21st JOINT WORKSHOP ON ELECTRON CYCLOTRON EMISSION
(ECE) AND ELECTRON CYCLOTRON RESONANCE HEATING
(ECRH), 20 - 24 June 2022, ITER Organization

Evaluation of the O-X mode conversion rate of the finite width wave in two dimensional systems

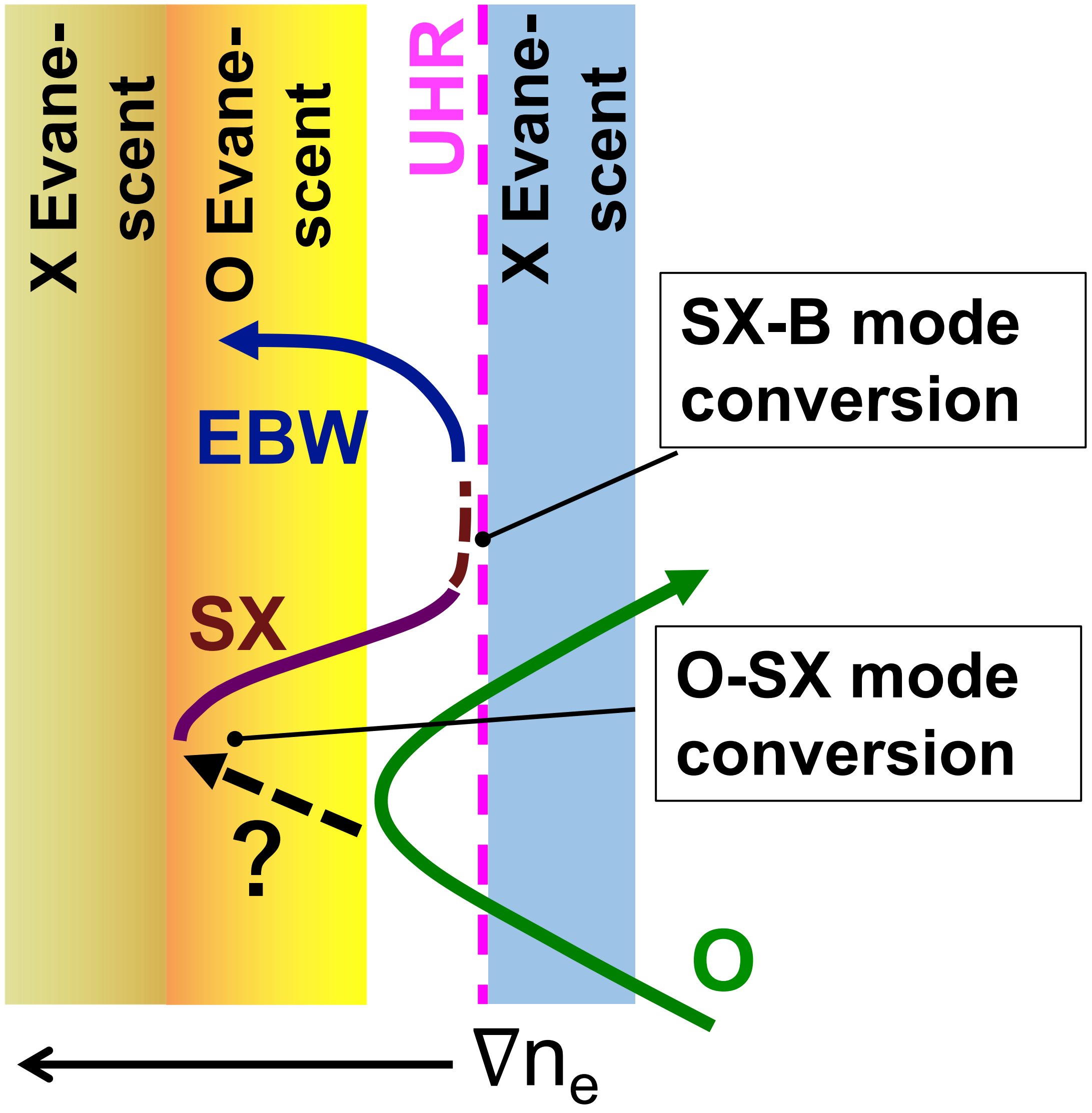
H. Igami[A], A. Fukuyama [B], K. Nagasaki [C]
[A]*NIFS,* [B]*GSE, Kyoto Univ.,* [C]*IAE, Kyoto Univ.*

Contents

- Background and Motivation
- Configuration of the 2D wave optical calculation with using TASK/WF2D
- Calculation results
- Summary

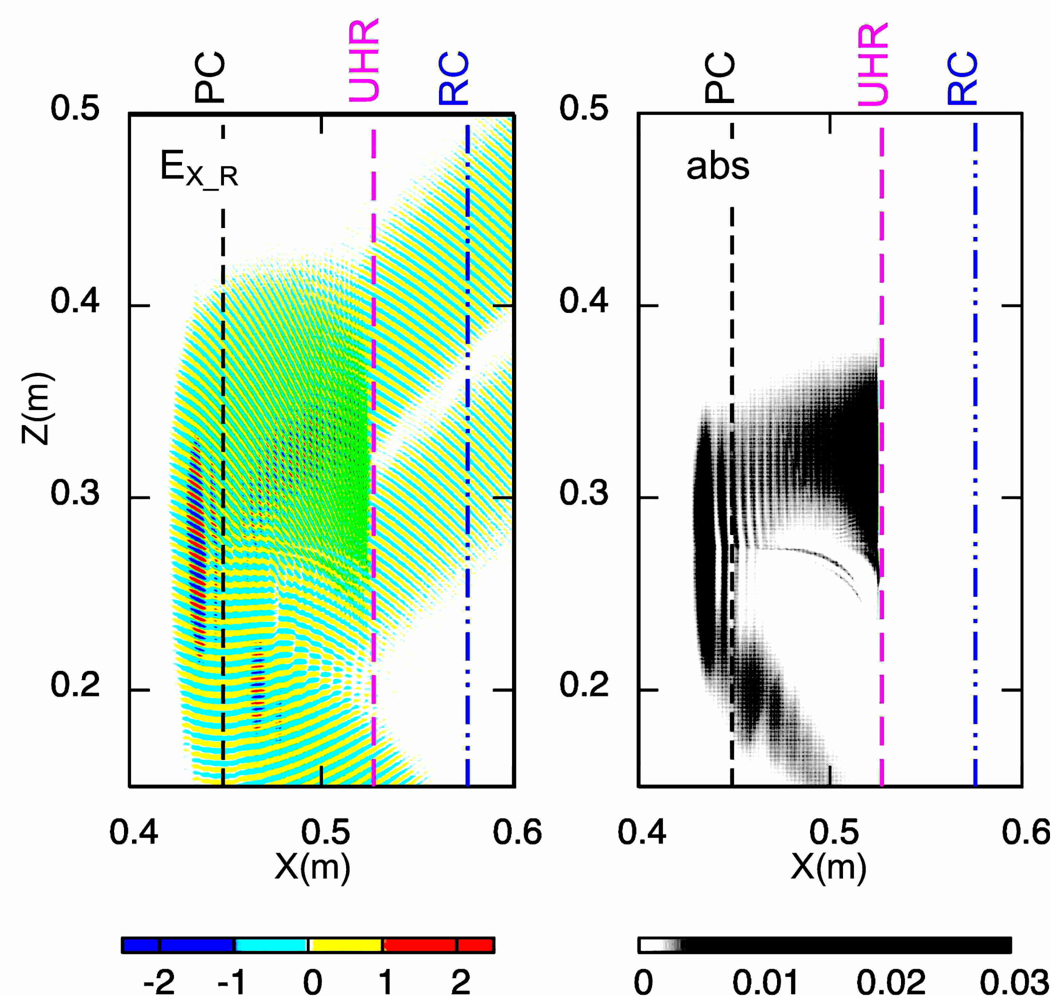
Background and Motivation

Mode conversions across the evanescent region



- For ECRH in the “over-dense” plasma, the electrostatic EBW should be excited via the mode conversion process from the SX wave at the UHR
- Waves launched from the low magnetic field side transmit the evanescent region to couple with the SX-mode
- The ray-tracing cannot treat the propagation across the evanescent region
- **Wave optical full wave analysis is required**

Broadening of the launched finite width beam affects the O-X mode conversion process

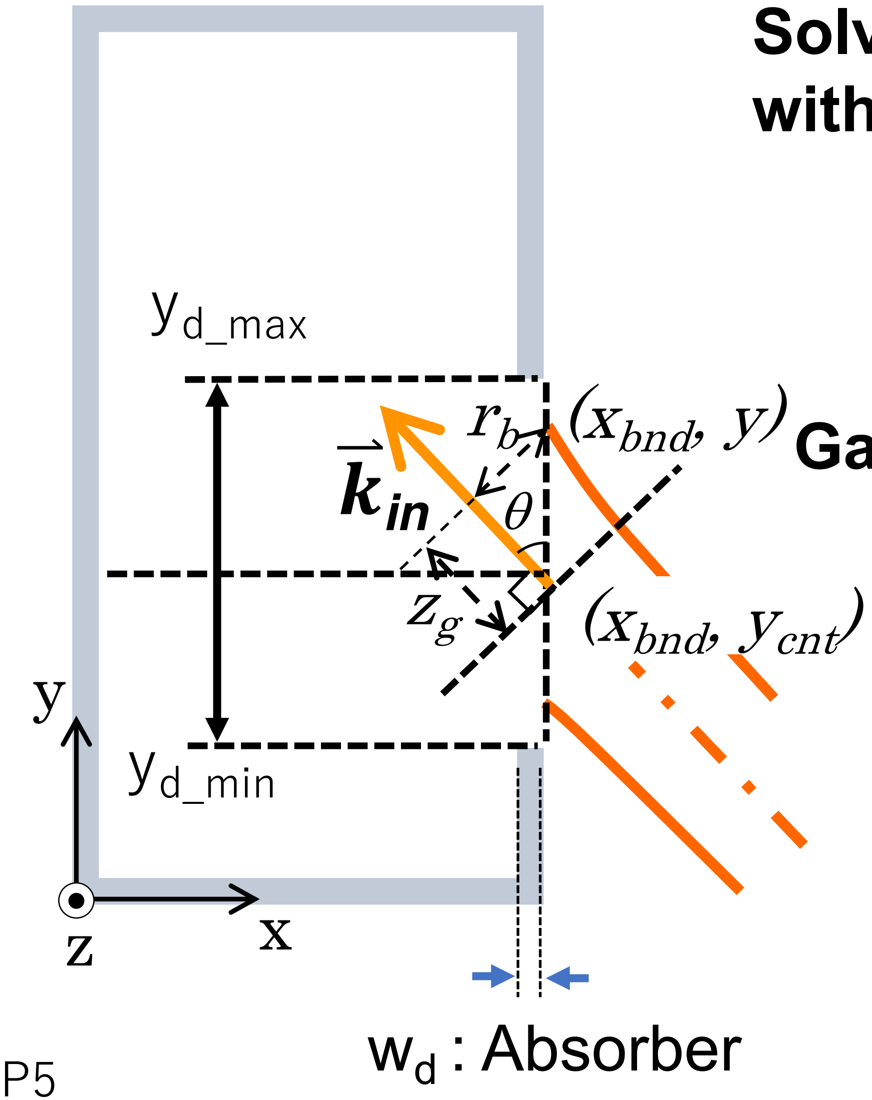


H. Igami. et al.
Plasma and Fusion Research, Volume 11, 2403098(2016)

- In 2D configuration, a finite width electromagnetic wave launched from the waveguide at $\vartheta_{inj} = \vartheta_{opt} = \arccos(N_{//opt})$ is **not fully mode converted** to the SX mode
- At the edge of the launched beam, $\vartheta_{inj} = \vartheta_{opt} \pm 1.5^\circ$ in this case

➢ **In this study, we’ve performed 2D wave optical full wave analyses to provide a guideline for designing the launching beam condition for ECRH by EBW**

Configuration of the 2D calculation



Solving Maxwell equation by finite element method with TASK/WF2D code*)

$$\nabla \times \nabla \times \boldsymbol{E} - (\omega^2/c^2)\,\boldsymbol{\varepsilon}\cdot\boldsymbol{E} = i\omega\mu_0\boldsymbol{j}_{ext}$$

*) A. Fukuyama, et. al 41st European Physical Society Conference on Plasma Physics (EPS 2014) P4.014 (2014)

Gaussian like beam is defined along the *y* axis at *x=x*$_{ant}$

$$E(x_{bnd}, y) = E(x_{bnd}, y_{cnt})\frac{w_0}{w_z}\, exp\left\{-\left(\frac{r_b}{w_z}\right)^2 - i|k_{in}|\left(z_p + \frac{r_b^2}{2R_c}\right) + i\zeta\right\}$$

$$w_z = w_0\sqrt{1 + \left(z_g/z_r\right)^2} \qquad r_b = |y - y_{cnt}|\sin\theta$$

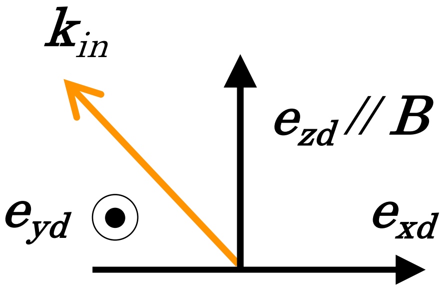
$$z_p = (y - y_{cnt})\cos\theta \qquad z_r = \left(\frac{\omega}{2c}\right){w_0}^2$$

$$\zeta = \tan^{-1}\left(z_g/z_r\right) \qquad z_g = z_0 + z_p$$

$$R_c = z_g\left(1 + \left(\frac{z_r}{z_g}\right)^2\right)$$

Calculation of the electric field at (x_{ant}, y_{ant})

1. Coordinate transformation $(x, y, z) \Longrightarrow (x_d, y_d, z_d)$



$$\left(N_x, N_y, N_z\right) \Longrightarrow \left(N_{xd}, N_{yd}, N_{zd}\right)$$

$$\vec{e}_{zd} \parallel \vec{B} \qquad \vec{e}_{xd} = \vec{e}_{zd} \times \vec{e}_{yd}$$

$$\vec{e}_{yd} = \vec{N} \times \vec{e}_{zd} / \left|\vec{N} \times \vec{e}_{zd}\right|$$

2. Solving the Maxwell equations

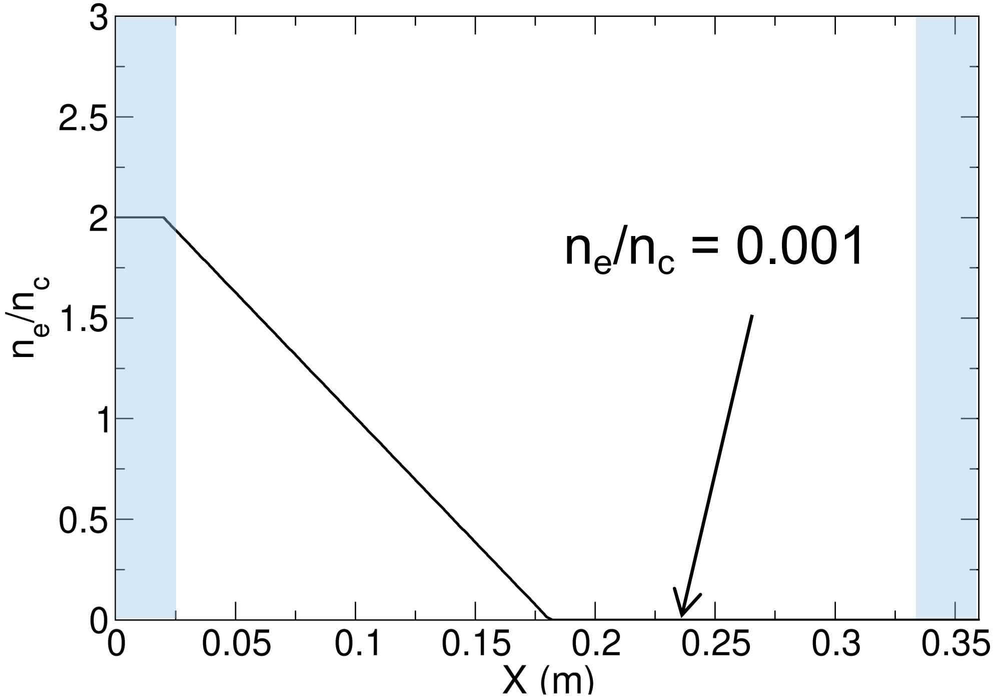
$$\vec{N}_d \times \left(\vec{N}_d \times \vec{E}_d\right) + \overleftrightarrow{\varepsilon} \cdot \vec{E}_d = 0 \qquad \overleftrightarrow{\varepsilon} = \begin{pmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{pmatrix}$$: Cold dielectric tensor

$$\begin{pmatrix} S - \left(N_{yd}^2 + N_{zd}^2\right) & -iD + N_{xd}N_{yd} & N_{zd}N_{xd} \\ iD + N_{xd}N_{yd} & S - \left(N_{zd}^2 + N_{xd}^2\right) & 0 \\ N_{zd}N_{xd} & N_{yd}N_{xz} & P - \left(N_{xd}^2 + N_{yd}^2\right) \end{pmatrix} \begin{pmatrix} E_{xd} \\ E_{yd} \\ E_{zd} \end{pmatrix} = 0$$

3. Coordinate transformation $\left(E_{xd}, E_{yd}, E_{zd}\right) \Longrightarrow \left(E_x, E_y, E_z\right)$

Calculation Parameters

- Frequency : $f = \omega/2\pi = 28\text{GHz}$
- Waist size : $w_0 = 0.0373\text{m}$ ($=3.48\lambda_0$)
- Mesh size = 0.25mm (~ λ_0 /40)
- thickness of the absorber : $w_d =$ 0.025m
- Permittivity in the absorber : $\varepsilon_{\text{damp}} = \varepsilon \times 2$
- Normalized collision frequency : $\nu_e/\omega = 0.001$, $\nu_p/\omega = 0.001$, $\nu_{abs}/\omega = 0.2$

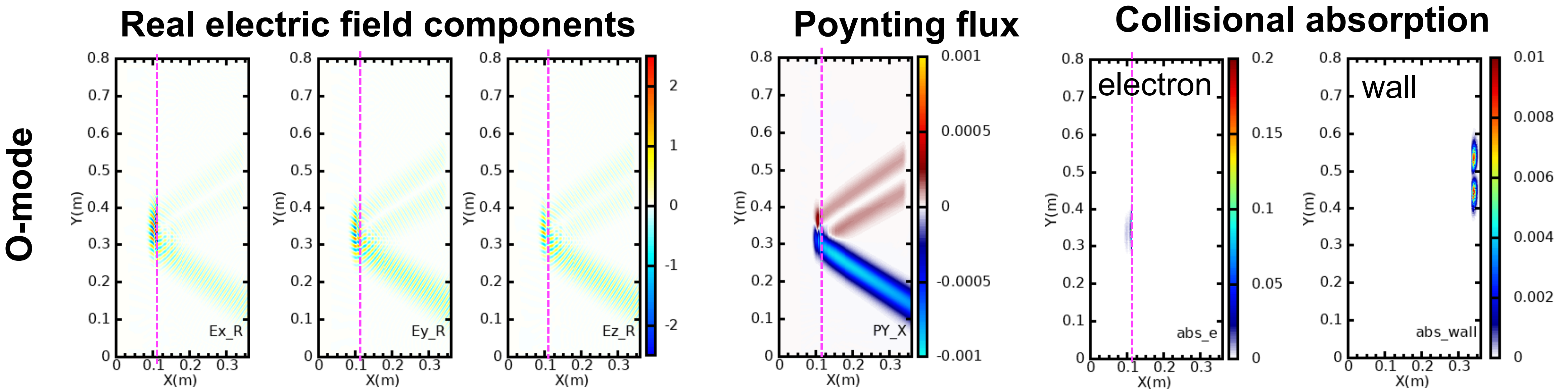


Density scale-length at the plasma cutoff,
$L_n=7.5^* \lambda_0$ (λ_0: vacuum wavelength)
Constant magnetic field : $|B| = 0.4\text{T}$,
$(\vec{B} \parallel \hat{z}$) $\omega_{ce}/\omega = 0.4$

Intel Xeon Silver 4216 CPU @ 2.10GHz, 16C/32T x 2
Memory: 768GB
Number of element : 12096000 (typical)
CPU time : 3196.55 sec (for FEM calculation, typical)

An example of the calculation

Launching of the Gaussian like beam with $N_{//} = N_{//\,opt}$ ($\theta_{//\,opt}$ =57.7deg.)



Real electric field components

Poynting flux

Collisional absorption

- ✓ **Cold dielectric tensor with weak collisional damping for UH resonant absorption**
- ✓ The artificial dielectric tensor is adopted in FEM calculation inside the artificial “absorption wall ” to make the wave number gradually diverge to cause the resonant absorption like in the UHR
- ✓ The “absorption wall” has an aperture around the launched wave

Based on the cold plasma resonant absorption model T_{OX} can be calculated from the outputs of TASK/WF2D

From the collisionally absorbed power

T_{OX} : O-X mode conversion rate

$$P_{abs} = \vec{j}^{*} \cdot \vec{E} = \left(\sigma \ \cdot \vec{E}\right)^{*} \cdot \vec{E}$$

$$T_{OX} = \frac{\sum_{X=0}^{X=Xant} P_{abs\_e}}{\sum_{X=0}^{X=Xant}\left(P_{abs\_e} + P_{abs\_i} + P_{abs\_wall}\right)}$$

From Poynting fluxes

$$T_{OX} = 1 - \sum_{Y=0}^{Y=Y_{max}} \left|\vec{P}_{X\_ref}\right| / \left|\vec{P}_{X\_in}\right|$$

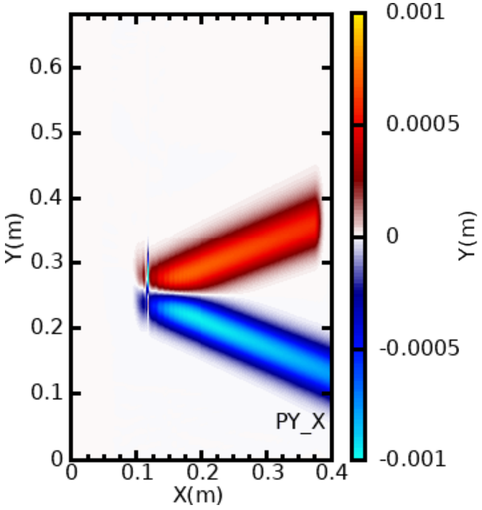
$$\begin{aligned}\vec{P} &= Re\left(\vec{E}e^{-i\omega t}\right) \times Re\left(\vec{B}e^{-i\omega t}\right)/u_0 \\ &= Re\left(\vec{E}e^{-i\omega t}\right) \times Re\left(\frac{\left(\nabla \times \vec{E}e^{-i\omega t}\right)}{i\omega u_0}\right) \\ &= \frac{1}{4\omega u_0}\begin{bmatrix} \left\{E_y^{*}\left(\partial_x E_y - \partial_y E_x\right) - E_z^{*}\left(-\partial_x E_z\right)\right\} - c.c. \\ \left\{E_z^{*}\left(\partial_y E_z\right) - E_x^{*}\left(\partial_x E_y - \partial_y E_x\right)\right\} - c.c. \\ \left\{E_x^{*}\left(-\partial_x E_z\right) - E_y^{*}\left(\partial_y E_z\right)\right\} - c.c. \end{bmatrix}\end{aligned}$$

time average

$$\left\langle E_i \partial E_j \right\rangle = \left\langle E_i^{*} \partial E_j^{*} \right\rangle = 0$$

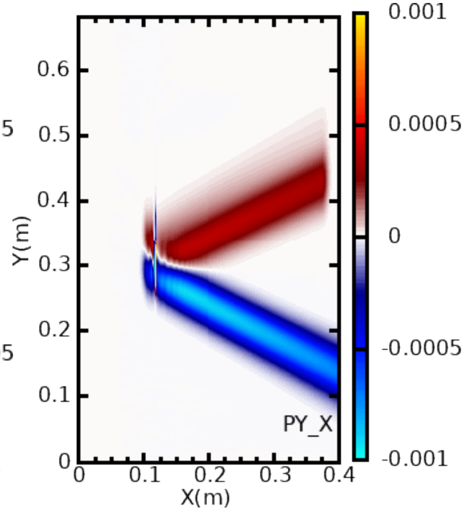
Poynting fluxes of various launching angle cases

$\theta = \theta_{// opt}$ +10deg.



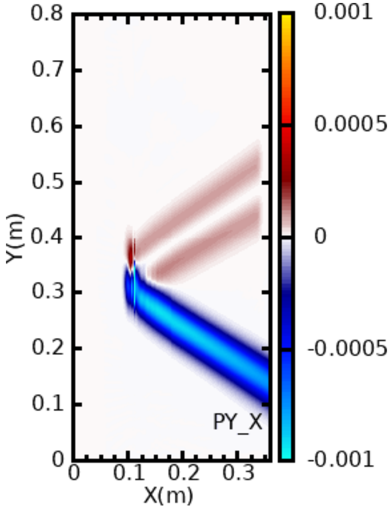
T_{OX}=0.15

$\theta = \theta_{// opt}$ +5deg.



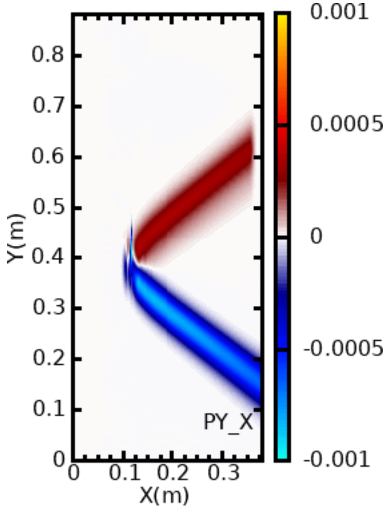
T_{OX}=0.46

$\theta = \theta_{// opt}$



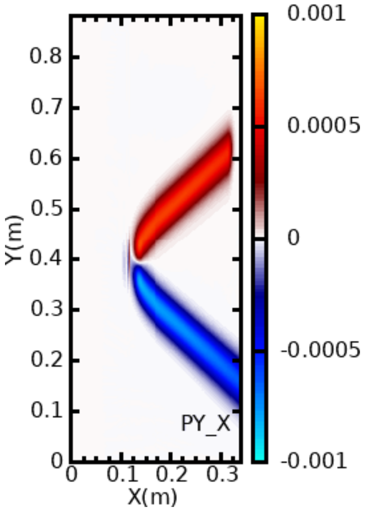
T_{OX}=0.81

$\theta = \theta_{// opt}$ -5deg.



T_{OX}=0.46

$\theta = \theta_{// opt}$ -10deg.



T_{OX}=0.13

- ✓ The incident Gaussian like beam is focused on the plasma cutoff for each case

$$E(x_{bnd}\ y) = E(x_{bnd}, y_{cnt})\frac{w_0}{w_z}exp\left\{-\left(\frac{r_b}{w_z}\right)^2 - i|k_{in}|\left(z_p + \frac{r_b^2}{2R_c}\right) + i\zeta\right\}$$

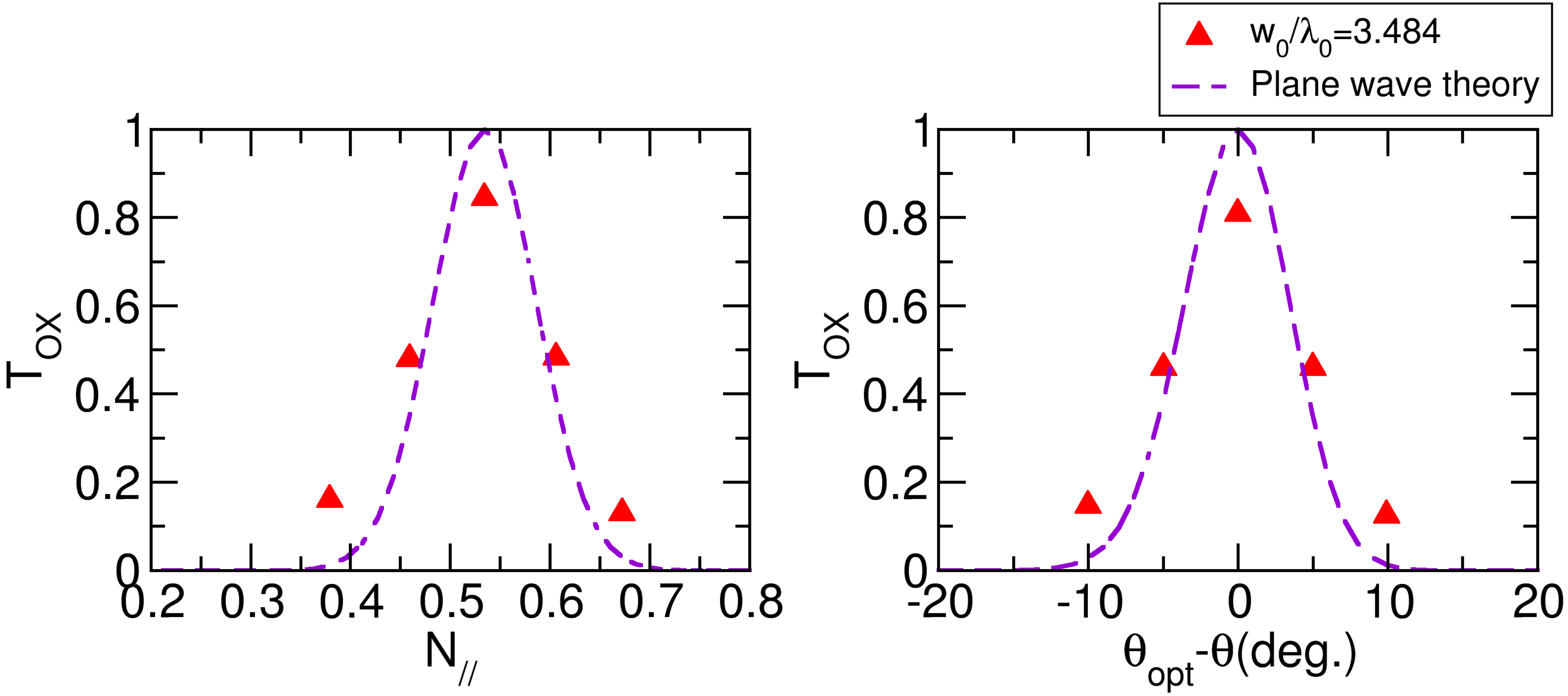
$$w_z = w_0\sqrt{1 + (z/z_r)^2}$$

$$z_r = \left(\frac{\omega}{2c}\right){w_0}^2 \quad z_p = (y - y_{cnt})\cos\theta$$

Calculation results

T_{ox} estimated for various propagation angles

T_{ox} is less than plane wave theory at $\theta = \theta_{//\,opt}$, greater than $\theta \neq \theta_{//\,opt}$

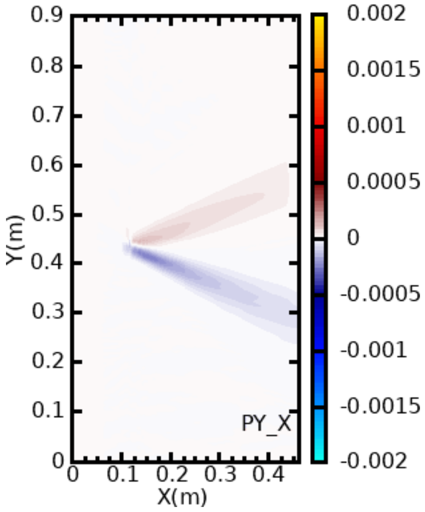


plane wave theory : $T_{OX} = \exp\{-\pi k_0 L_n (Y/2)^{1/2}[2(1+Y)(N_{//}-N_{//opt})^2 + N_y^2]\}$,

$Y = (\Omega_{ce}/\omega)$, $N_{zc} = \{Y/(1+Y)\}^{-1/2}$ *Mjolhus (1984)*

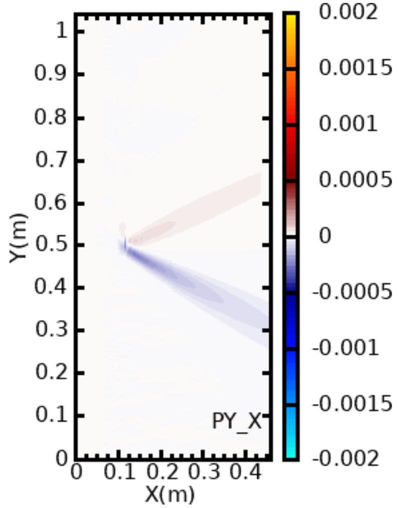
Propagation of waves of a smaller waist size $w_0 = 0.017\text{m}$ $(=1.59\lambda_0)$

$\theta = \theta_{//\,\mathrm{opt}}$ +10deg.



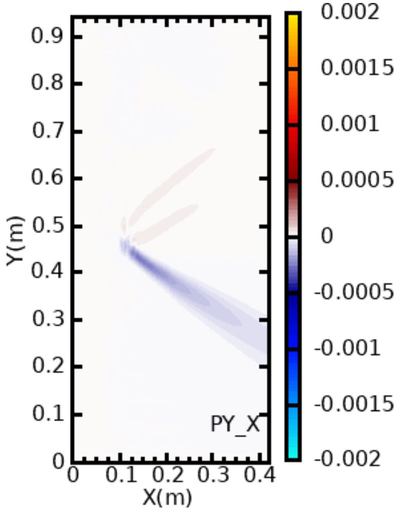
T_{OX}=0.226

$\theta = \theta_{//\,\mathrm{opt}}$ +5deg.



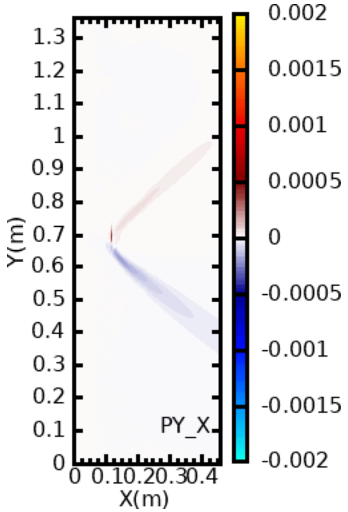
T_{OX}=0.454

$\theta = \theta_{//\,\mathrm{opt}}$



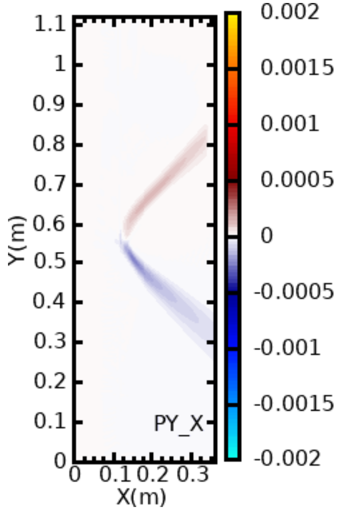
T_{OX}=0.576

$\theta = \theta_{//\,\mathrm{opt}}$ -5deg.



T_{OX}=0.471

$\theta = \theta_{//\,\mathrm{opt}}$ -10deg.



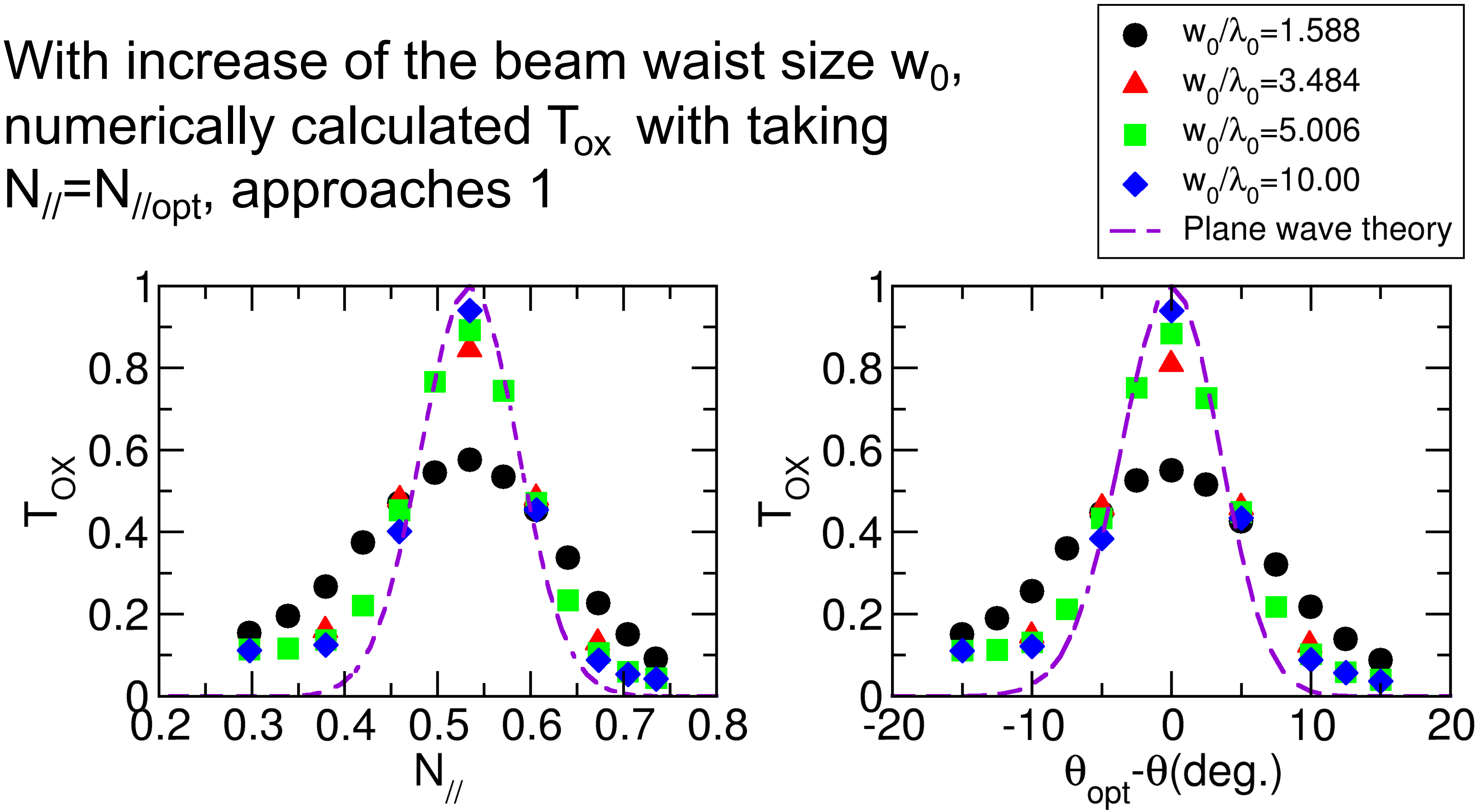
T_{OX}=0.267

$\theta = \theta_{//\,\mathrm{opt}}$: T_{OX} decreases

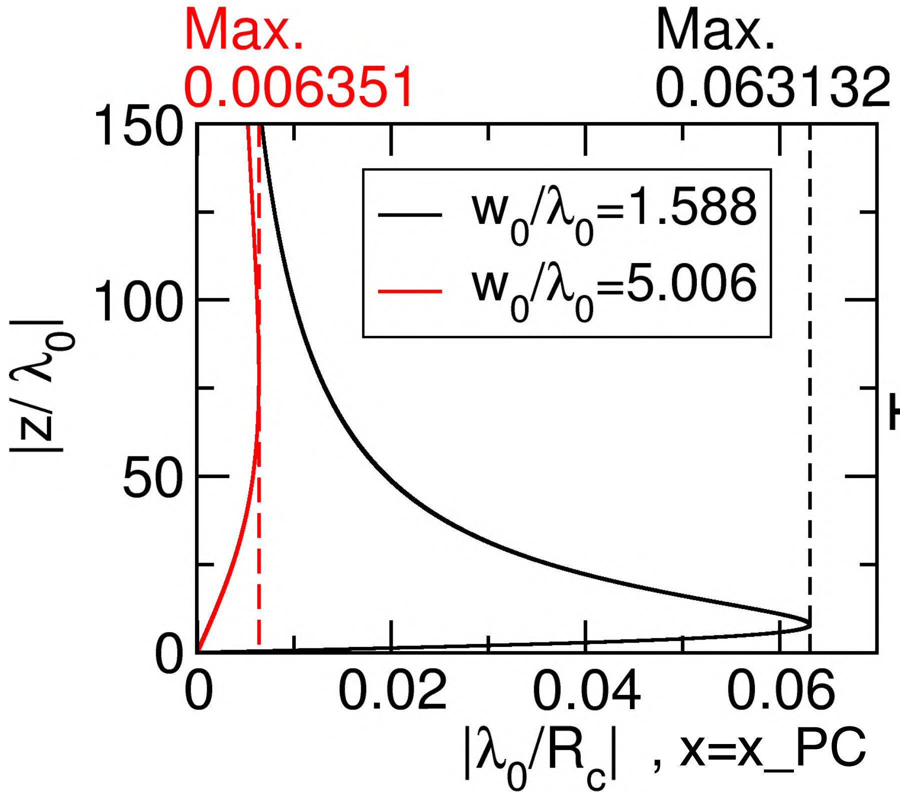
$\theta = \theta_{//\,\mathrm{opt}} \pm 10$ deg. : T_{OX} increases

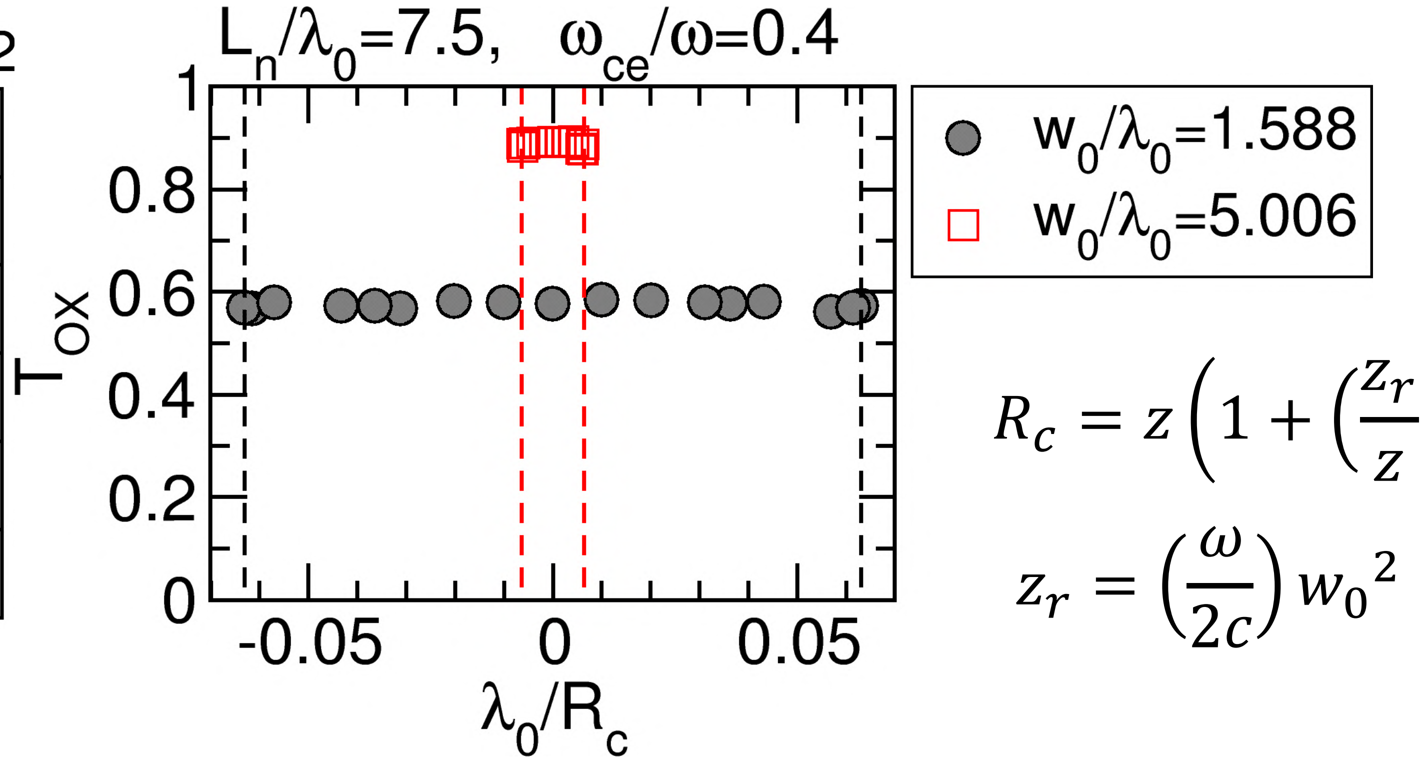
T_{ox} estimated for various beam waist sizes for various propagation angles

With increase of the beam waist size w_0, numerically calculated T_{ox} with taking $N_{//}=N_{//opt}$, approaches 1



The effect of the beam curvature radius R_c on the T_{ox} is investigated with changing the beam focal point





$$R_c = z\left(1+\left(\frac{z_r}{z}\right)^2\right)$$

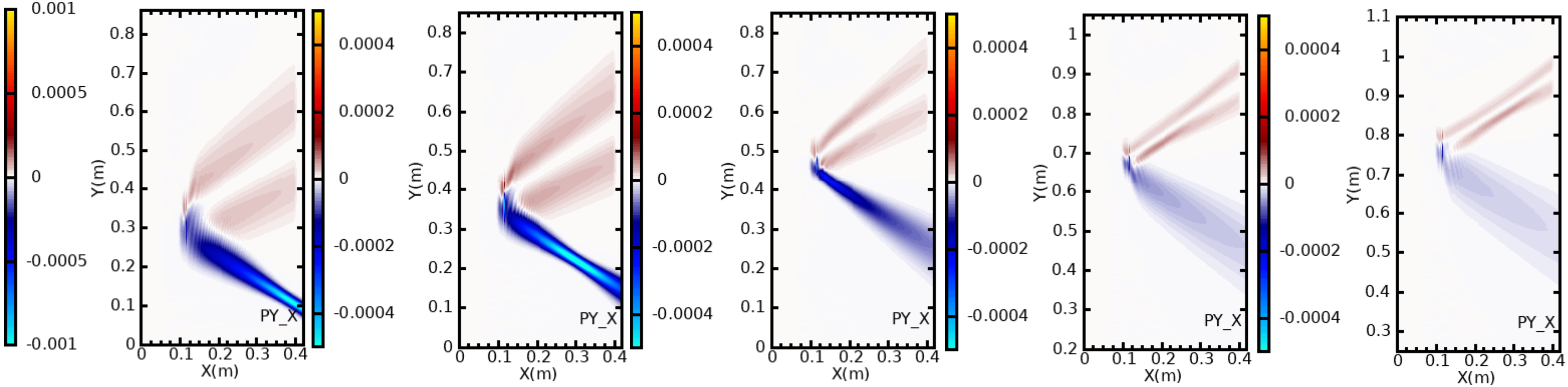
$$z_r = \left(\frac{\omega}{2c}\right){w_0}^2$$

- ✓ T_{OX} remains to be constant for the same beam waist size w_0 since the $N_{//}$ Fourier spectrum is conserved in the slab plasma in the uniform magnetic field.
- ✓ With introducing the magnetic shear, T_{OX} can change because the $N_{//}$ spectrum can change during the wave propagation

$L_n=7.5^* \lambda_0$, $w_0/\lambda_0=1.59$

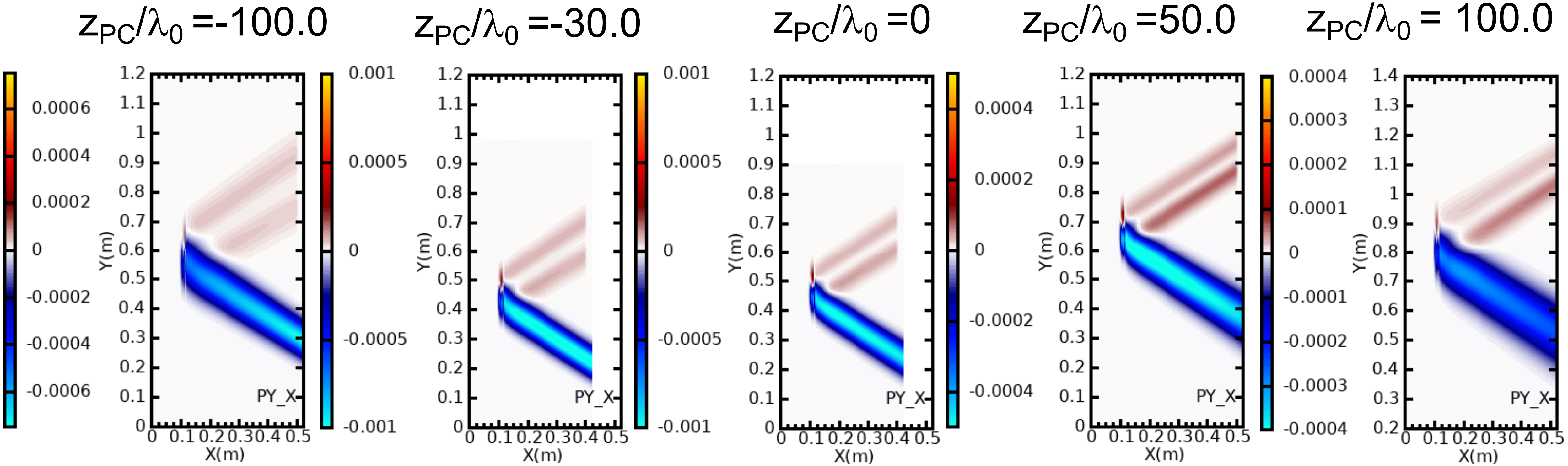
z_{PC} : Distance between the beam focal point ant the plasma cutoff

$z_{PC}/\lambda_0=-30$ $z_{PC}/\lambda_0=-20$ $z_{PC}/\lambda_0=0$ $z_{PC}/\lambda_0=20$ $z_{PC}/\lambda_0=30$



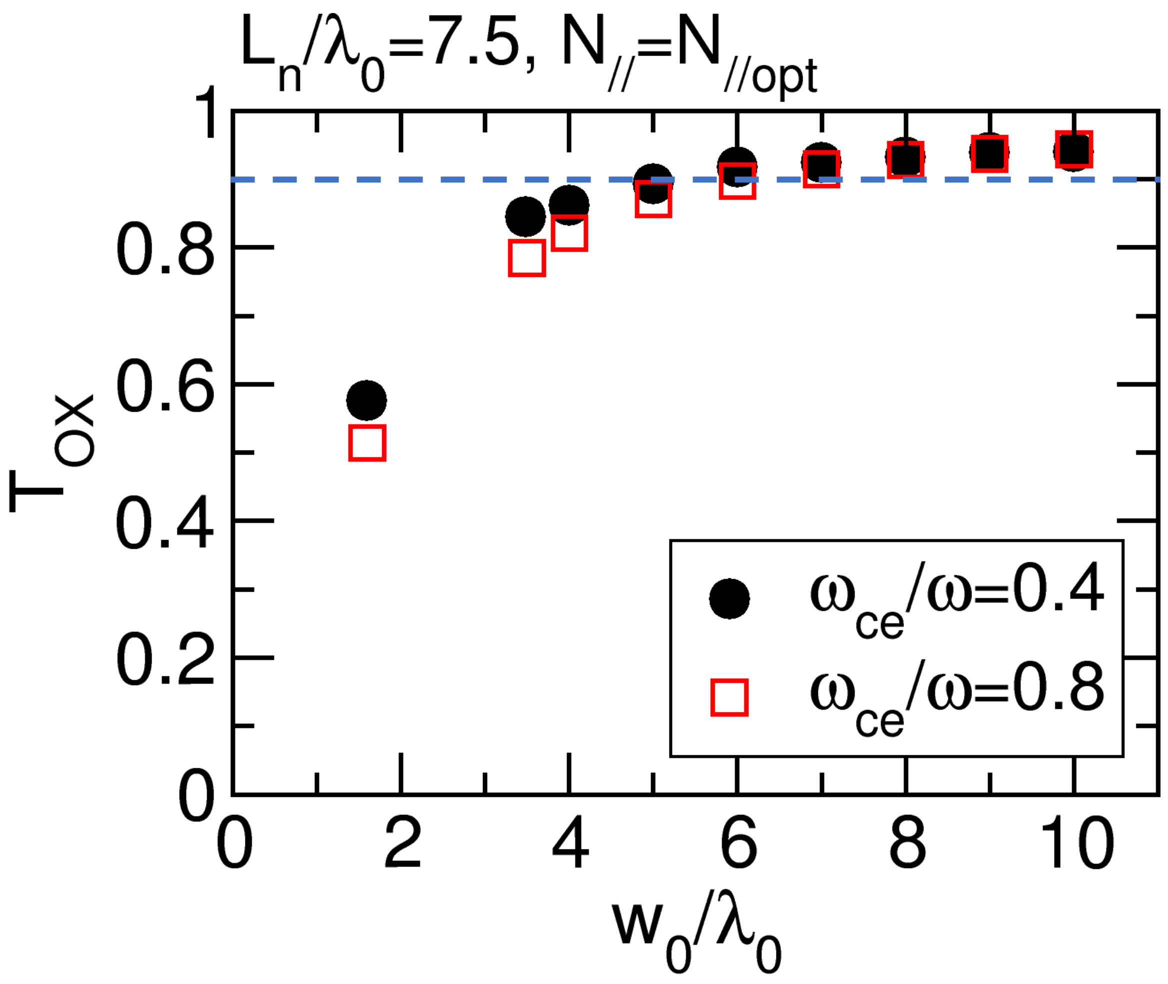
- ✓ The phase and amplitude along the y-axis vary with the propagation
- ✓ On the other hand, the $N_{//}$ Fourier spectrum is conserved

$L_n=7.5^* \lambda_0$, $w_0 /\lambda_0 =5.006$


z_{PC}/λ_0 =-100.0 z_{PC}/λ_0 =-30.0 z_{PC}/λ_0 =0 z_{PC}/λ_0 =50.0 z_{PC}/λ_0 = 100.0

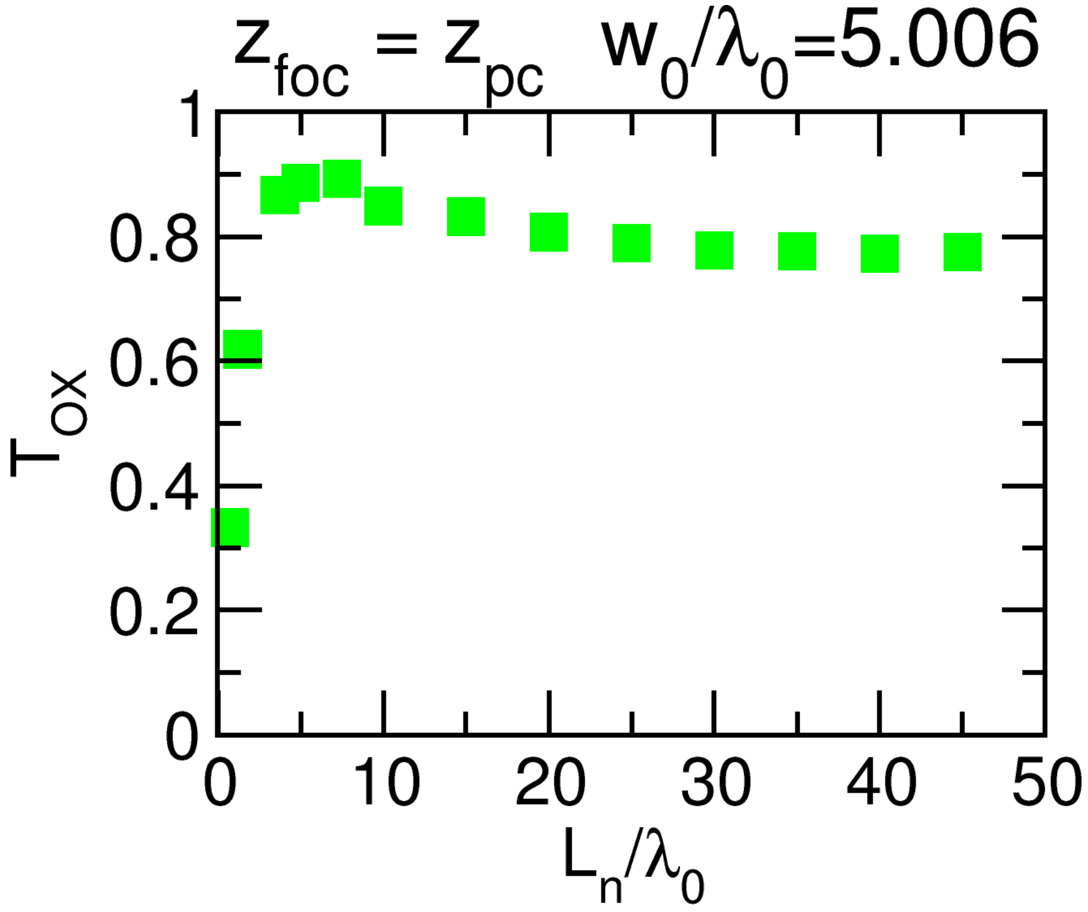
- ✓ With adopting a larger beam waist size w_0, the spreading of the $N_{//}$ Fourier spectrum can be reduced to obtain higher T_{OX}

T_{ox} with change of the beam waist size w_0 for the case of ω_{ce}/ω = 0.4 and 0.8

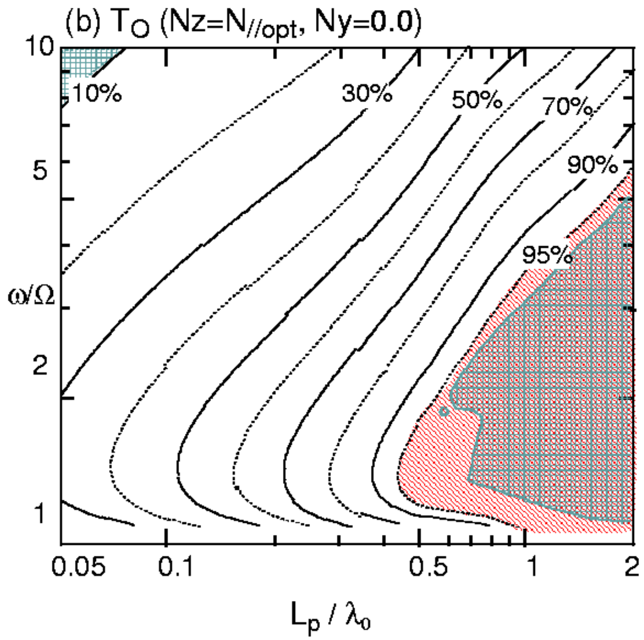


- ✓ **w_0/λ_0 > 6 is required for T_{ox} > 90 % with L_n/λ_0 =7.5**
- ✓ Higher T_{ox} can be obtained with lower ω_{ce}/ω for the small waist size beam

Estimation of T_{ox} with change of the density gradient



T_{ox} decreases as increase of L_n, however, it converges with 80% for L_n/λ_0 > 30



Decrement of Tox in low L_n/λ_0 region might cause by transmission of the X-mode toward the lower density side after the O-X mode conversion as the 1D calculation suggested

H. Igami et al., Plasma Phys. Control. Fusion **48** (2006) 573–598

Summary : purpose and method

- **To provide a guideline for designing the launching beam for ECRH by EBW,** wave optical analyses were performed for various normalized density scale lengths and beam parameters with using of TASK/WF2D code

- T_{OX} is estimated based on the cold plasma UH resonant absorption model in slab plasmas in the uniform magnetic field

Summary : results and future works

- **The beam waist size w_0 affects T_{ox}**
- **With adopting sufficiently large w_0/λ_0, higher T_{OX} can be obtained so that the spreading of the $N_{//}$ Fourier spectrum can be reduced**
  - consistent with the previous 2D numerical analysis (Y. Oka, et al., COMSOL Conference 2020, K. Nagasaki et al., EPS 2020)
- **Though T_{ox} decreases as increase of L_n, it converges with 80 % for $L_n/\lambda_0 > 30$ with adopting sufficiently large w_0/λ_0 ($\gtrsim 5$)**

Future works

- Effect of the magnetic shear on for finite width waves
- Propagation characteristics during the X-B mode with introducing integral from of dielectric tensor