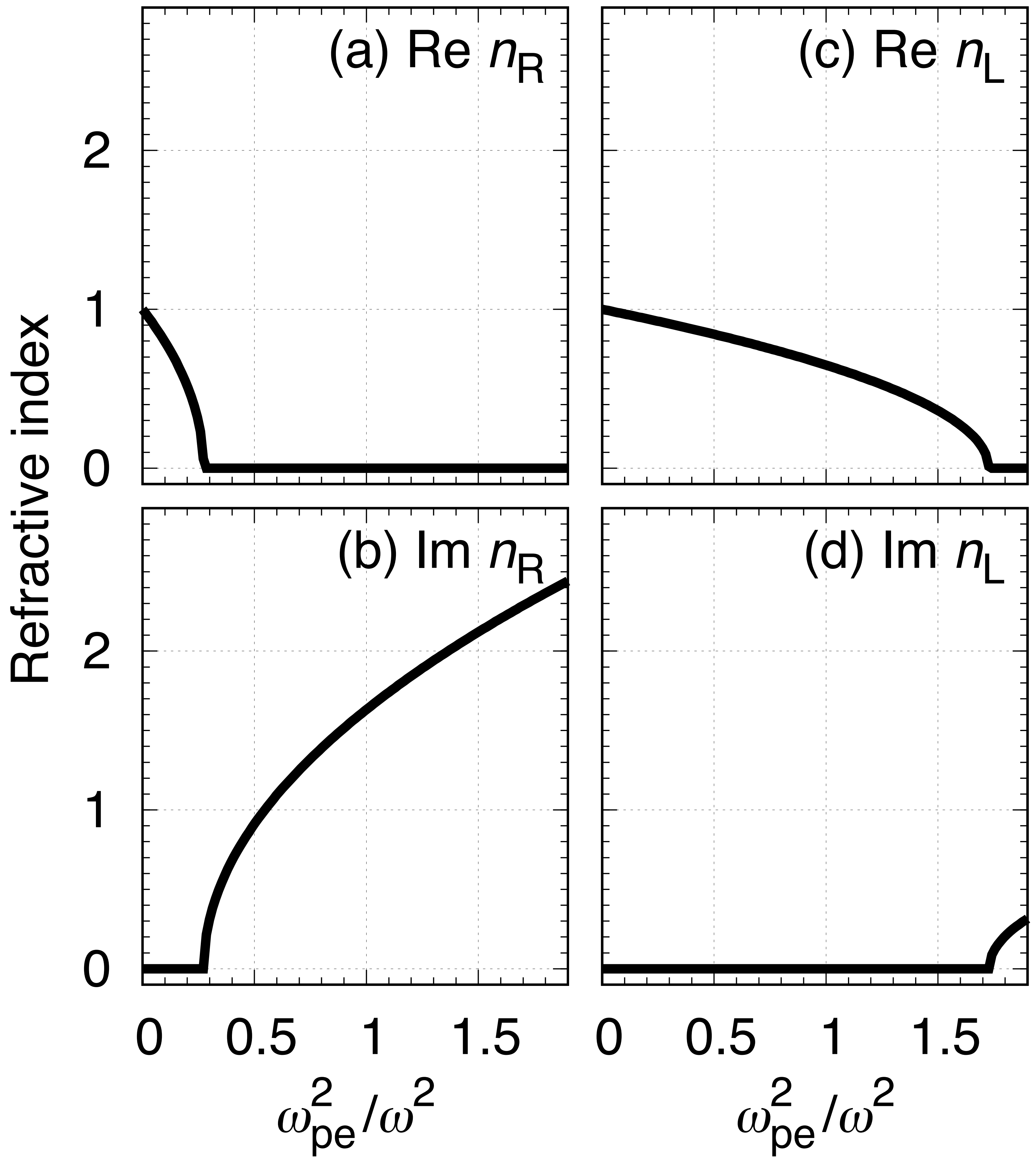
Refractive index $n_{z'} = n_z$

$$n_z^2 = R\,(\equiv S + D), \quad L\,(\equiv S - D)$$

right-handed (R) circularly polarized wave
left-handed (L) circularly polarized wave

Same as a plane wave

ω_{ce}/ω = 0.73 (f = 77 GHz, B_0 = 2 T)



T. I. Tsujimura & S. Kubo, Phys. Plasmas **28**, 012502 (2021)

Electric field polarizations are different and expressed in 3D

“vortex” R mode

$$\tilde{\boldsymbol{E}}_\mathrm{R} = [1,\ \mathrm{i},\ 0]\tilde{E}_x \quad (l \geq 0),$$

Pure R polarization

$$\tilde{\boldsymbol{E}}_\mathrm{R} = \left[1,\ \mathrm{i}\frac{PD + n_l^2(P - n_\mathrm{R}^2)}{PD - n_l^2(P - n_\mathrm{R}^2)},\quad 2\mathrm{i}\frac{n_l n_\mathrm{R} D}{PD - n_l^2(P - n_\mathrm{R}^2)}\right]\tilde{E}_x \quad (l < 0)$$

not R polarization due to a parallel component

“vortex” L mode

$$\tilde{\boldsymbol{E}}_\mathrm{L} = \left[1,\ -\mathrm{i}\frac{PD - n_l^2(P - n_\mathrm{L}^2)}{PD + n_l^2(P - n_\mathrm{L}^2)},\quad 2\mathrm{i}\frac{n_l n_\mathrm{L} D}{PD + n_l^2(P - n_\mathrm{L}^2)}\right]\tilde{E}_x \quad (l \geq 0)$$

not L polarization due to a parallel component

$$\tilde{\boldsymbol{E}}_\mathrm{L} = [1,\ -\mathrm{i},\ 0]\tilde{E}_x \quad (l < 0).$$

Pure L polarization

$$\nabla \cdot \boldsymbol{D} \approx \frac{1}{2}\left[\mathrm{i}\boldsymbol{k} \cdot \left\{\varepsilon_0 \boldsymbol{\varepsilon}_r(\omega) \cdot \tilde{\boldsymbol{E}} s\right\} - \mathrm{i}\boldsymbol{k}^* \cdot \left\{\varepsilon_0 \boldsymbol{\varepsilon}_r^*(-\omega) \cdot \tilde{\boldsymbol{E}}^* s^*\right\}\right] = 0$$

satisfied

$$\tilde{\boldsymbol{E}}_\mathrm{R} \cdot \tilde{\boldsymbol{E}}_\mathrm{L}^* \neq 0$$

not orthogonal

T. I. Tsujimura & S. Kubo, Phys. Plasmas **28**, 012502 (2021)

Contents

Perpendicular propagation: $\theta = \pi/2$ & $\boldsymbol{k}_x \perp \boldsymbol{B}_0$

Solvability condition

$$\det\begin{pmatrix} S & -\mathrm{i}D + \mathrm{sgn}(l)n_l n_x & \mathrm{i}n_l n_x \\ \mathrm{i}D + \mathrm{sgn}(l)n_l n_x & S - n_x^2 + n_l^2 & \mathrm{i}\,\mathrm{sgn}(l)n_l^2 \\ \mathrm{i}n_l n_x & \mathrm{i}\,\mathrm{sgn}(l)n_l^2 & P - n_x^2 - n_l^2 \end{pmatrix} = 0$$

$$n_l \equiv \frac{c}{\omega}\frac{|l|}{r' e^{\mathrm{i}\,\mathrm{sgn}(l)\varphi'}}, \quad r' = \sqrt{y^2 + z^2}, \quad \varphi' = \tan^{-1}\frac{y}{-z}$$

Refractive index $n_{z'} = n_x$

$$n_x^4 + \alpha n_x^2 + \beta = 0,$$

$$\alpha \equiv -\left(P + \frac{RL}{S}\right) \underline{- n_l^2\left(\frac{P}{S} - 1\right)},$$

$$\beta \equiv \frac{PRL}{S} \underline{+ n_l^2\left(P - \frac{RL}{S}\right)},$$

$P - RL/S = P/S - 1$

$$\therefore n_x^2 = \frac{1}{2}\left(-\alpha \pm \sqrt{\alpha^2 - 4\beta}\right),$$

Relations of electric field components to calculate the polarization

$$\tilde{E}_x = \frac{1}{S}\{\mathrm{i}D - \mathrm{sgn}(l)n_l n_\sigma\}\tilde{E}_y - \mathrm{i}\frac{n_l n_\sigma}{S}\tilde{E}_z,$$

$$\tilde{E}_z = \frac{(D^2 + n_l^2 n_\sigma^2) - S(S - n_\sigma^2 + n_l^2)}{n_l\{Dn_\sigma - \mathrm{i}\,\mathrm{sgn}(l)n_l n_\sigma^2 + \mathrm{i}\,\mathrm{sgn}(l)n_l S\}}\tilde{E}_y,$$

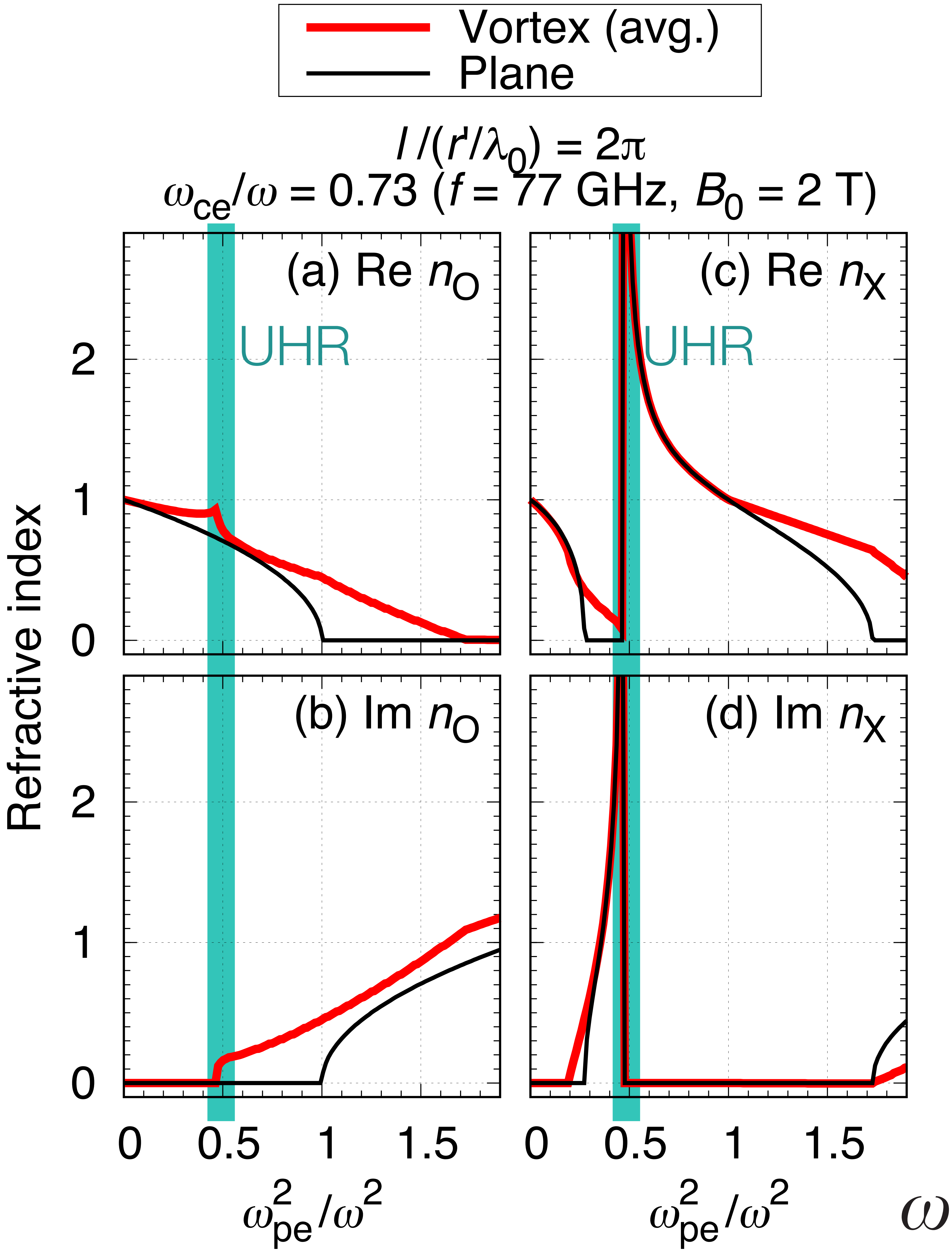
$$(\sigma = \mathrm{O}, \mathrm{X}),$$

- **The terms on n_l are additions in a plane wave.**
  - “vortex” O (ordinary) mode
  - “vortex” X (extraordinary) mode
  - noticeable when *l*/*r*’ is large

- **Modulated with the azimuthal angle ϕ’**
  - ▶ started with k_z a function of *r* and ϕ

T. I. Tsujimura & S. Kubo, Phys. Plasmas **28**, 012502 (2021)

Refractive indices of “vortex” O and X modes in the ideal limit ($l/r' = k_0$)

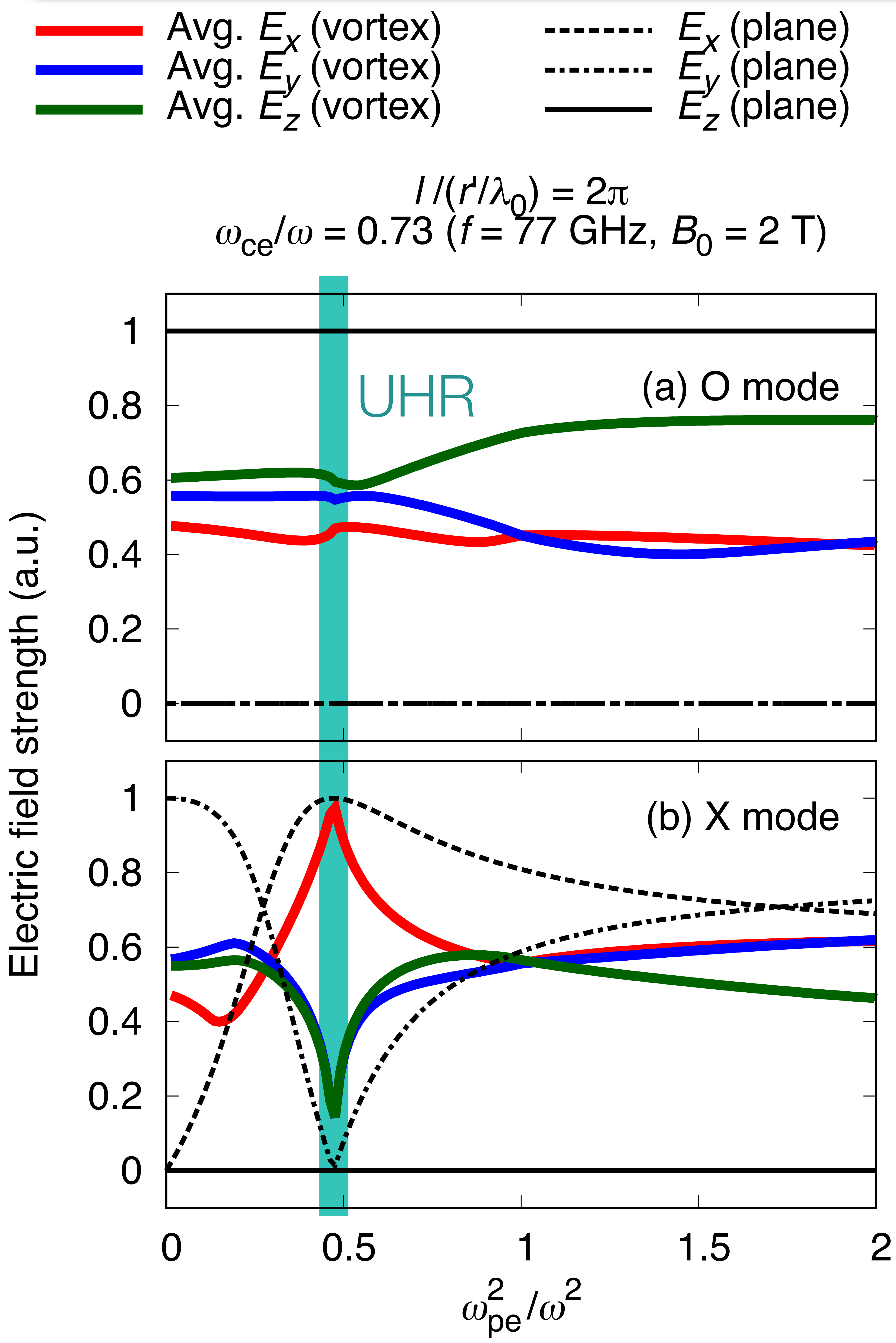


- **Both refractive indices deviate from those in plane wave.**
  - strongly modulated with ϕ
- The “vortex” O mode is **influenced by** the upper hybrid resonance (**UHR**) from the lower n_e side.
  - affected by $\boldsymbol{B}_0$
- The “vortex” X mode experiences UHR from the higher n_e side and can **propagate in the higher n_e region.**

$$\omega_{pe}^2/\omega^2 \propto n_e$$

T. I. Tsujimura & S. Kubo, Phys. Plasmas **28**, 012502 (2021)

Electric fields of “vortex” O and X modes in the ideal limit ($l/r' = k_0$)



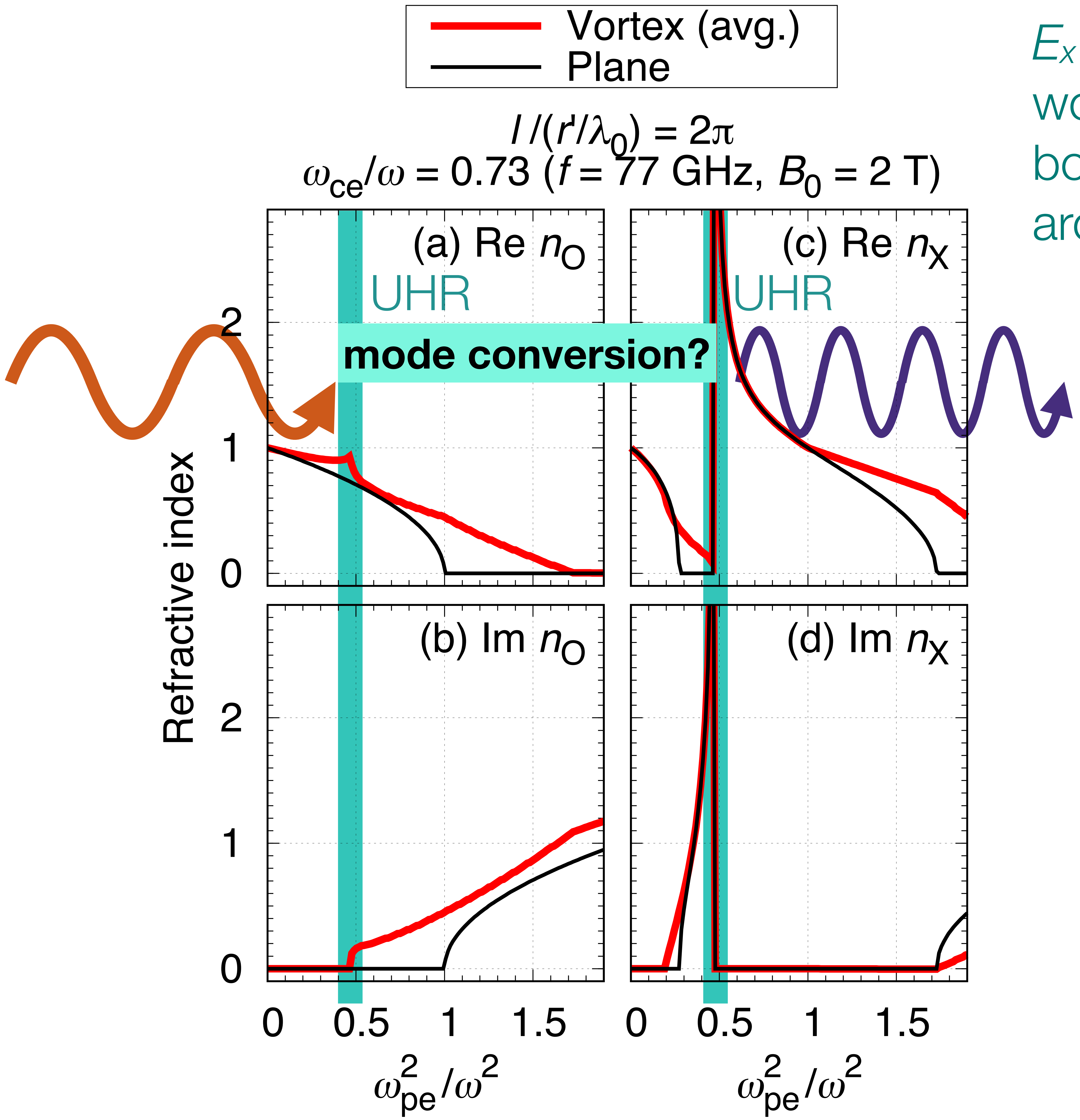
- **The *E* fields entirely deviate from those in a plane wave.**
- “Vortex” O mode
  - not pure linear polarization directed in $\boldsymbol{B}_0$
  - has a component parallel to the propagation direction
- “Vortex” X mode
  - has a component parallel to $\boldsymbol{B}_0$

- Expectation that the ***E*** fields of both modes become similar to each other around UHR when l/r' can be much larger.
  - $l/r' > k_0$ is not accessible in this theory.
  - accessible when the ordering assumptions can be relaxed to treat smaller r_0 and a PDE for a complex phase function can be solved → future work

T. I. Tsujimura & S. Kubo, Phys. Plasmas **28**, 012502 (2021)

Does more-advanced theory suggest direct mode conversion?

vortex O mode
excited from
the low-*B* &
low-n_e side



E_x parallel to the propagation direction would become the largest component for both modes with a high wavenumber around UHR.

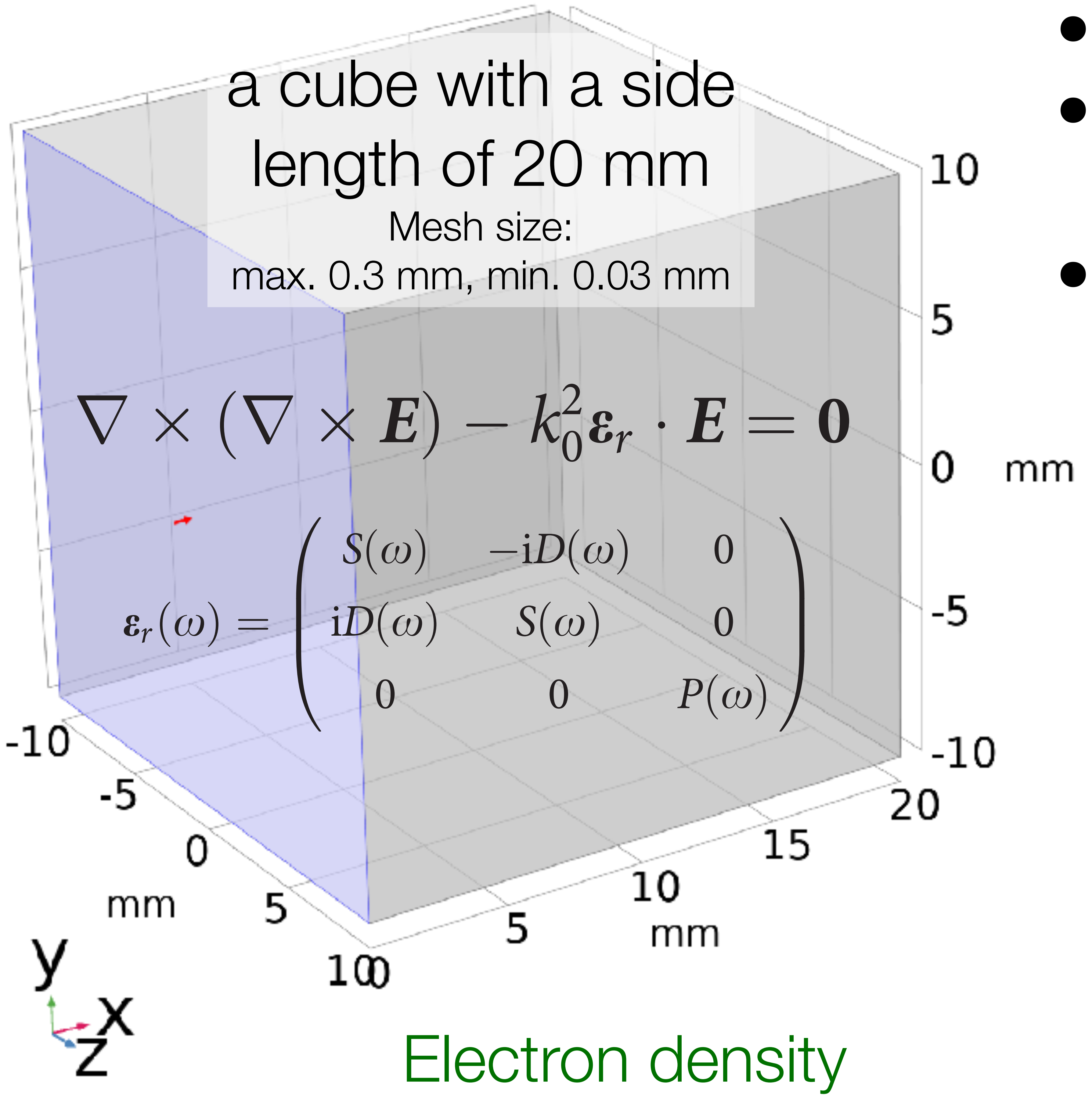
vortex X mode?
or vortex electron Bernstein mode?
propagate into high-*B* & high-n_e region
to heat plasma at ECR

→ **New tool to heat high-n_e plasma?**

future work

Contents

Propagation of an EC wave with a helical wavefront with 3D simulations



a cube with a side length of 20 mm

Mesh size:
max. 0.3 mm, min. 0.03 mm

$$\nabla\times(\nabla\times\boldsymbol{E})-k_0^2\boldsymbol{\varepsilon}_r\cdot\boldsymbol{E}=\boldsymbol{0}$$

$$\boldsymbol{\varepsilon}_r(\omega)=\begin{pmatrix} S(\omega) & -\mathrm{i}D(\omega) & 0\\ \mathrm{i}D(\omega) & S(\omega) & 0\\ 0 & 0 & P(\omega)\end{pmatrix}$$

$$P=1-\frac{\omega_{\mathrm{pe}}^2}{\omega(\omega+i\nu_0)}$$

$$R=1-\frac{\omega_{\mathrm{pe}}^2}{\omega(\omega+i\nu_0-\omega_{\mathrm{ce}})}$$

$$L=1-\frac{\omega_{\mathrm{pe}}^2}{\omega(\omega+i\nu_0+\omega_{\mathrm{ce}})}$$

$$S=\frac{1}{2}(R+L)$$

$$D=\frac{1}{2}(R-L)$$

- The theory suitable analytically as in a plane wave
- Wave amplitude restricted to a finite beam size for practical use
  - ▸ a Laguerre-Gaussian beam
- Commercial COMSOL Multiphysics with RF solver
  - finite element method
  - scattering boundary condition

Electric field

$$E_z(x,y,z)=E_0\left(\frac{r^2}{w^2(x)}\right)^{|l|}\frac{w_0}{w(x)}$$

$$\times\exp\left[-\frac{r^2}{w(x)^2}+\mathrm{i}\left\{-k_0\frac{r^2}{2R(x)}-l\varphi+(|l|+1)\zeta(x)\right\}\right]$$

$$\text{at}\quad x=0,$$

$$r^2=y^2+z^2,\quad \varphi=\tan^{-1}\frac{y}{-z},$$

$$w(x)=w_0\sqrt{1+\left(\frac{x-x_R}{x_R}\right)^2},\quad x_R=\frac{\pi w_0^2}{\lambda_0},$$

$$R(x)=(x-x_R)\left\{1+\left(\frac{x_R}{x-x_R}\right)^2\right\},\quad \zeta(x)=\tan^{-1}\frac{x-x_R}{x_R}$$

Electron density

$$n_{\mathrm{e}}(x)=n_{\mathrm{e,max}}x/L_n$$

$$n_{\mathrm{e,max}}=5\times10^{19}\ \mathrm{m}^{-3}$$

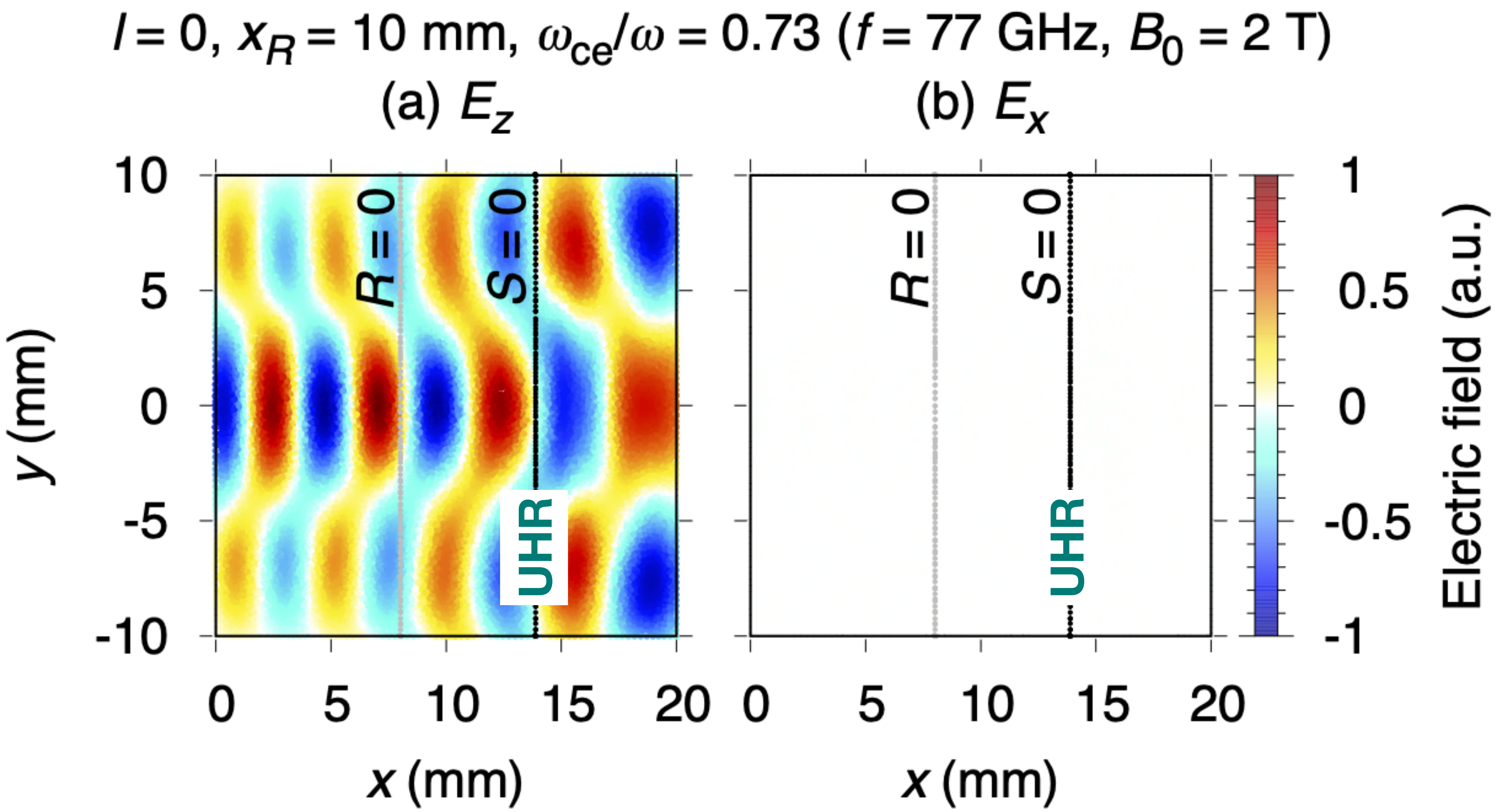
$$L_n=20\,\mathrm{mm}$$

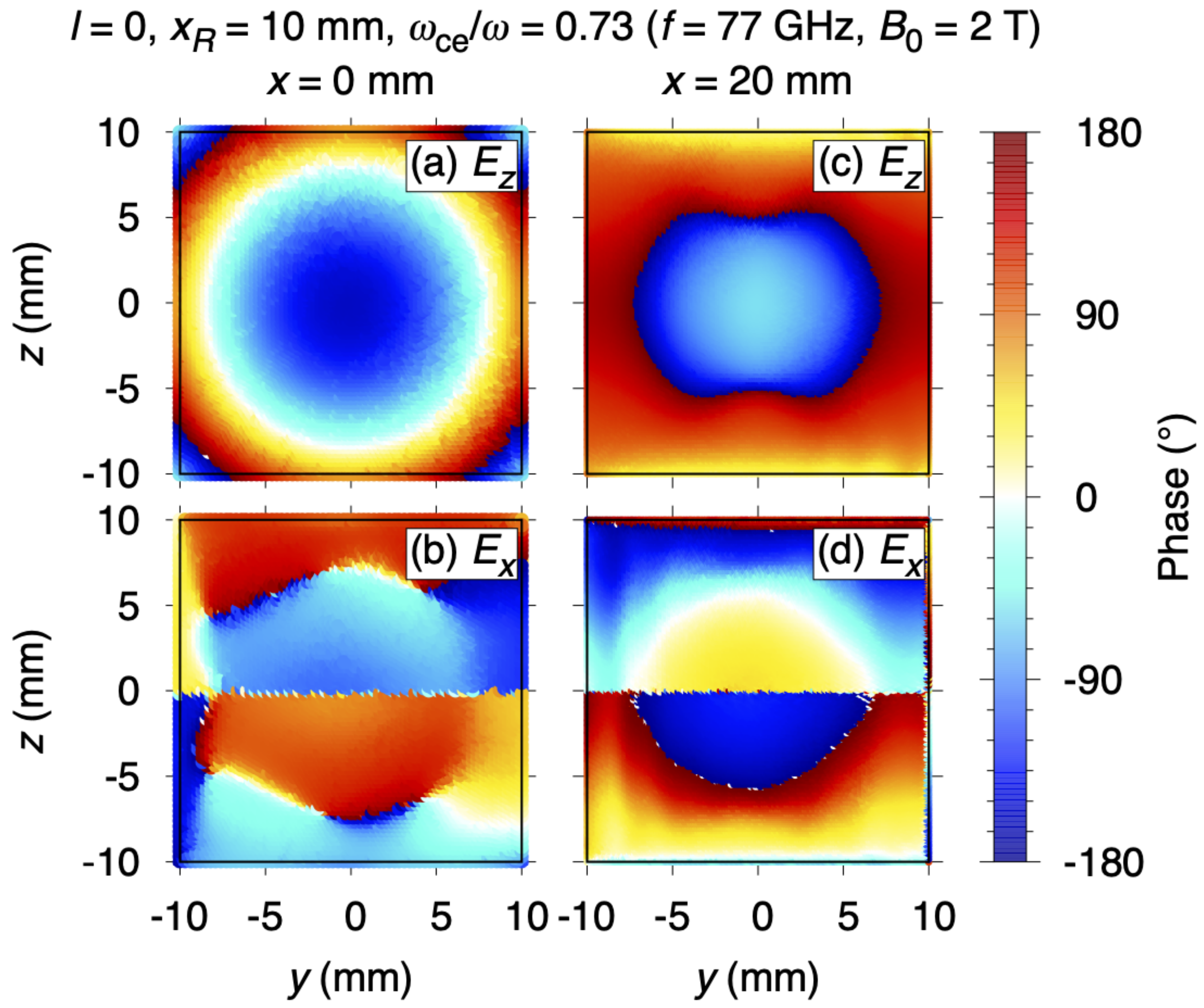
Uniform plasma in *y* and *z* directions

T. I. Tsujimura & S. Kubo, Phys. Plasmas **28**, 012502 (2021)

f = 77 GHz, $\nu_0 = 0.01\omega$, B_{z0} = 2 T, x_R = 10 mm, w_0 = 3.5 mm

In the case of $l = 0$, the O mode propagates



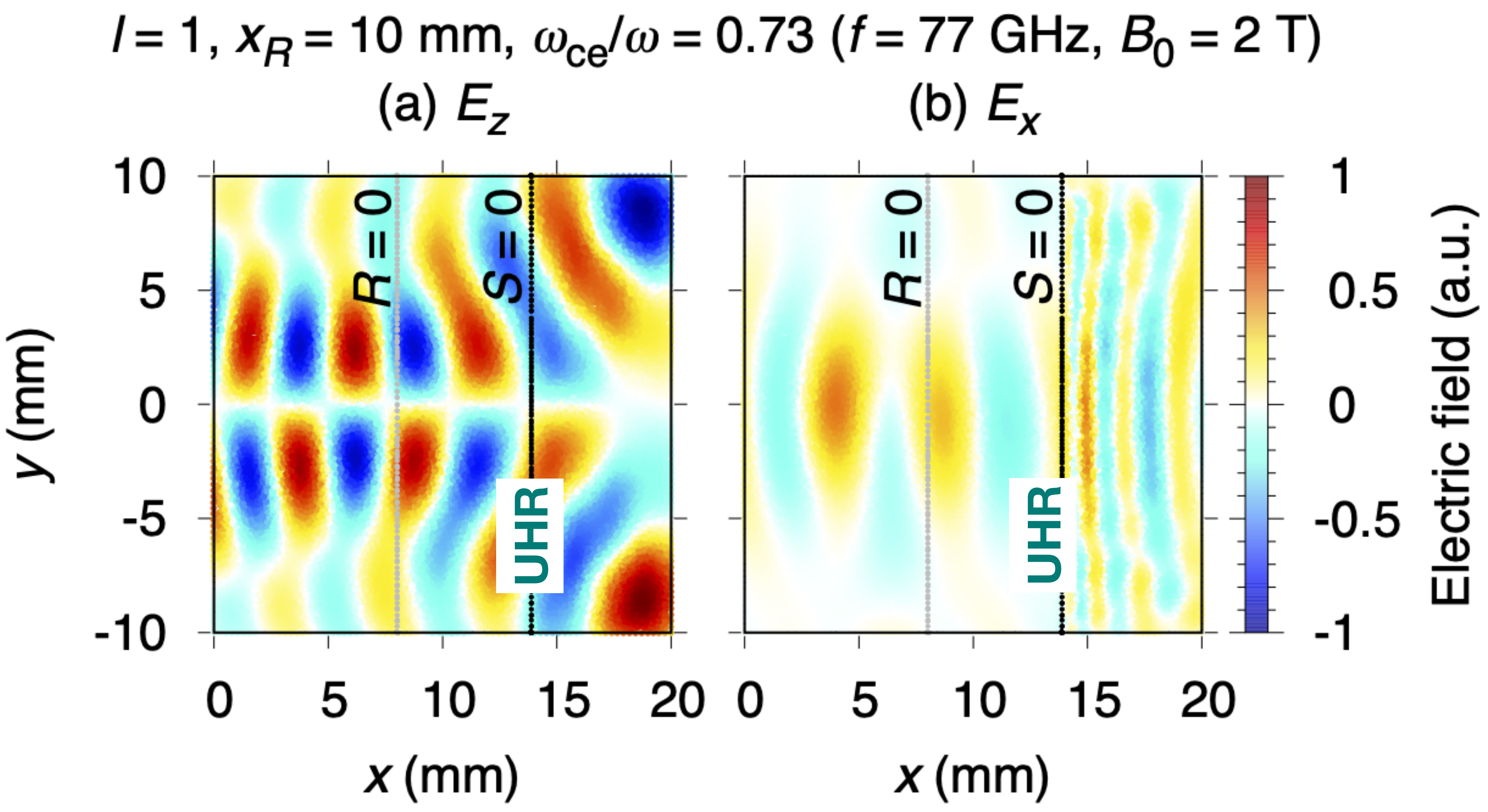


- The excited linearly polarized E_z parallel to $\boldsymbol{B}_0$ propagates in the x direction.
  - Negligible E_x

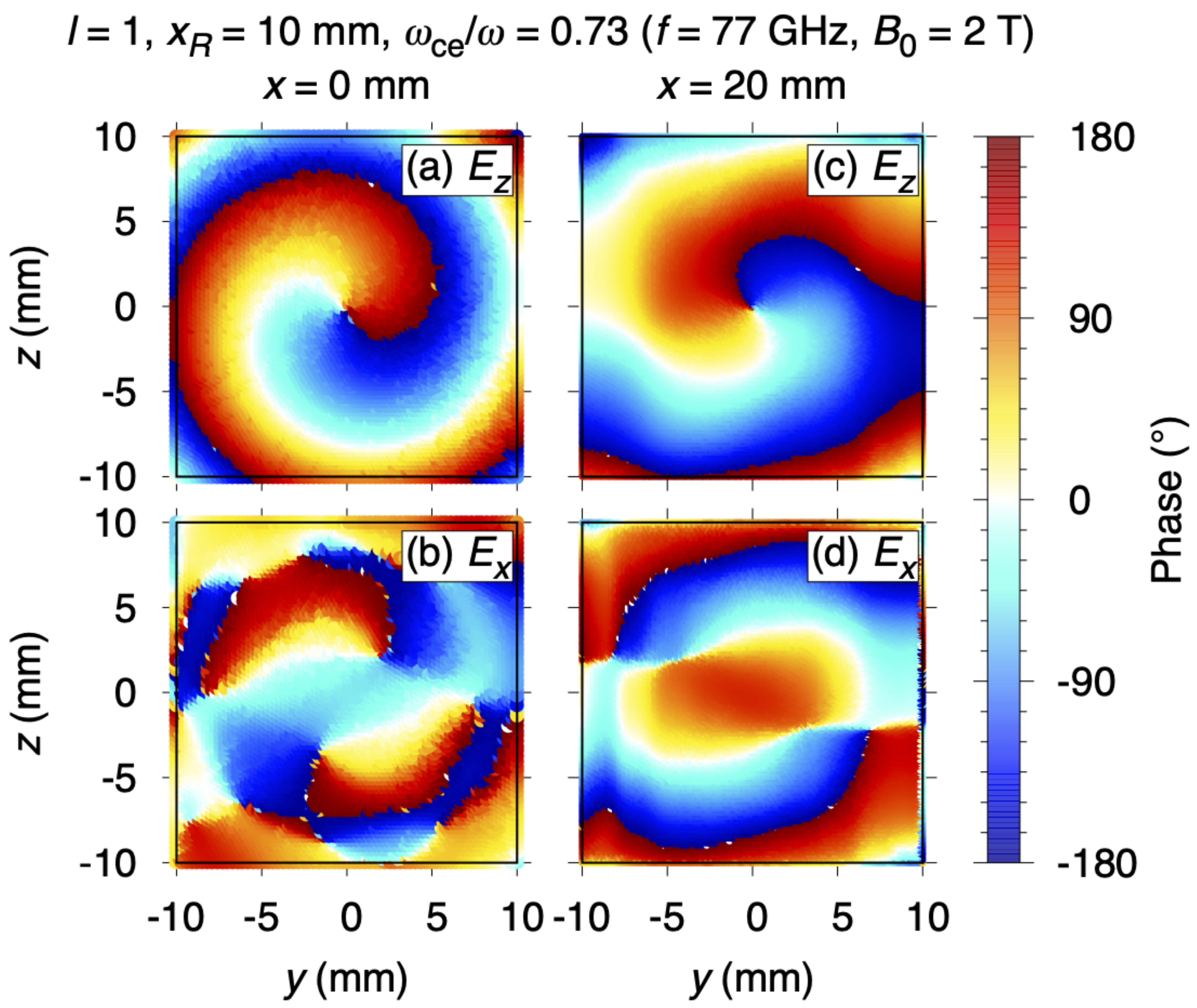
Almost axisymmetric phase E_z

T. I. Tsujimura & S. Kubo, Phys. Plasmas **28**, 012502 (2021)

In the case of $l = 1$, a part of the O mode is suggested to be converted to the high-wavenumber X mode

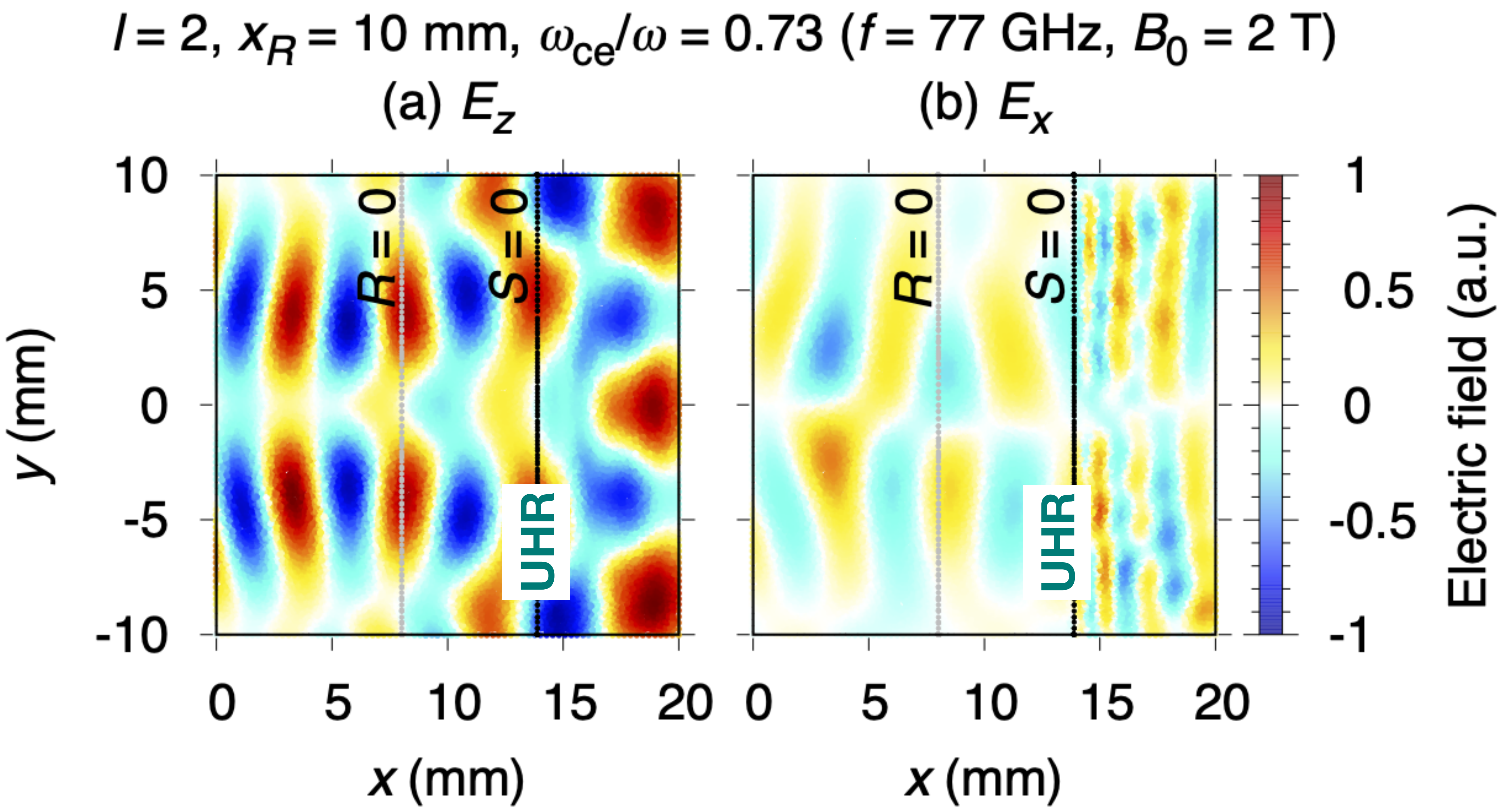


- The excited LG beam E_z propagates to UHR.
  - Diffuse outward over the UHR layer
  - E_x parallel to the propagation direction with higher wavenumber is excited at UHR and propagates to the higher n_e region.
  - The amplitude E_x is larger around the optical axis.
- **A part of E_z with the O-mode polarization is converted into E_x with the X-mode polarization.**

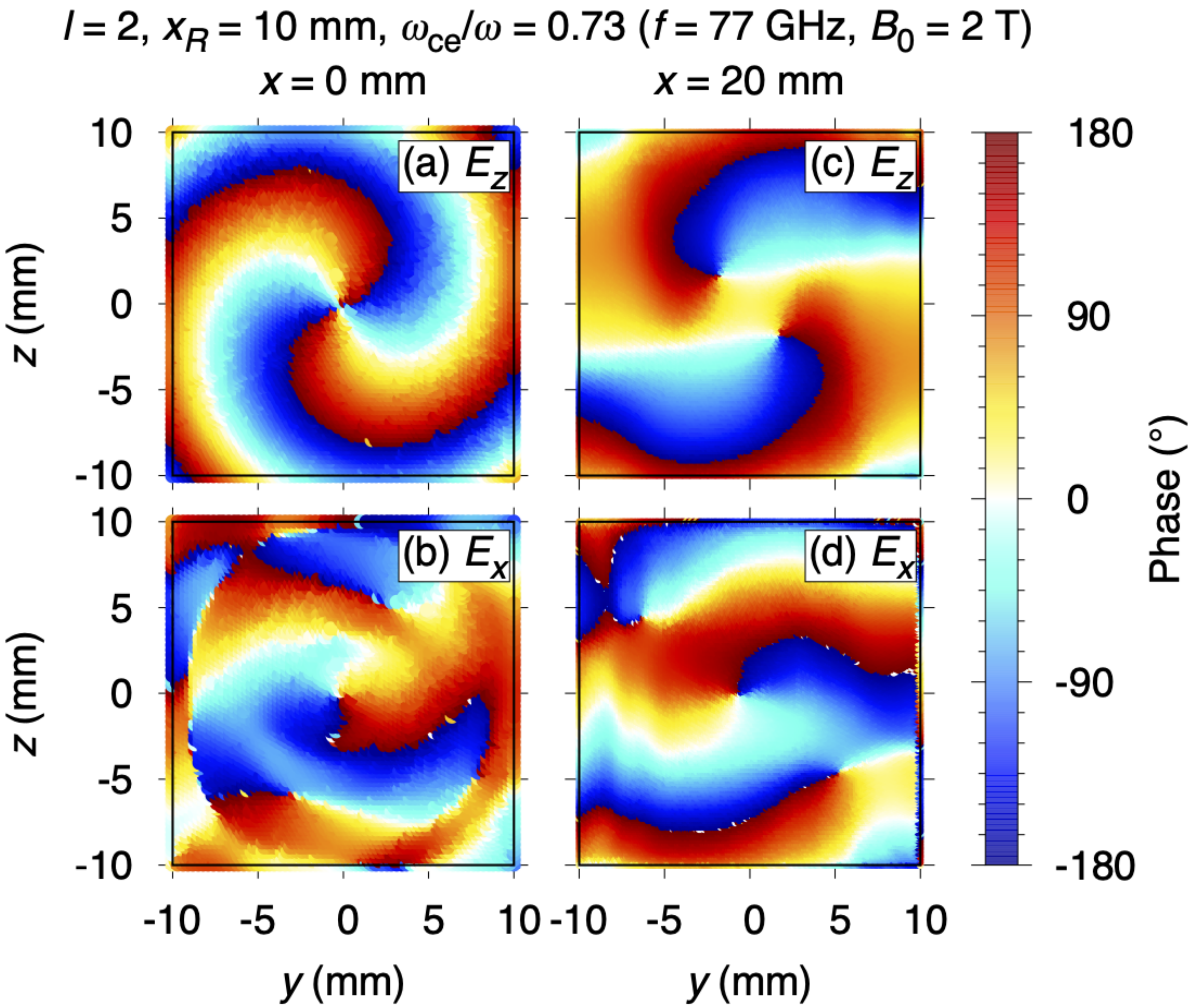


- $l = 1$ in E_z, $l = 0$ in E_x
  - The topological charge of E_x is one smaller than that of E_z due to the factor of $e^{-i\phi}$ in the theory.

Similarly, in the case of *l* = 2, a part of the O mode is suggested to be converted to the high-wavenumber X mode



E_x with the high wavenumber is excited at UHR.



The topological charge of *E_x* is *l* = 1 around the optical axis.

T. I. Tsujimura & S. Kubo, Phys. Plasmas **28**, 012502 (2021)

Contents

Summary

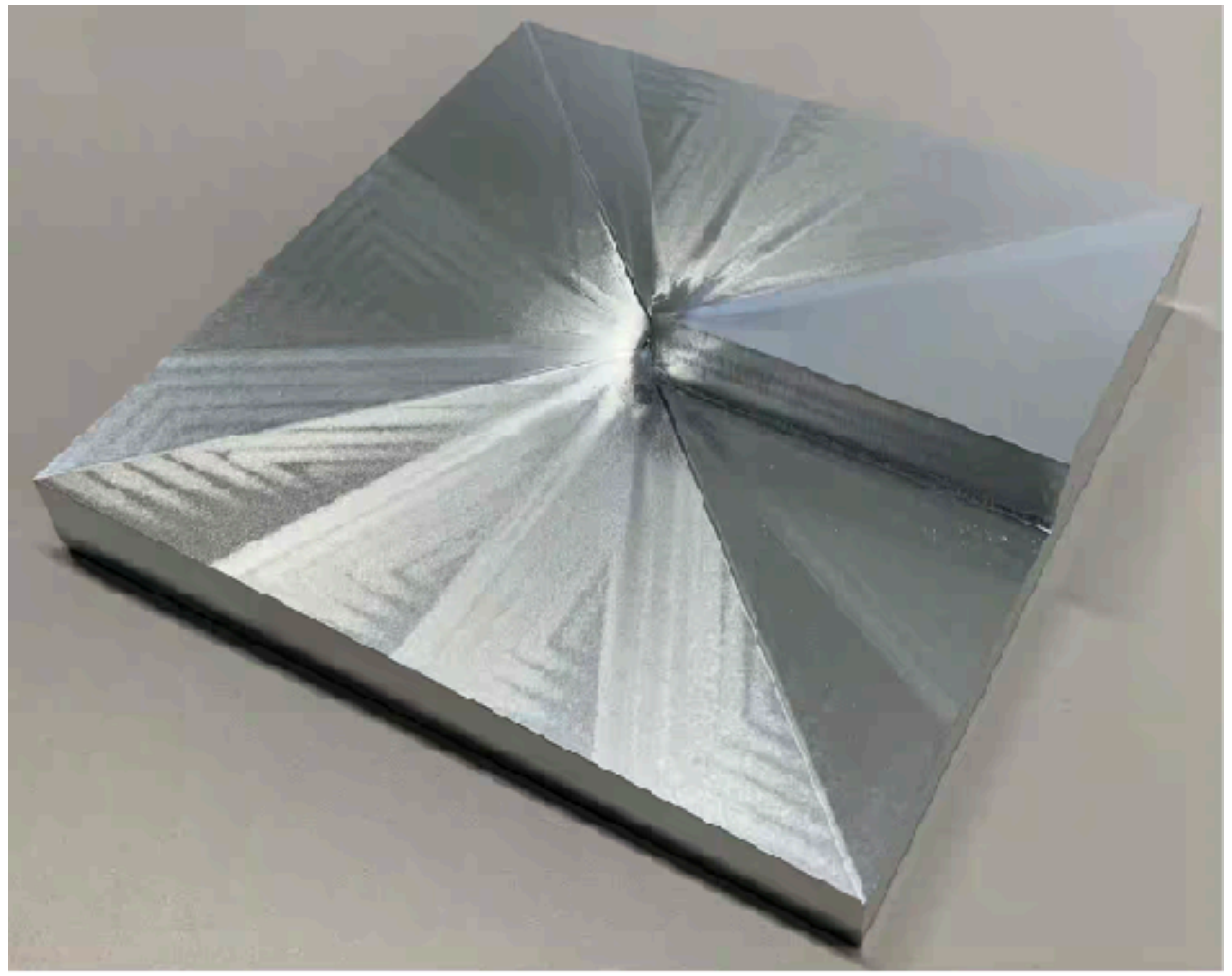
- **Propagation properties of EC waves with helical wavefronts are investigated theoretically in cold uniform magnetized plasma.**
  - The effects of the helical wavefronts on the wave fields are described.
  - These effects become significant as the topological charge of the vortex EC wave increases or the distance from the optical axis becomes small.
- **The different properties of propagation are also confirmed in COMSOL simulations with LG beams.**
  - It is found that a part of the O-mode LG beam with the topological charge l excited at the lower n_e region is converted into the high-wavenumber X-mode LG beam with $l - 1$ at UHR.

T. I. Tsujimura & S. Kubo, Phys. Plasmas **28**, 012502 (2021)

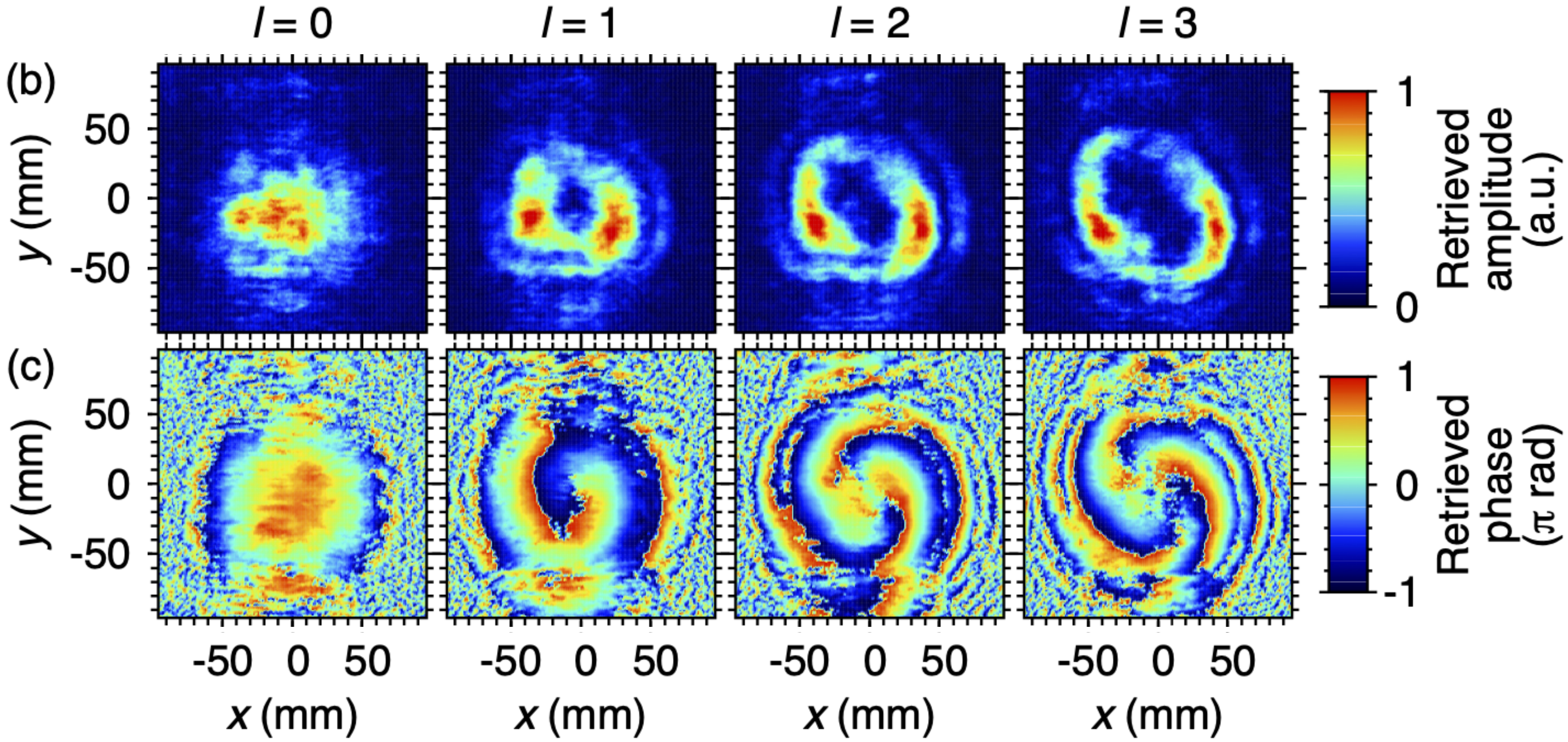
Outlook

To demonstrate the new propagation properties of vortex EC waves in plasma heating experiments, **off-axis spiral-phase mirrors were developed** to generate an optical vortex with designed *l* in **millimeter waves**.

- generated vortex mm waves will be injected into fusion plasma
- to verify whether an optical vortex can be **a new tool to efficiently heat high-n_e plasma**



spiral phase mirror



T. I. Tsujimura *et al*., Rev. Sci. Instrum. **93**, 043507 (2022)