



A model of non-Maxwellian electron distribution for the analysis of ECE in JET discharges

G. Giruzzi (CEA, IRFM, France)

The logo for the Joint European Torus (JET), consisting of the letters "JET" in a large, bold, blue, italicized sans-serif font.



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- What **is seen** by ECE harmonics and by Thomson scattering
- Evidence of **non-Maxwellian electron distributions** in JET high- T_e database (M. Fontana's talk)
- **ECE sets constraints** on the electron distribution function
- **Toy model** of perturbed electron distribution function: a data analysis tool
- Model-experiment **comparison**
- Conclusions. Possible **interpretations**

Note: this work continues and develops previous analyses made by E. De la Luna and V. Krivenski:

- *E. de la Luna et al., Rev. Sci. Instr. **74**, 1414 (2003)*
- *V. Krivenski, Fus. Eng. Des. **53**, 23 (2001)*

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Radiation temperature

$$T_{rad}(\omega) = \int_{R_0-a}^{R_0+a} dR \beta(R) \exp\left(-\int_R^{R_0+a} \alpha(R') dR'\right)$$

Optical depth

$$\tau = \int_{R_0-a}^{R_0+a} \alpha(R) dR$$

absorption coefficient: $\alpha \propto \int d\vec{p} p_{\perp}^{2n-1} \frac{\partial f}{\partial p_{\perp}} \delta\left(\gamma - n \frac{\omega_c}{\omega}\right)$

emission coefficient: $\beta \propto \int d\vec{p} \frac{p_{\perp}^{2n}}{\gamma} f \delta\left(\gamma - n \frac{\omega_c}{\omega}\right) \quad \vec{p} = m\gamma\vec{v}$

for a **Maxwellian:** $\beta = T_e \alpha$ (**Kirchhoff's law** \rightarrow plasma is a black body)

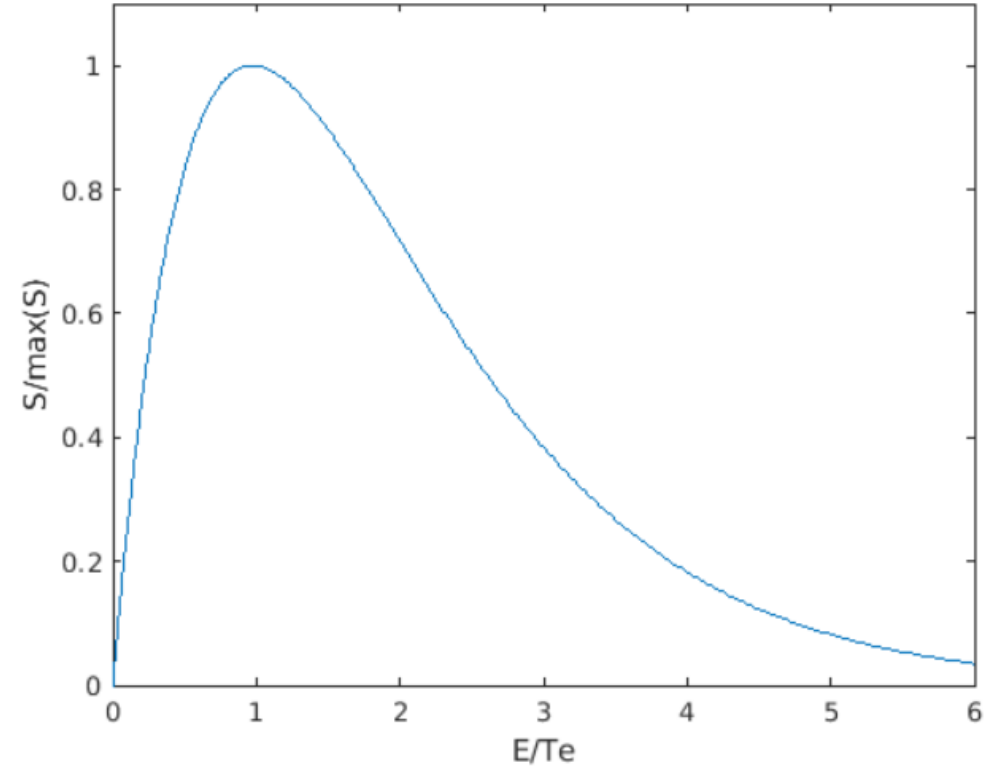
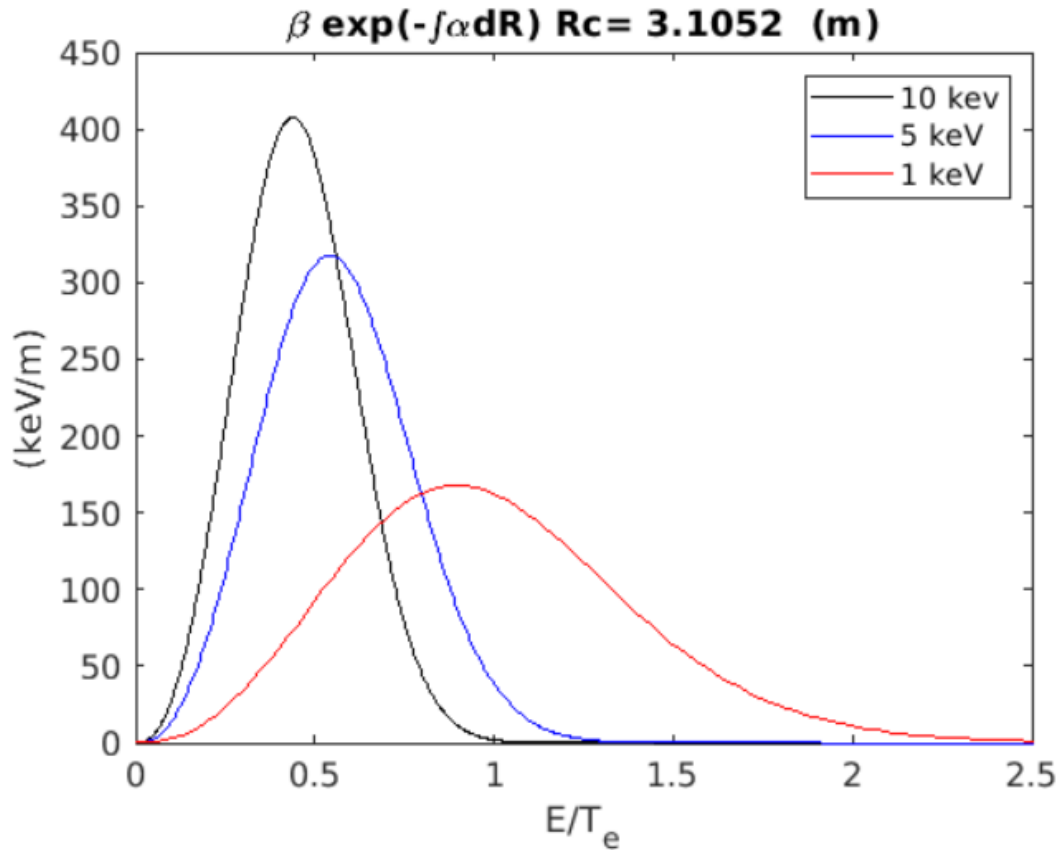
$$\omega = \frac{n\omega_c(R)}{\gamma}, \quad \gamma = \sqrt{1 + \frac{p^2}{(mc)^2}}, \quad \text{cold resonance position: } n\omega_c(R_c) = \omega$$

kinetic energy: $E_k = mc^2(\gamma - 1) = mc^2 \left(\frac{R_c}{R} - 1\right)$

What is seen by 2nd harmonic ECE and by Thomson (in energy)

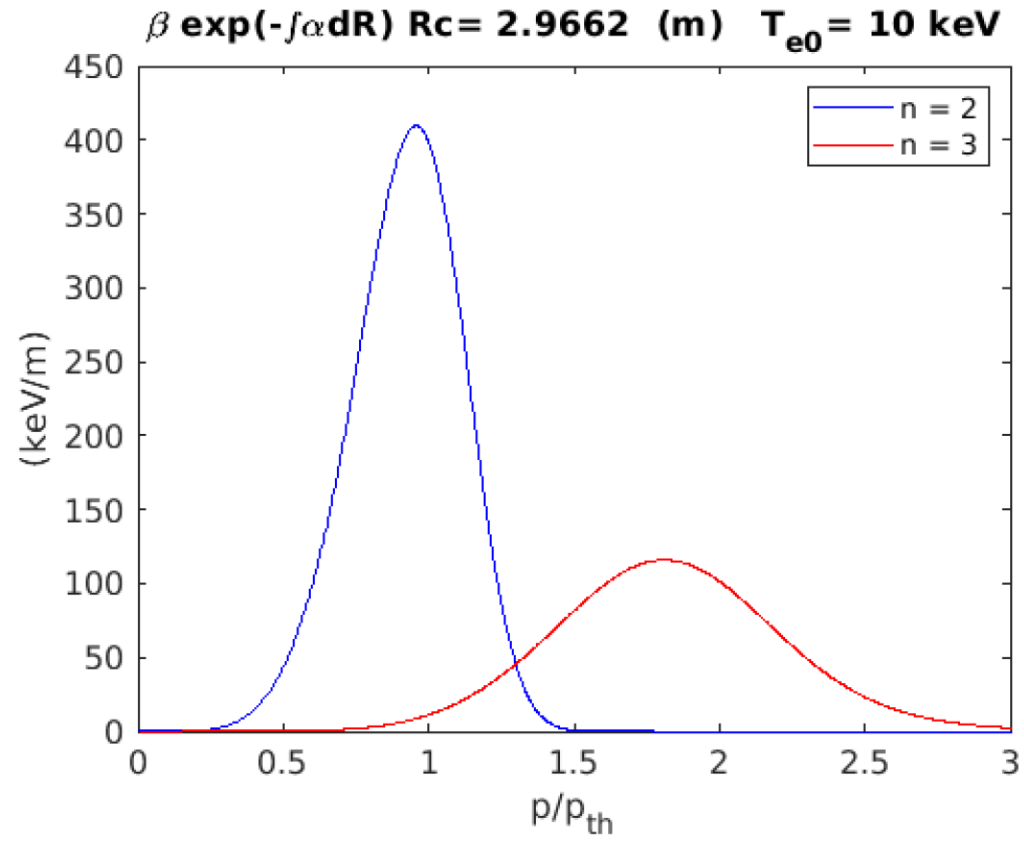


From S.L. Prunty 2014 *Phys. Scr.* **89** 128001, Eqs. 5.8, 5.9 (neglecting depolarisation effect):

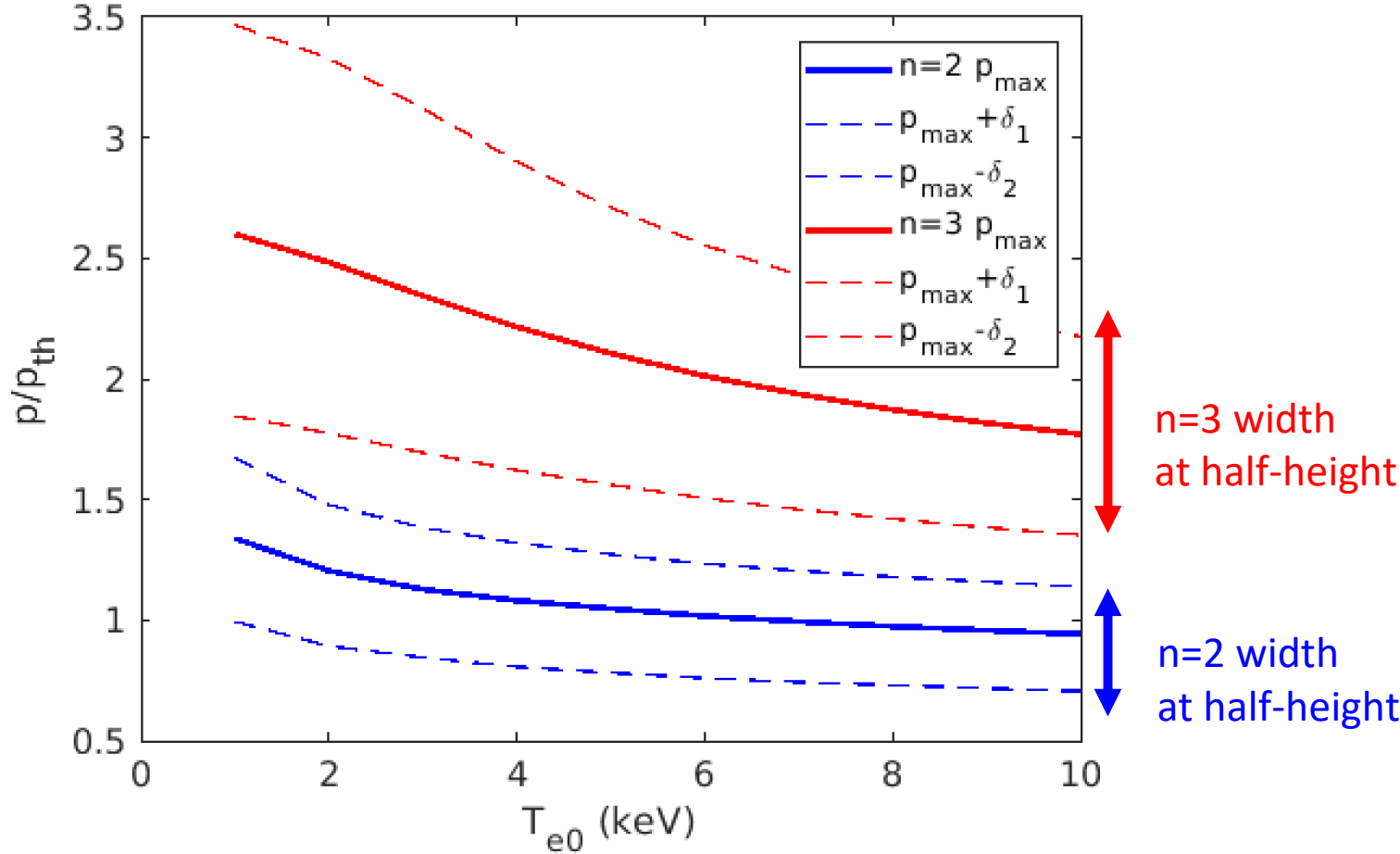


$$S \propto \frac{\left(\frac{v}{c}\right)^2}{\left(1 - \left(\frac{v}{c}\right)^2\right)^{3/2}} \exp\left(-\frac{mc^2/T_e}{\left(1 - \left(\frac{v}{c}\right)^2\right)^{1/2}}\right)$$

2nd & 3rd harmonics see different momenta → constraints on $f(p)$

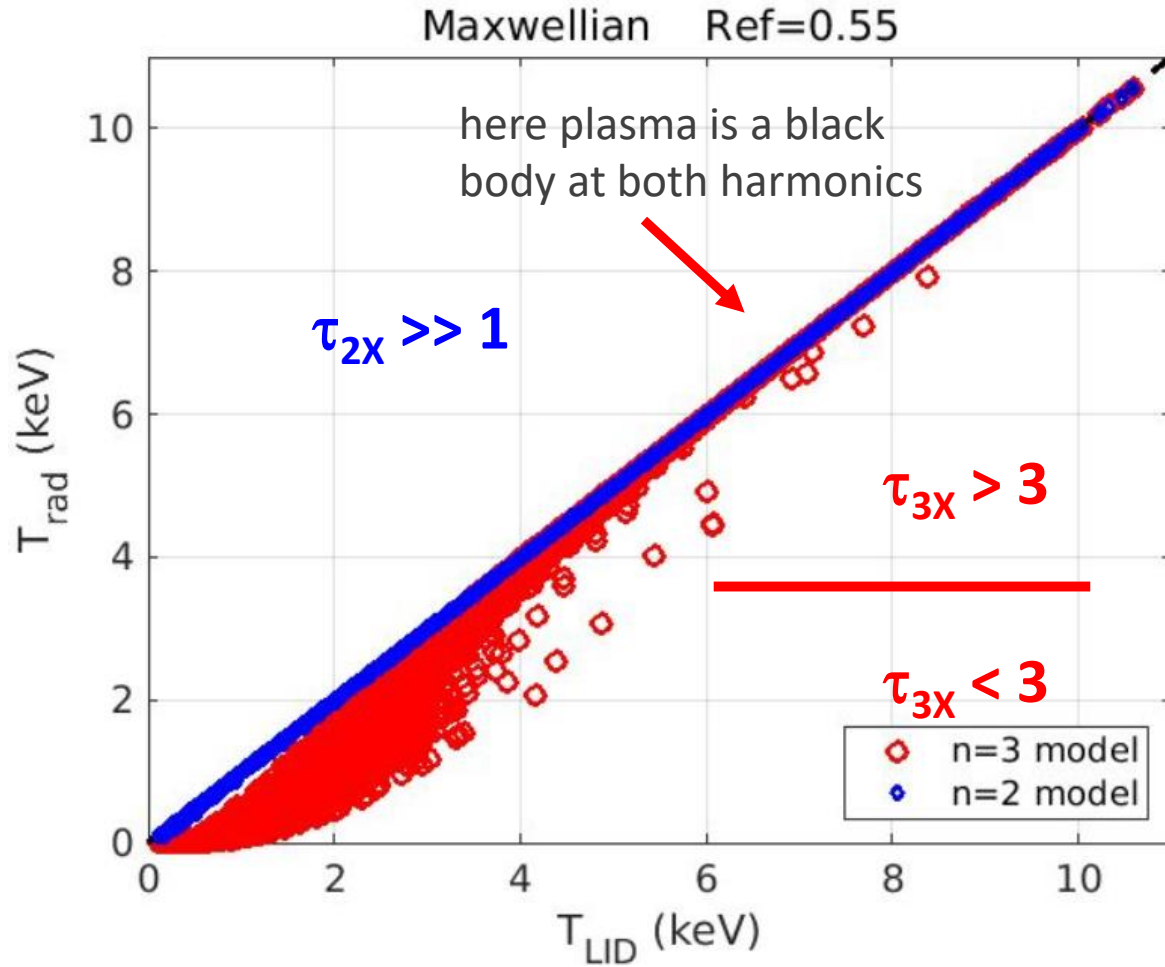


$$p_{th} = mv_{th}$$



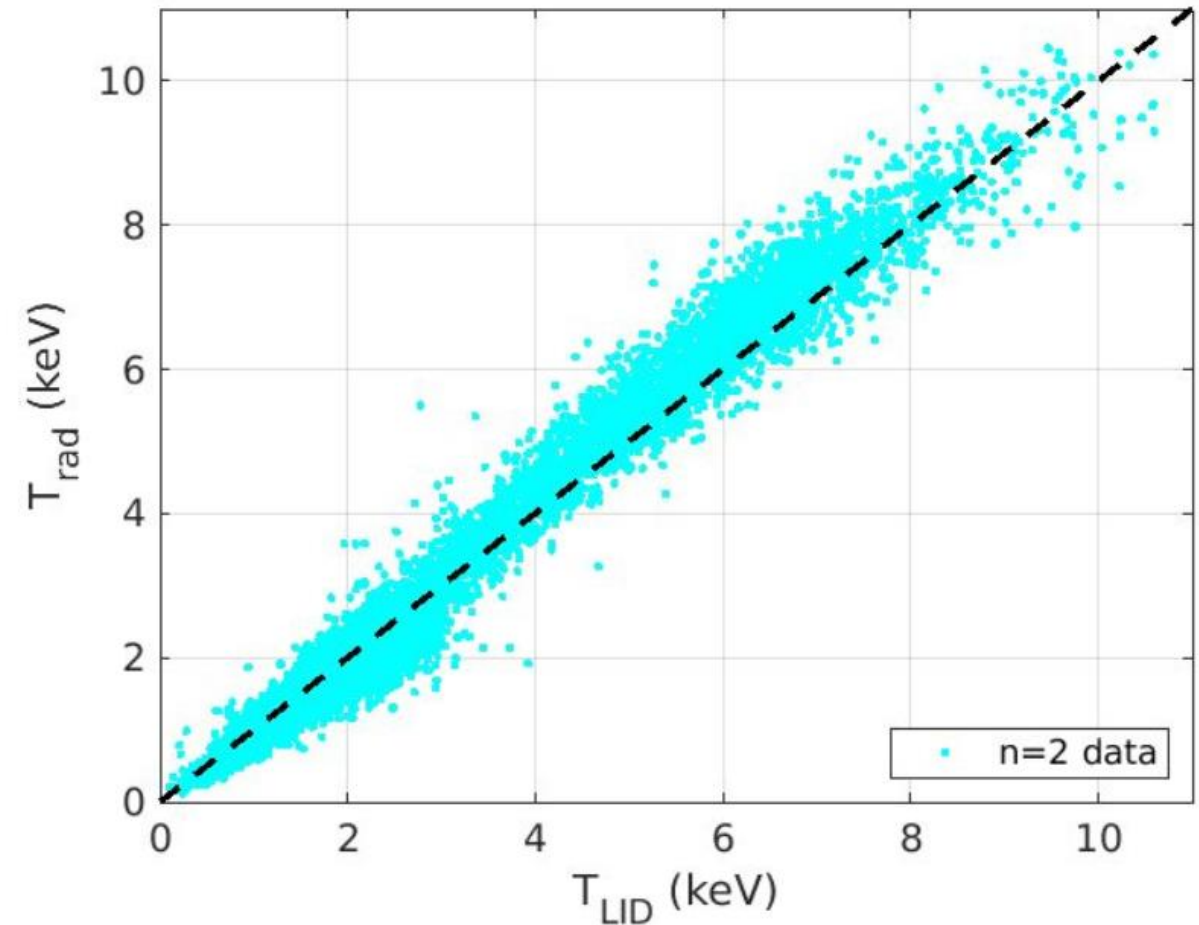
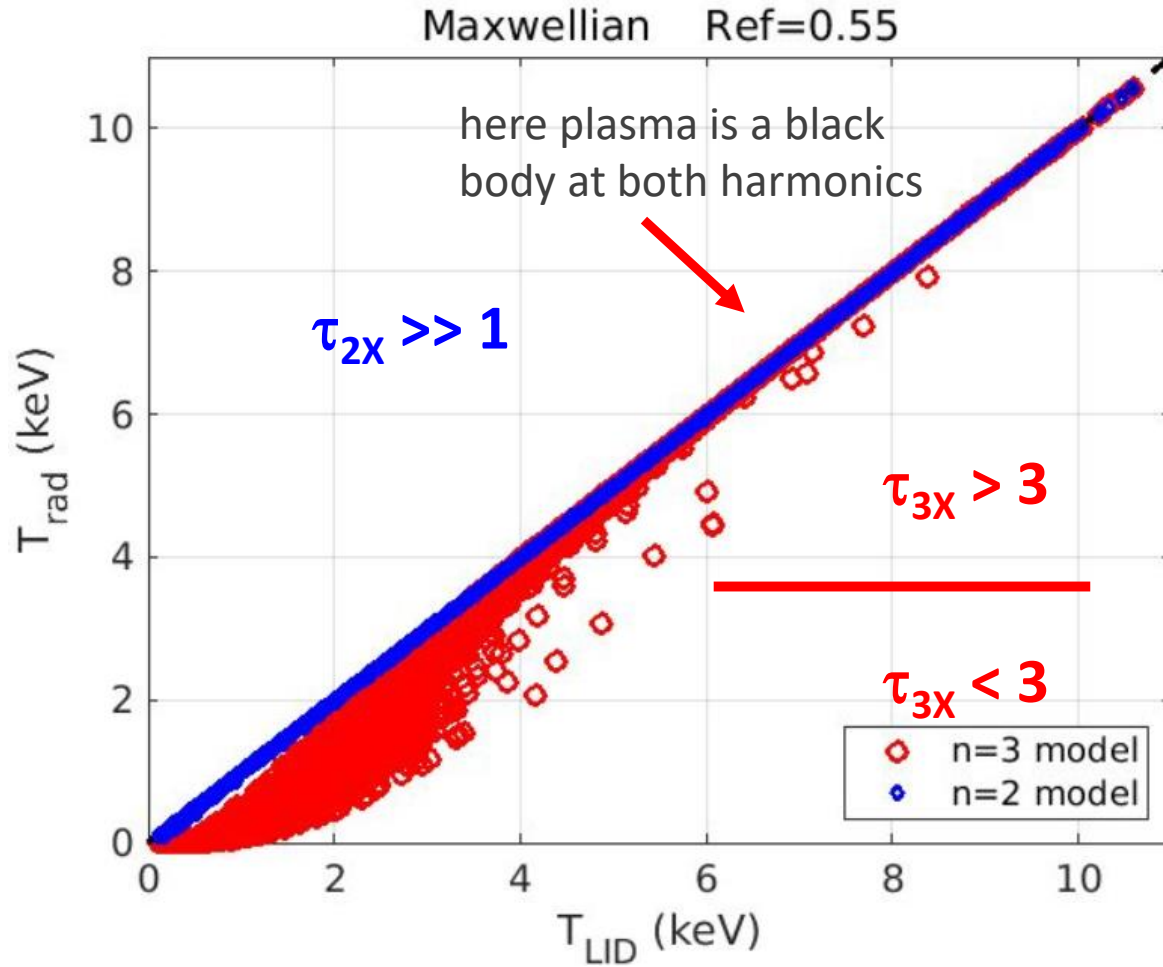


Maxwellian predictions and data



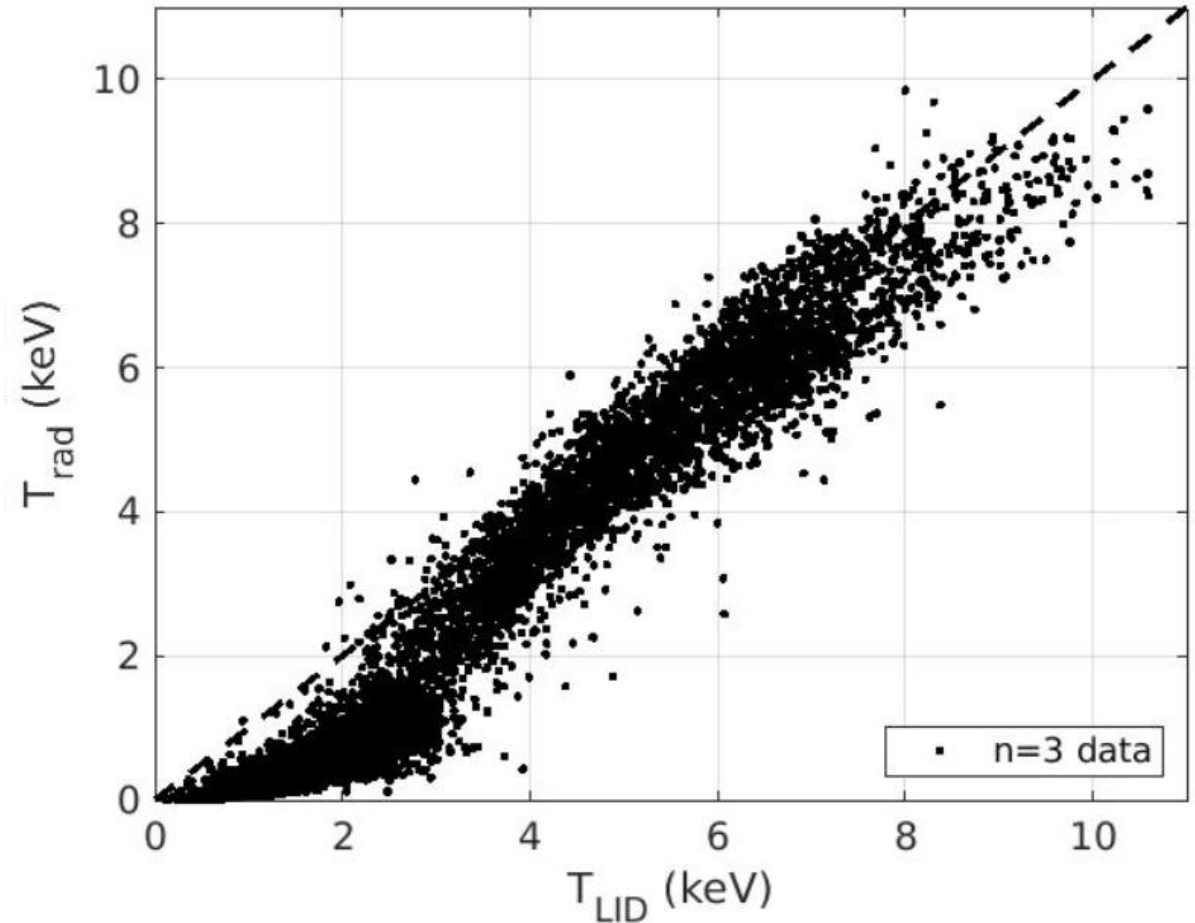
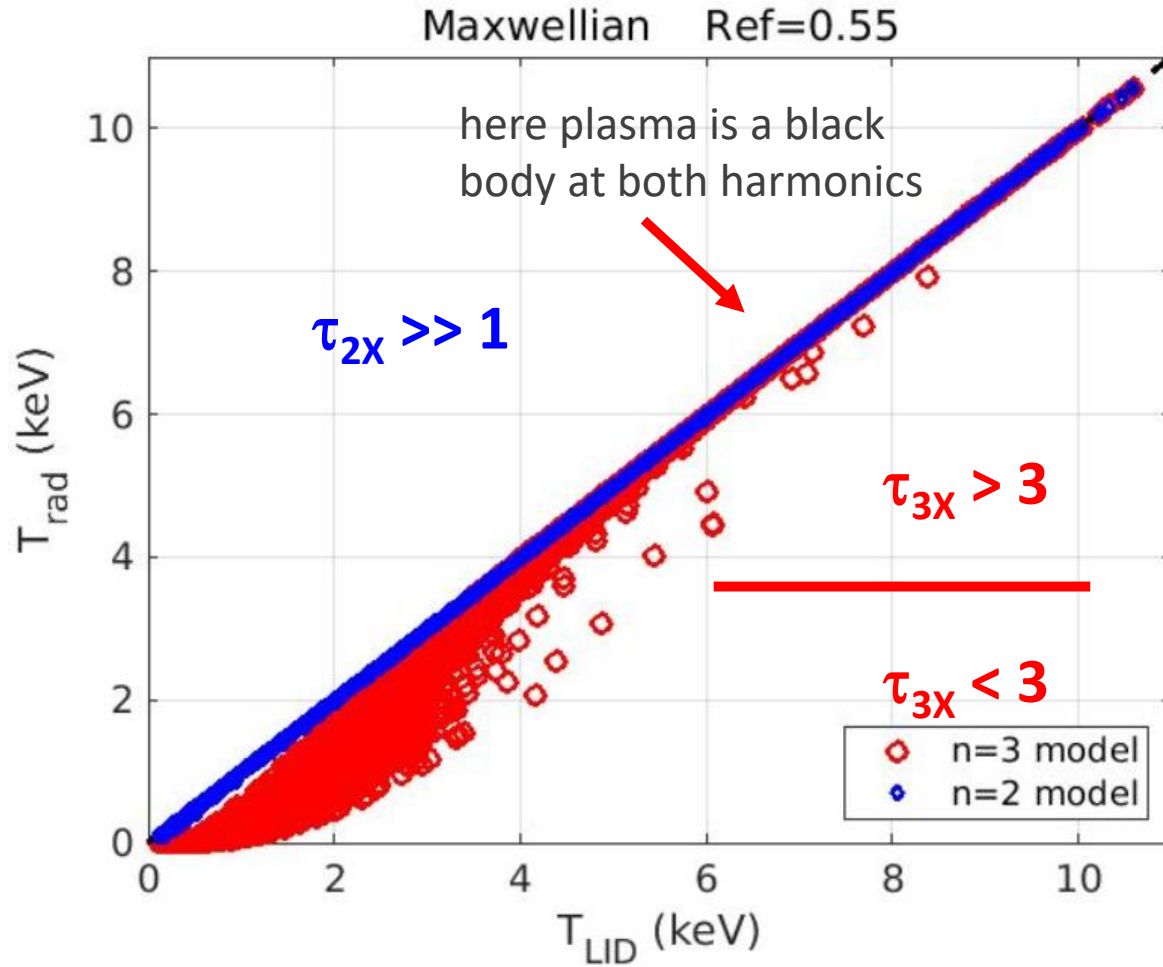


Maxwellian predictions and data



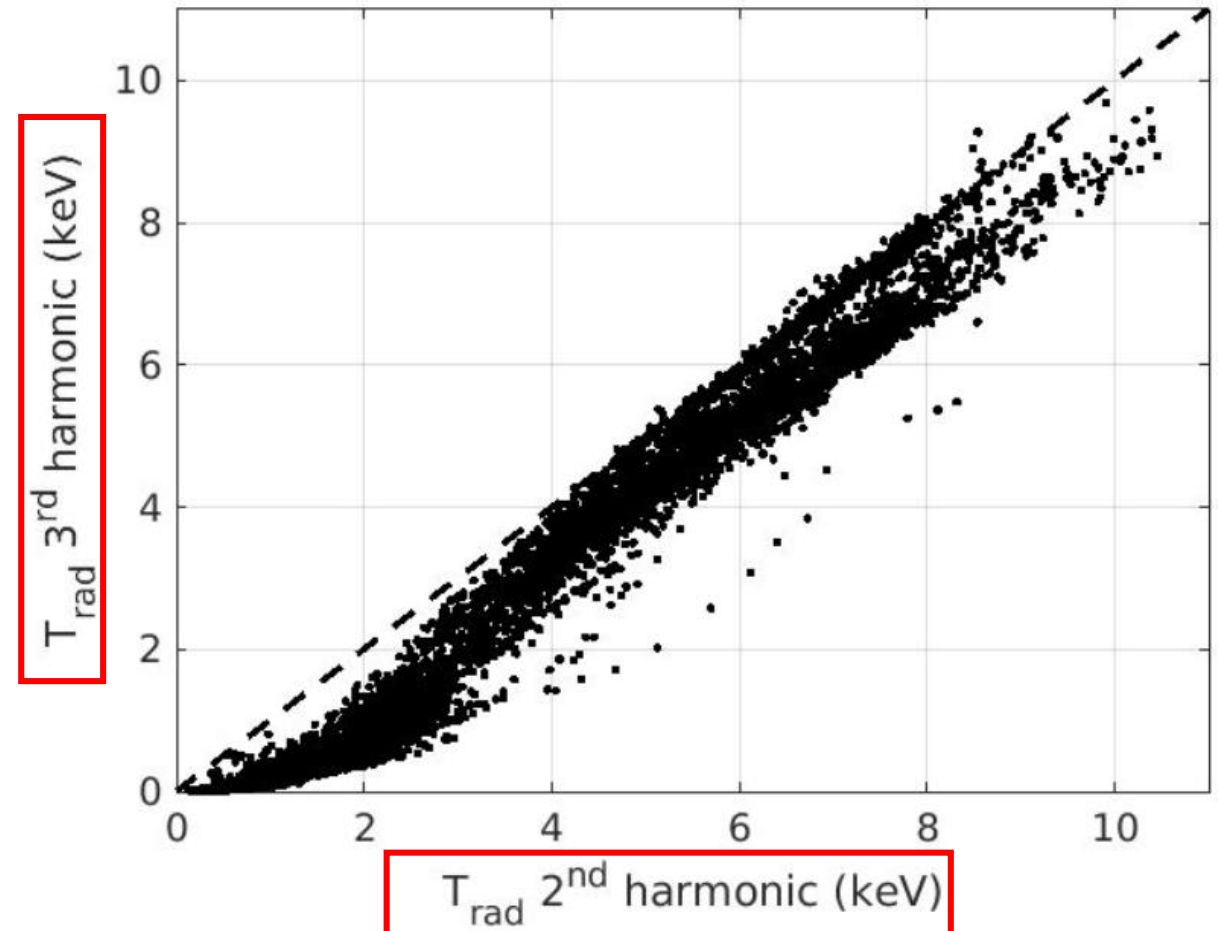
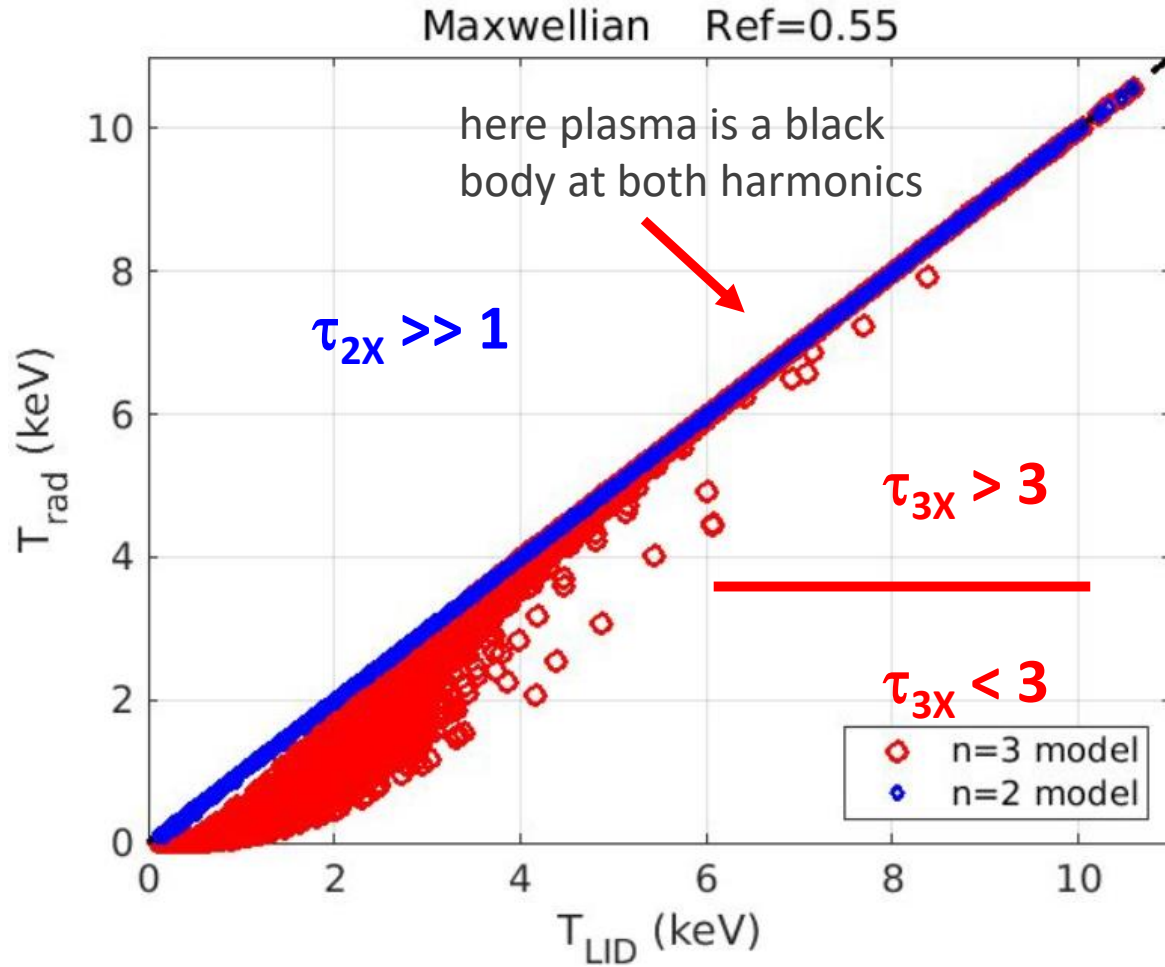


Maxwellian predictions and data



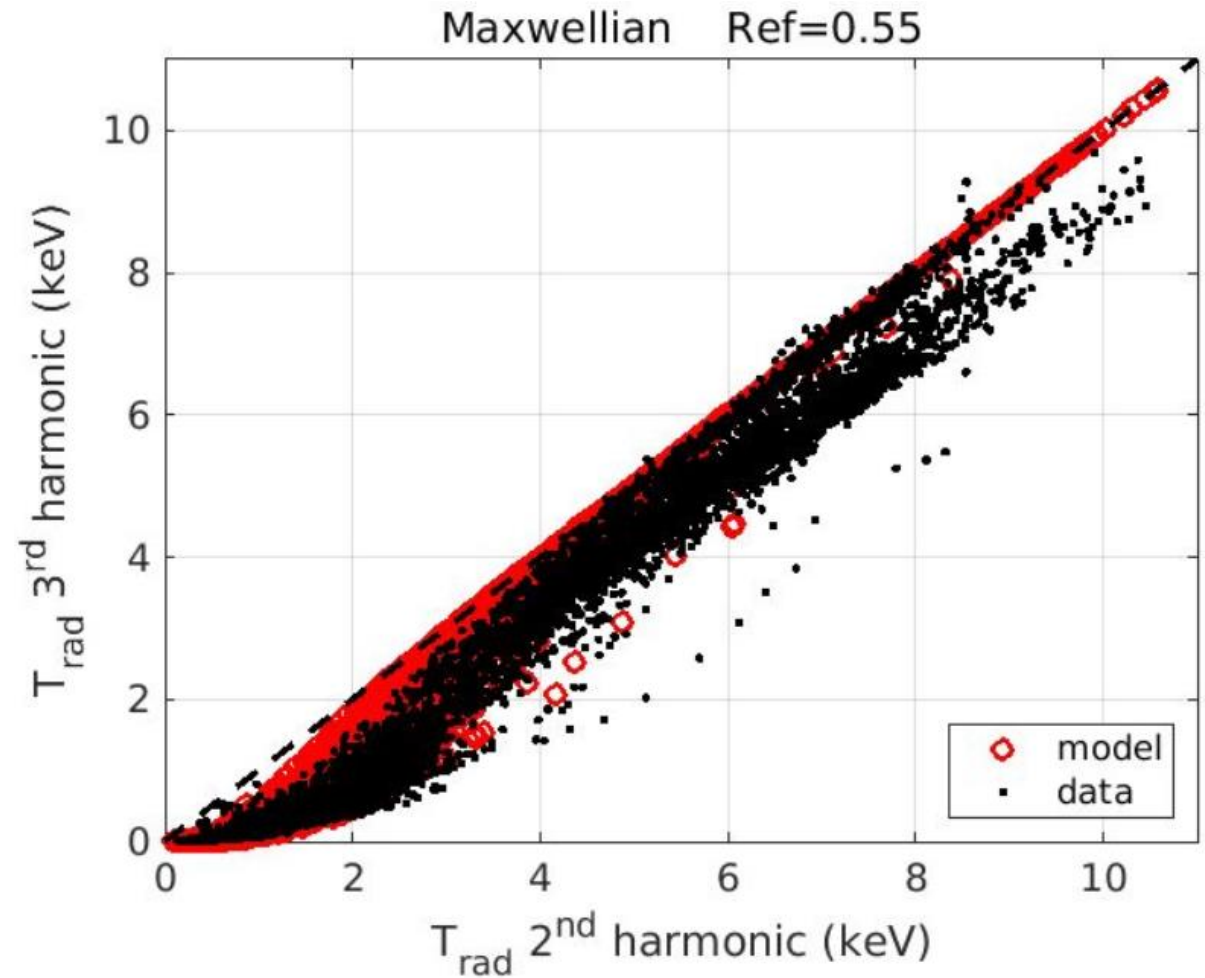
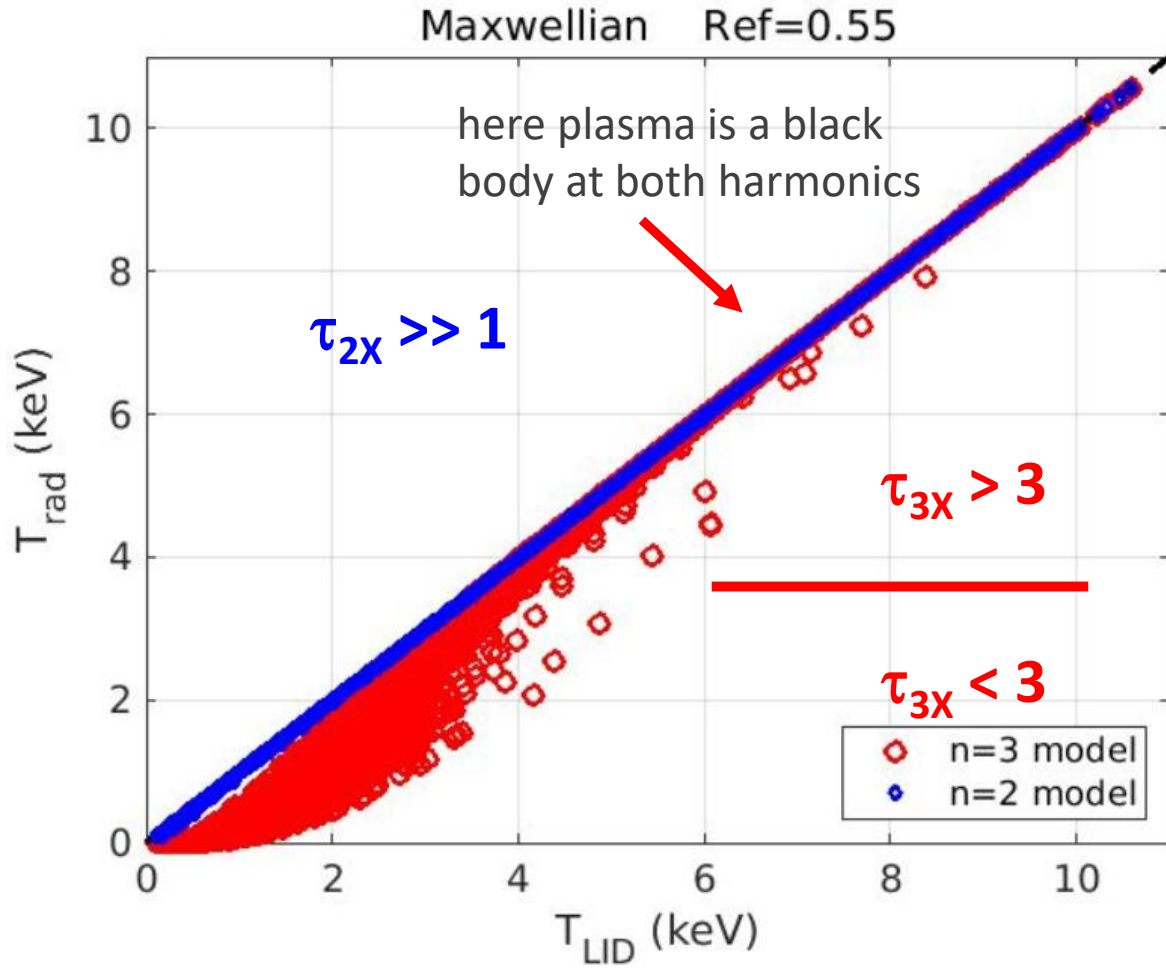


Maxwellian predictions and data





Maxwellian predictions and data



Toy model of perturbed electron distribution function



Relativistic Maxwellian distribution in momentum p :

$$f_M = Ae^{-\mu(\gamma-1)} \quad A = \frac{\mu e^{-\mu}}{4\pi K_2(\mu)(mc)^3} \quad \mu = \frac{mc^2}{T_e}$$

Perturbed distribution:

$$f = A(e^{-\mu(\gamma-1)} + f_1)$$

Bipolar isotropic perturbation:

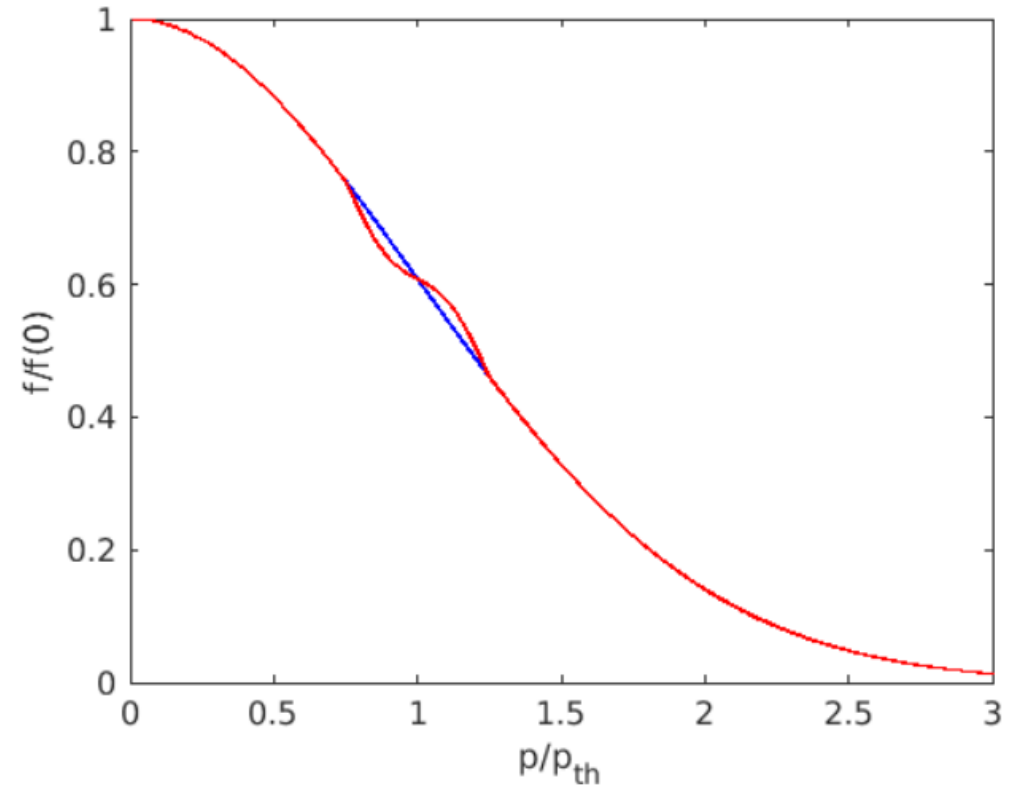
$$f_{1u} = f_0 \sin \left[\frac{\pi}{\delta} (p - p_0) \right] \quad \text{for } p_0 - \delta < p < p_0 + \delta$$

Some anisotropic perturbations (θ = pitch-angle), examples:

$$\begin{aligned} f_{1s} &= f_{1u} \sin \theta & f_{1c2} &= f_{1u} \cos^2 \theta \\ f_{1s2} &= f_{1u} \sin^2(2\theta) & f_{1c} &= f_{1u} \cos \theta \end{aligned}$$

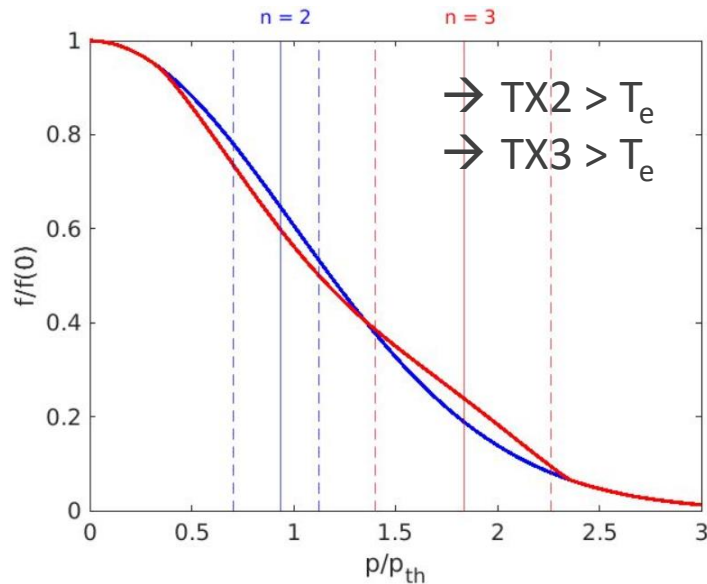
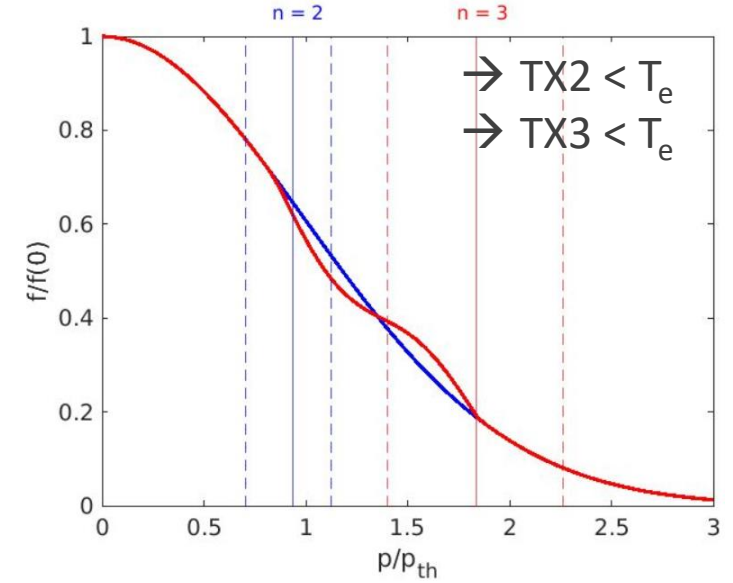
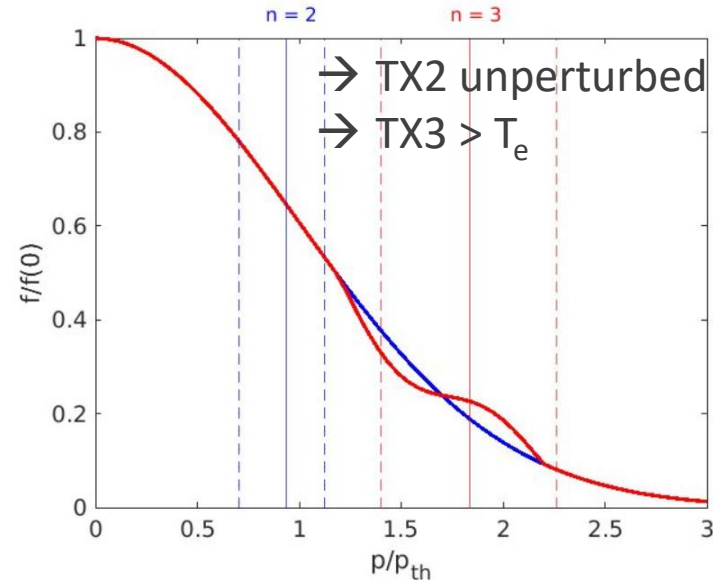
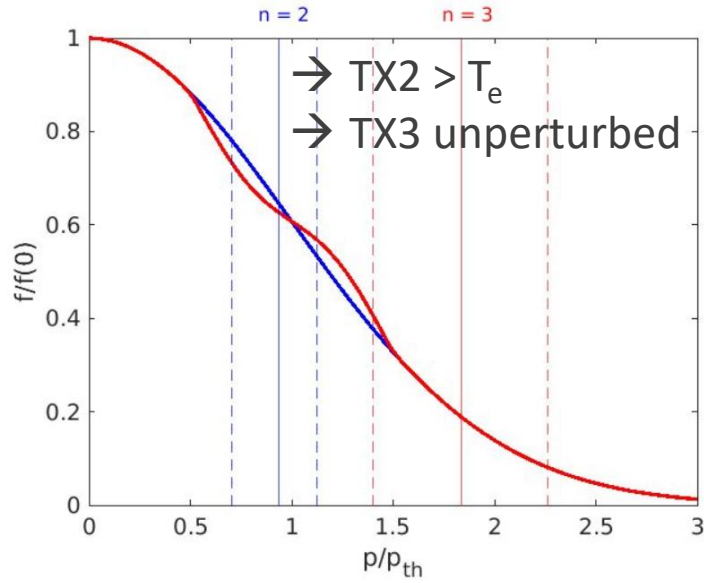
All these functions allow analytical calculation of EC absorption and emission coefficients.

In the following, only the isotropic perturbation is used



$f_0 = 0.03$ $p_0/p_{th} = 1$ $\delta/p_{th} = 0.25$
↑ ↑ ↑
intensity **location** **width**

Different perturbation locations and widths \rightarrow different effects



Various possibilities depending on perturbation location and width:

- TX2 $>$ T_e and TX3 $<$ T_e (most usual one)
- TX2 Maxwellian and TX3 non-Maxwellian
- TX2 non-Maxwellian and TX3 Maxwellian
- both TX2 and TX3 $<$ T_e
- both TX2 and TX3 $>$ T_e

Absorption and emission coefficients of X mode



$$\alpha = A_0 \sum_n \alpha_n \quad \alpha_n = A_n \left[\frac{\mu u_n}{n\omega_c/\omega} e^{-\mu(\frac{n\omega_c}{\omega}-1)} - Q_n h(u_n) \frac{\pi}{\delta/mc} f_0 \cos\left(\frac{\pi}{\delta/mc}(u_n - u_0)\right) \right]$$

$$A_0 = \frac{2\pi^2}{N_X} \frac{\omega}{c} \frac{\omega_p^2}{\omega^2} \left| 1 - \frac{i\varepsilon_{12}}{\varepsilon_{11}} \right|^2 \frac{\mu e^{-\mu}}{4\pi K_2(\mu)} \quad A_n = \mu^{n-1} \frac{n\omega_c}{\omega} u_n^{2n} \left(\frac{N_X \omega}{\sqrt{\mu} \omega_c} \right)^{2(n-1)} \frac{B(n+1, 1/2)}{[2^n (n-1)!]^2}$$

$$\mu = \frac{mc^2}{T_e} \quad u_n = \left[\left(\frac{n\omega_c}{\omega} \right)^2 - 1 \right]^{1/2} \quad u_0 = \frac{p_0}{mc} \quad h(u_n) = H\left(u_n - u_0 + \frac{\delta}{mc}\right) H\left(u_0 + \frac{\delta}{mc} - u_n\right)$$

$N_X, \varepsilon_{11}, \varepsilon_{12}$ → cold plasma X-mode refractive index and dielectric tensor elements

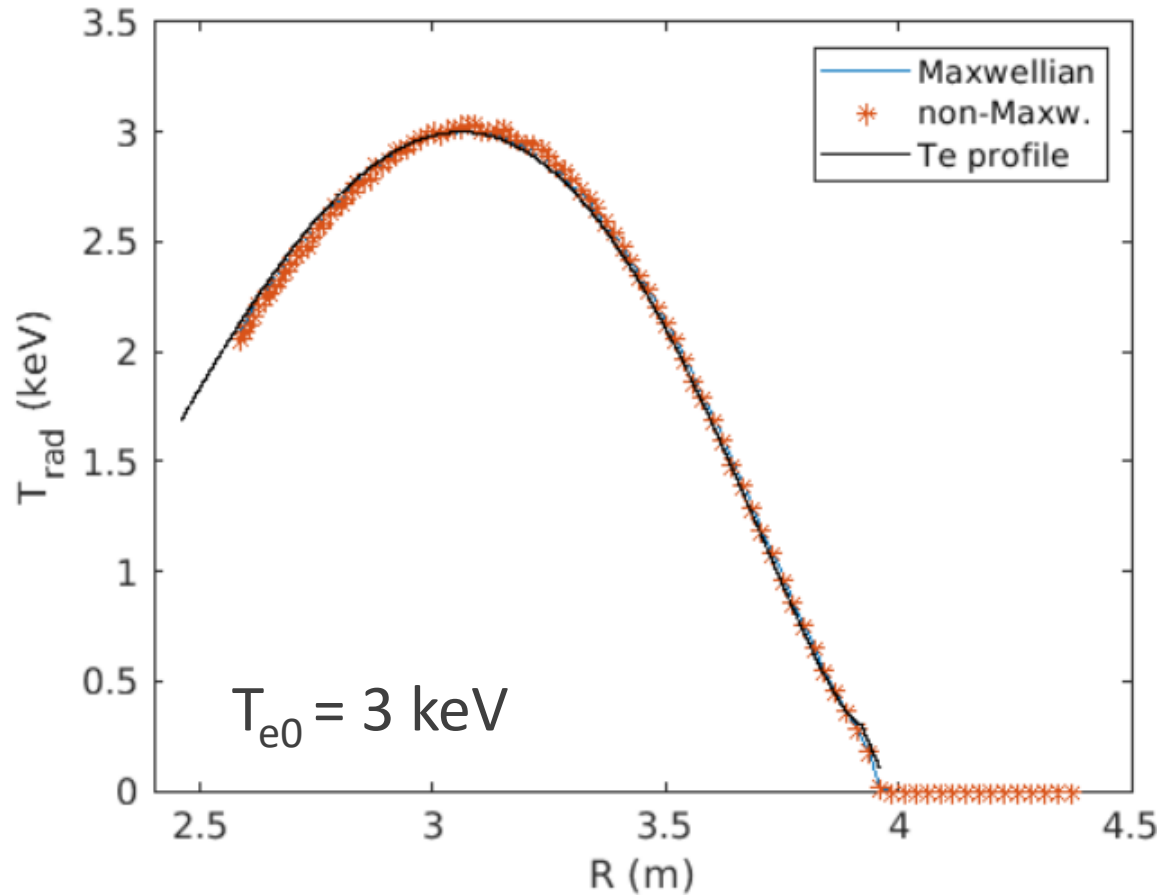
$B(x, y)$ = Beta function $K_2(x)$ = modified Bessel function $H(x)$ = Heaviside function

$$\beta = A_0 \sum_n \beta_n \quad \beta_n = A_n \frac{mc^2 u_n}{n\omega_c/\omega} \left[e^{-\mu(\frac{n\omega_c}{\omega}-1)} + Q_n h(u_n) f_0 \sin\left(\frac{\pi}{\delta/mc}(u_n - u_0)\right) \right]$$

$$f_1 = f_{1u} \rightarrow Q_n = 1 \quad f_1 = f_{1u} \sin\theta \rightarrow Q_n = \frac{B(n+3/2, 1/2)}{B(n+1, 1/2)} \quad f_1 = f_{1u} \cos^2\theta \rightarrow Q_n = \frac{B(n+1, 3/2)}{B(n+1, 1/2)}$$

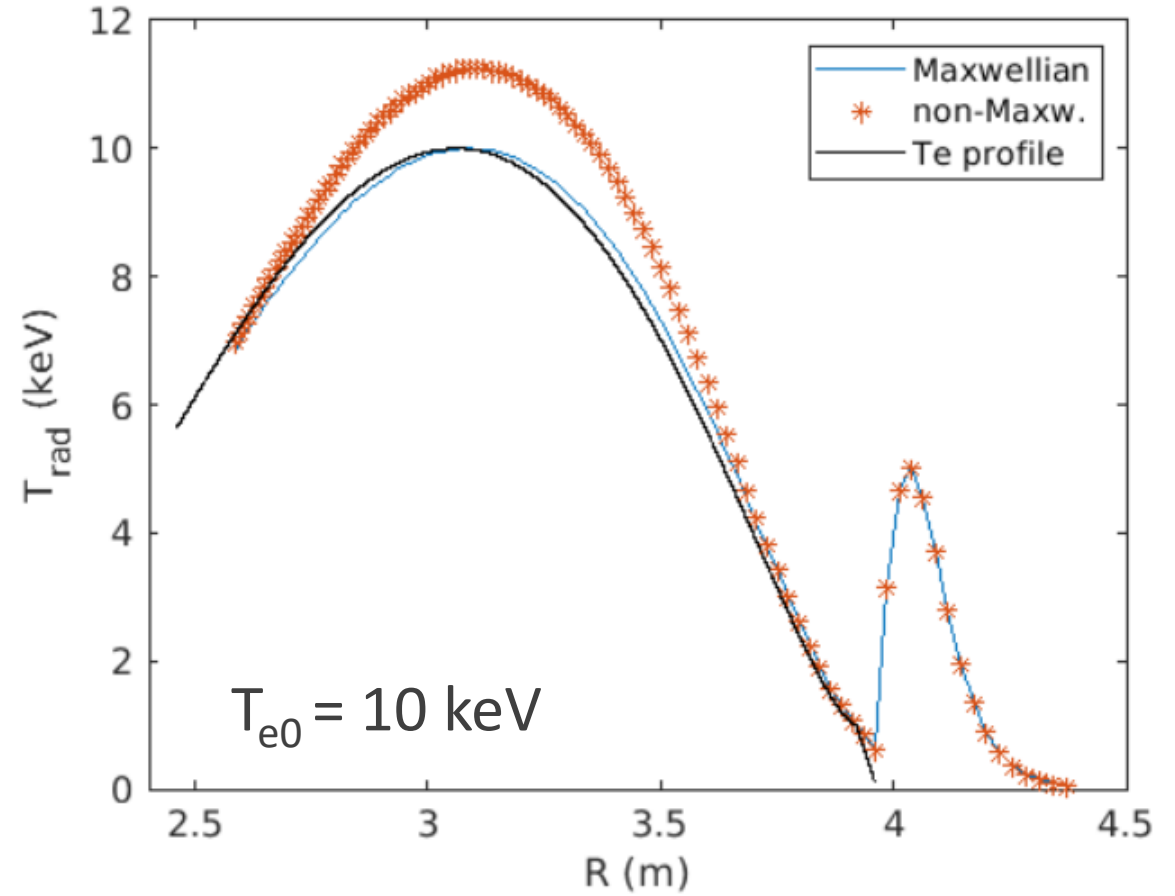
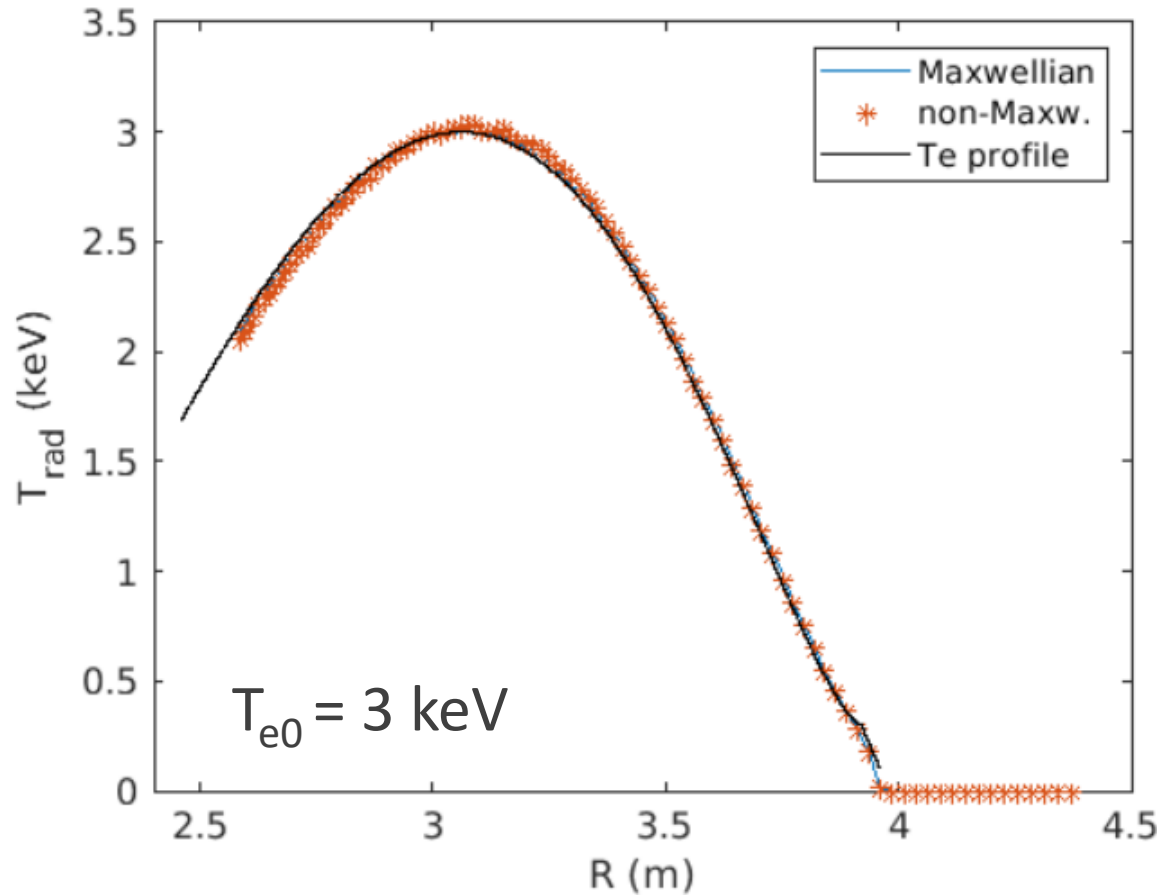
$$f_1 = f_{1u} \sin^2(2\theta) \rightarrow Q_n = 4 \frac{B(n+2, 3/2)}{B(n+1, 1/2)} \quad f_1 = f_{1u} \cos\theta \rightarrow Q_n = \frac{B(n+1, 1)}{B(n+1, 1/2)}$$

ECE X2 profiles affected at high temperature only



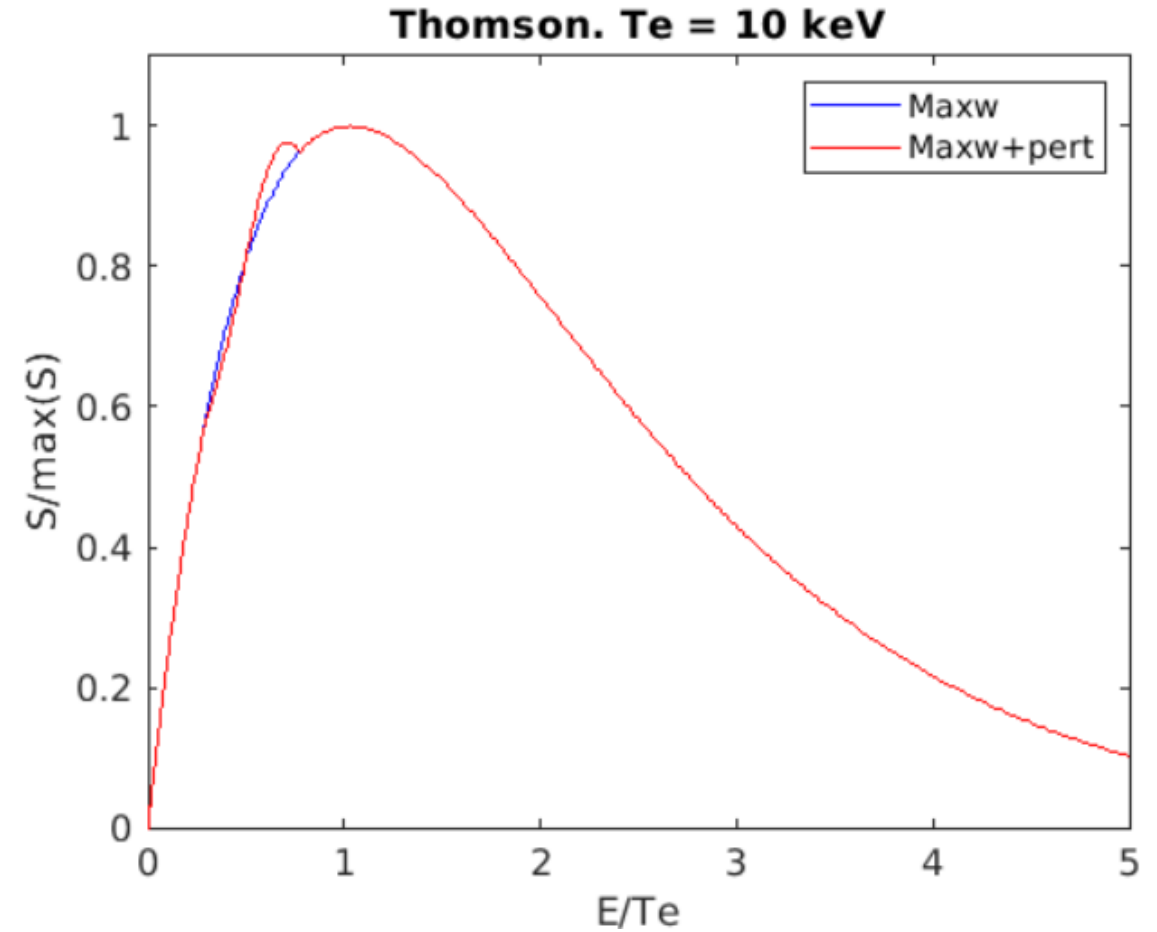
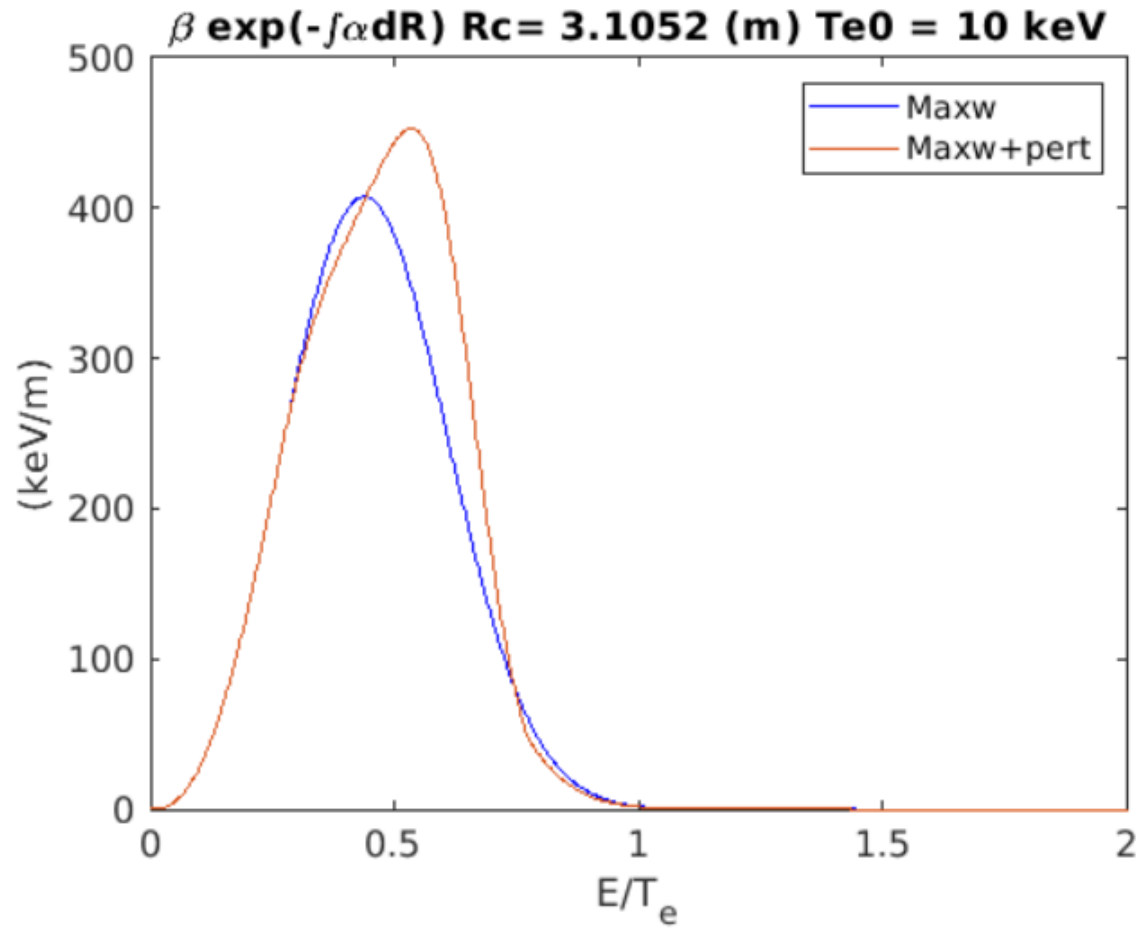
isotropic perturbation, $f_0 = 0.03$ $p_0/p_{\text{th}} = 1$ $\delta/p_{\text{th}} = 0.25$

ECE X2 profiles affected at high temperature only



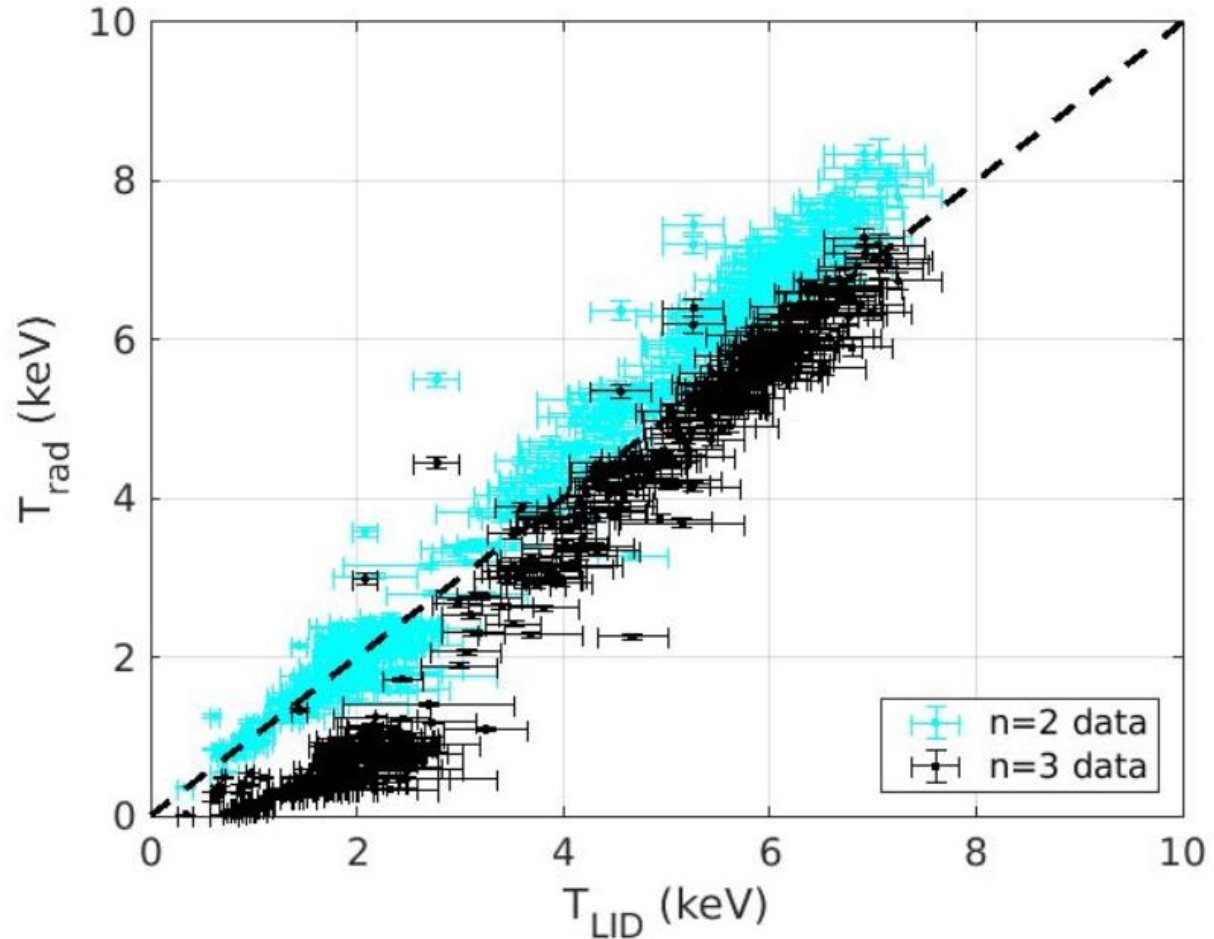
isotropic perturbation, $f_0 = 0.03$ $p_0/p_{\text{th}} = 1$ $\delta/p_{\text{th}} = 0.25$

Effect of perturbation on ECE X2 and Thomson spectra



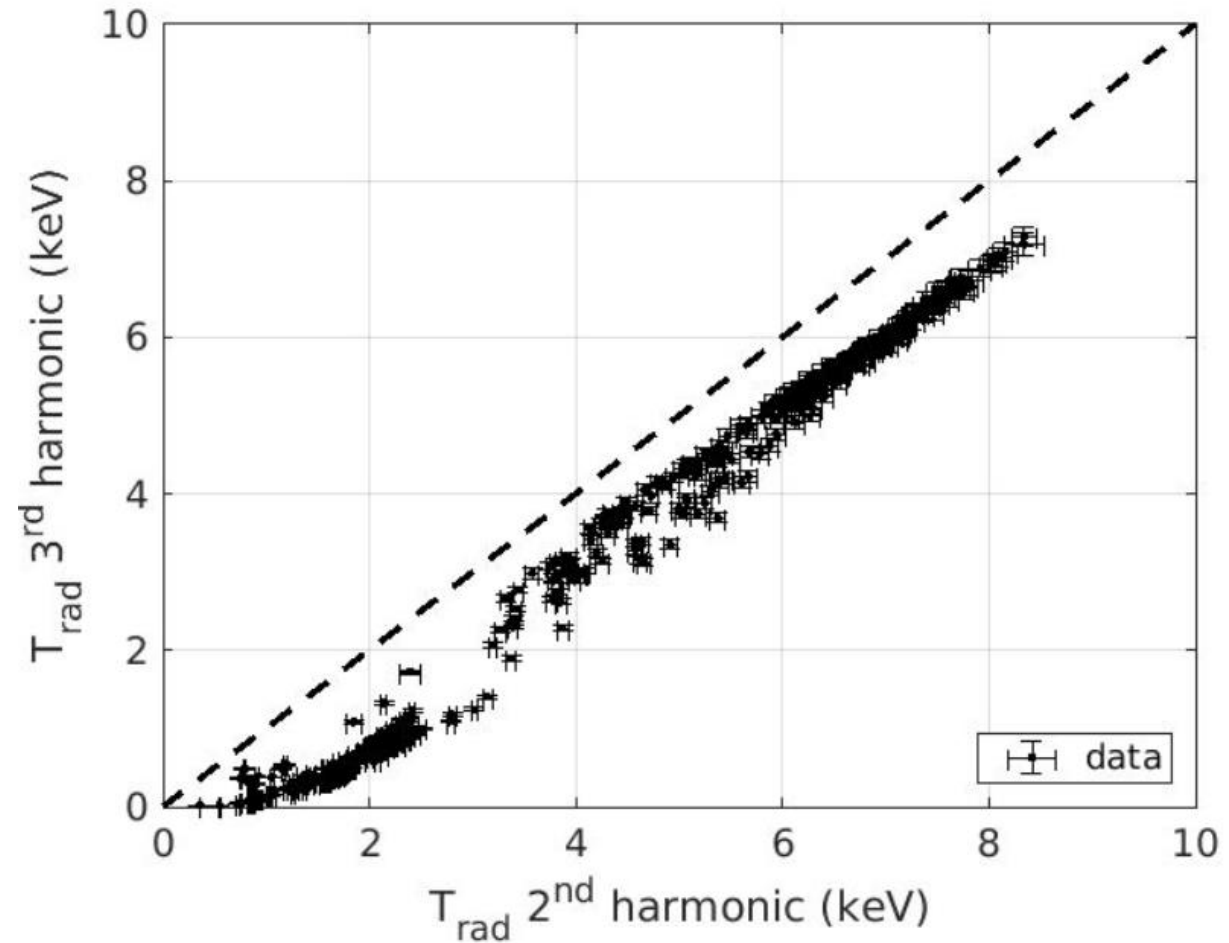
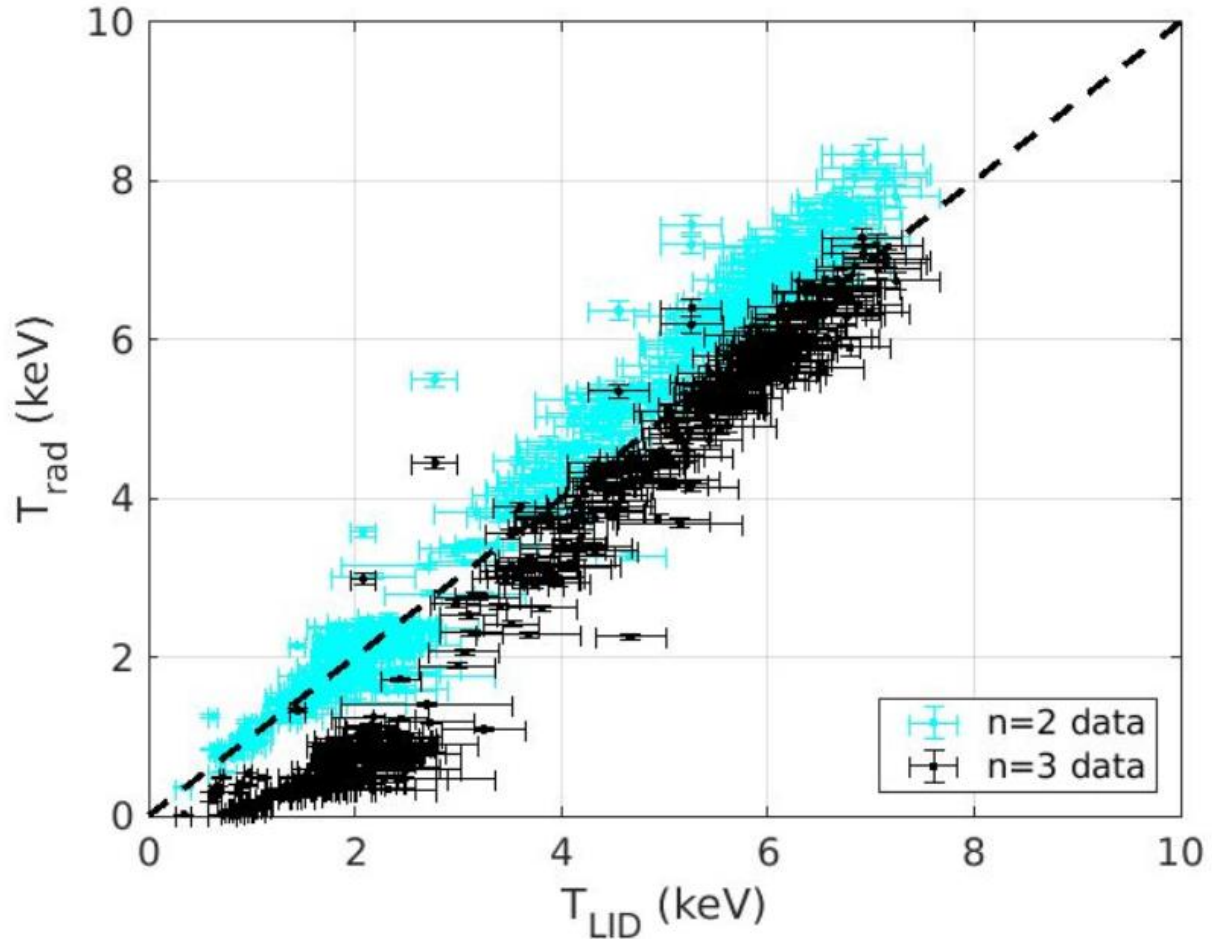


96990, 96992, 96993, 96994, 96996, 96998, 96999





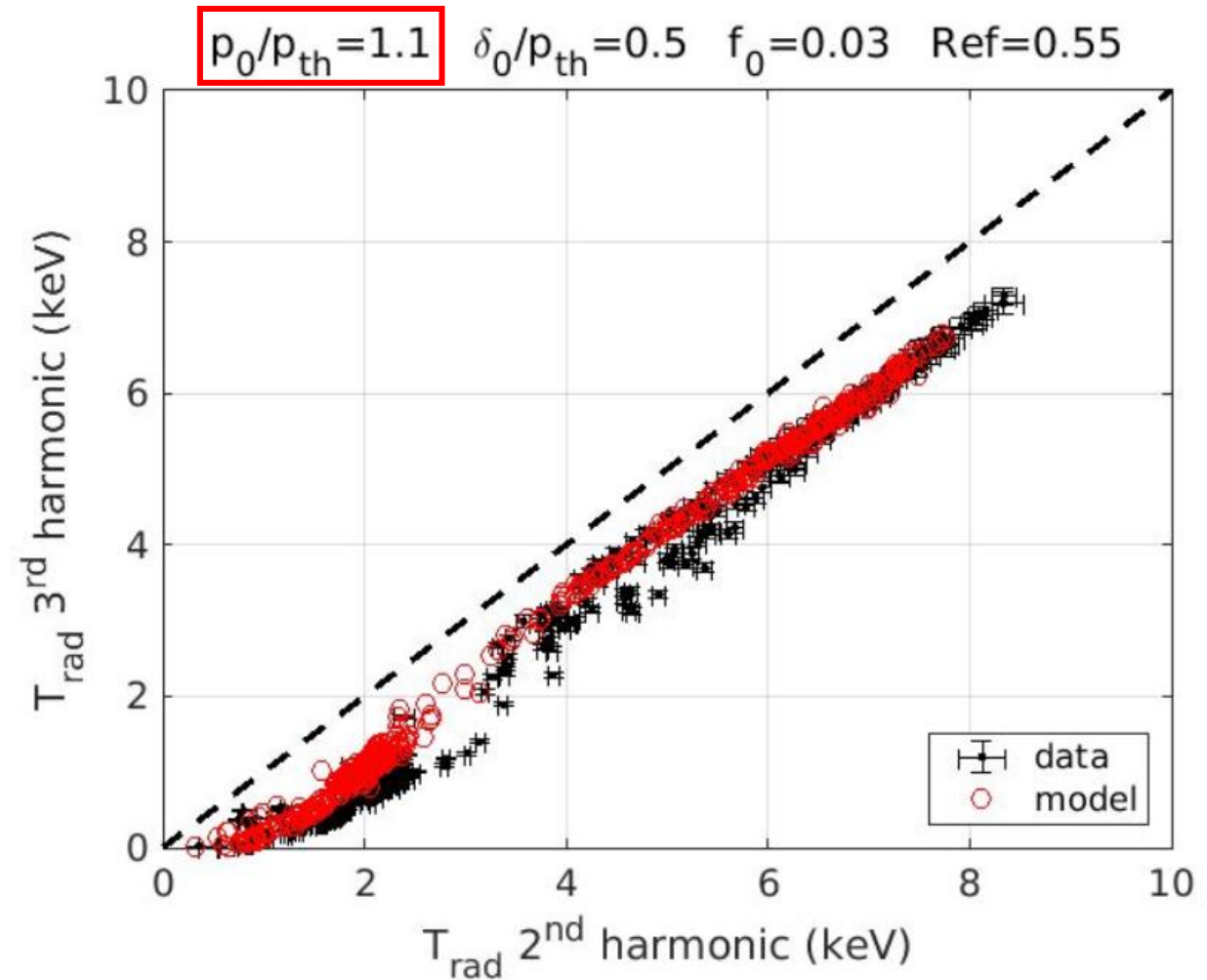
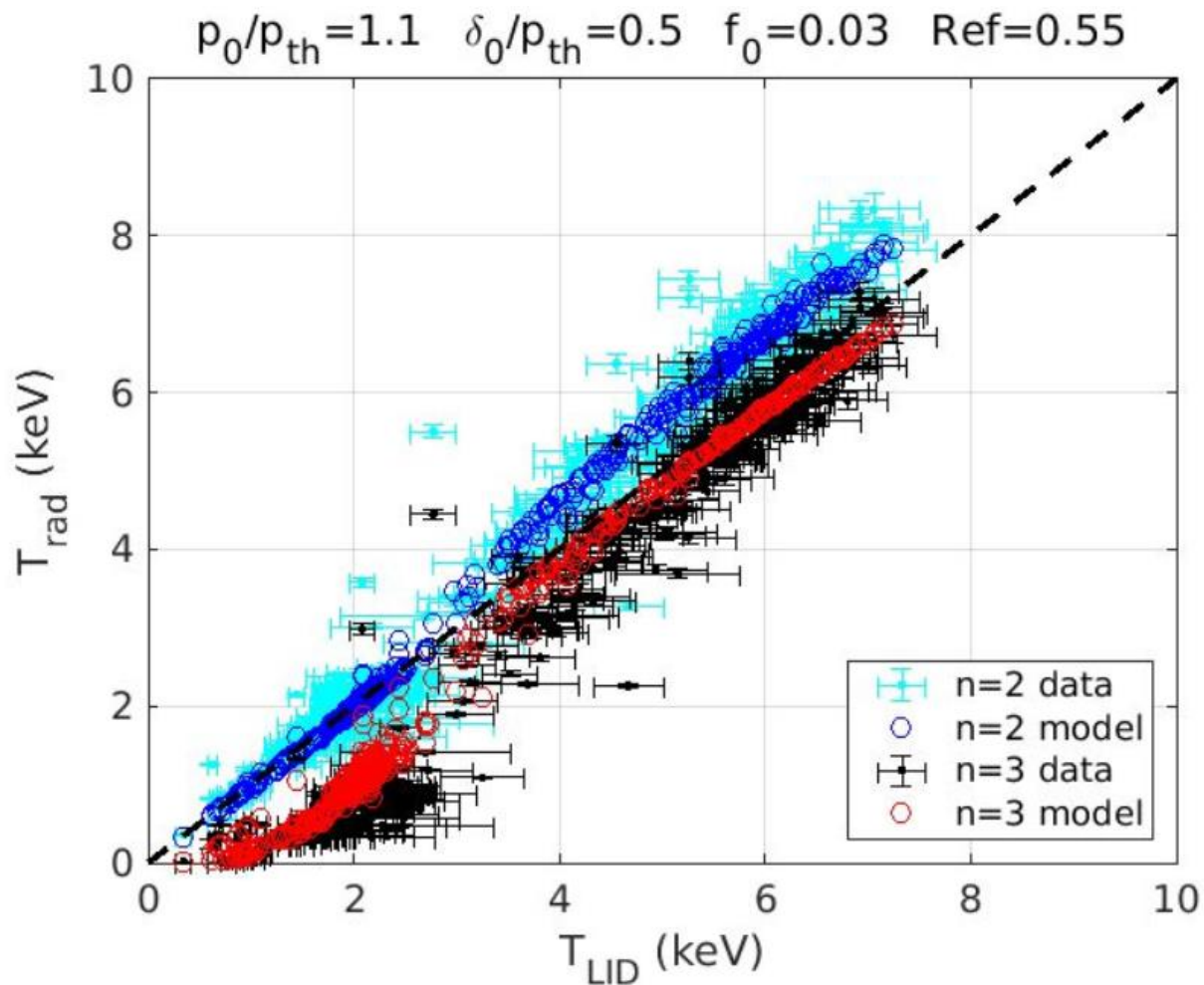
96990, 96992, 96993, 96994, 96996, 96998, 96999



DD baseline database "with low gas+pellets+small Neon injection"



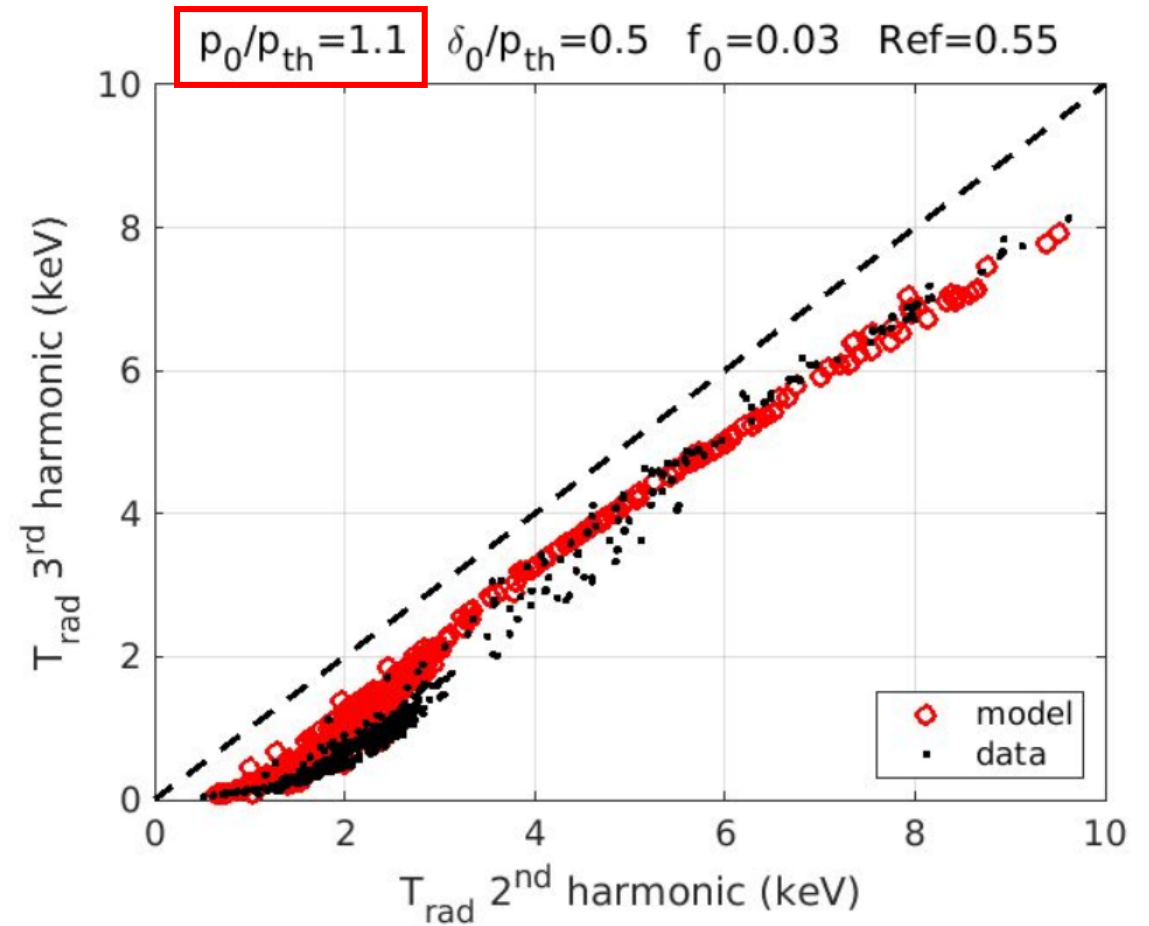
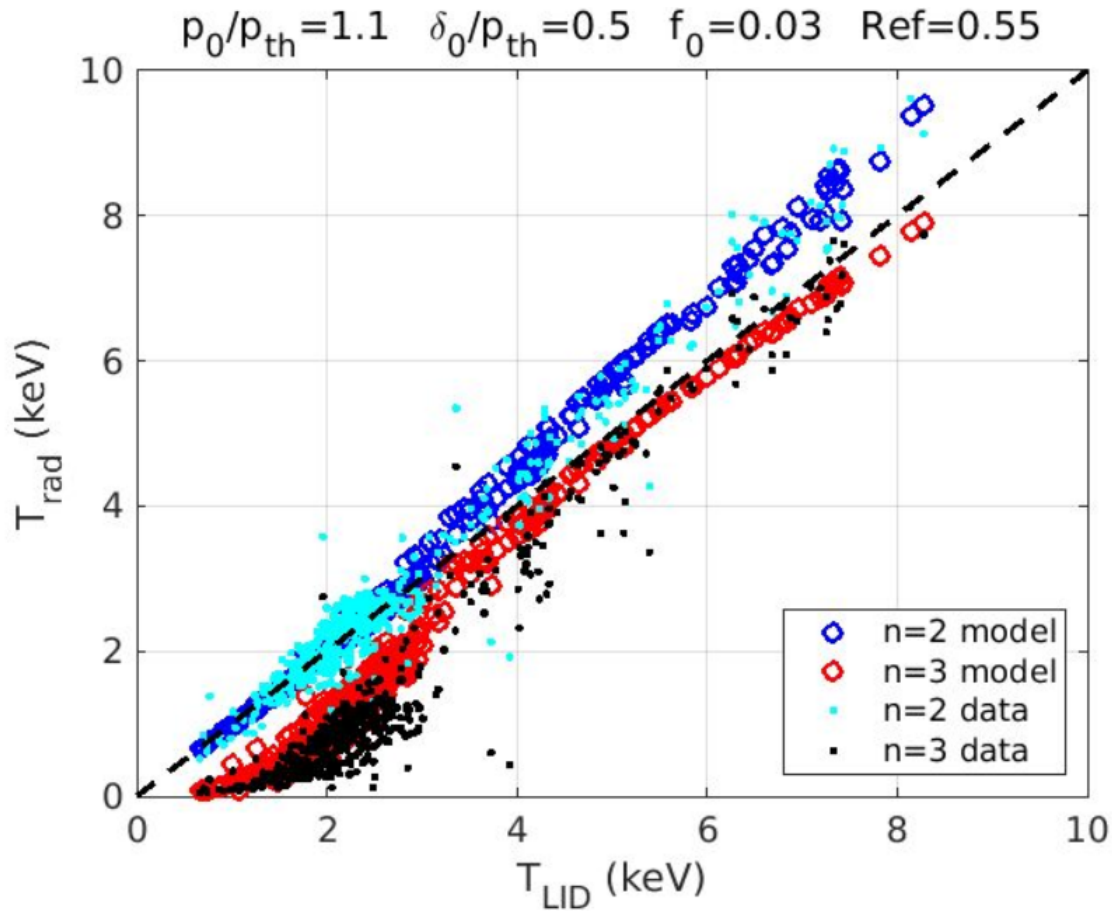
96990, 96992, 96993, 96994, 96996, 96998, 96999



DT baseline database



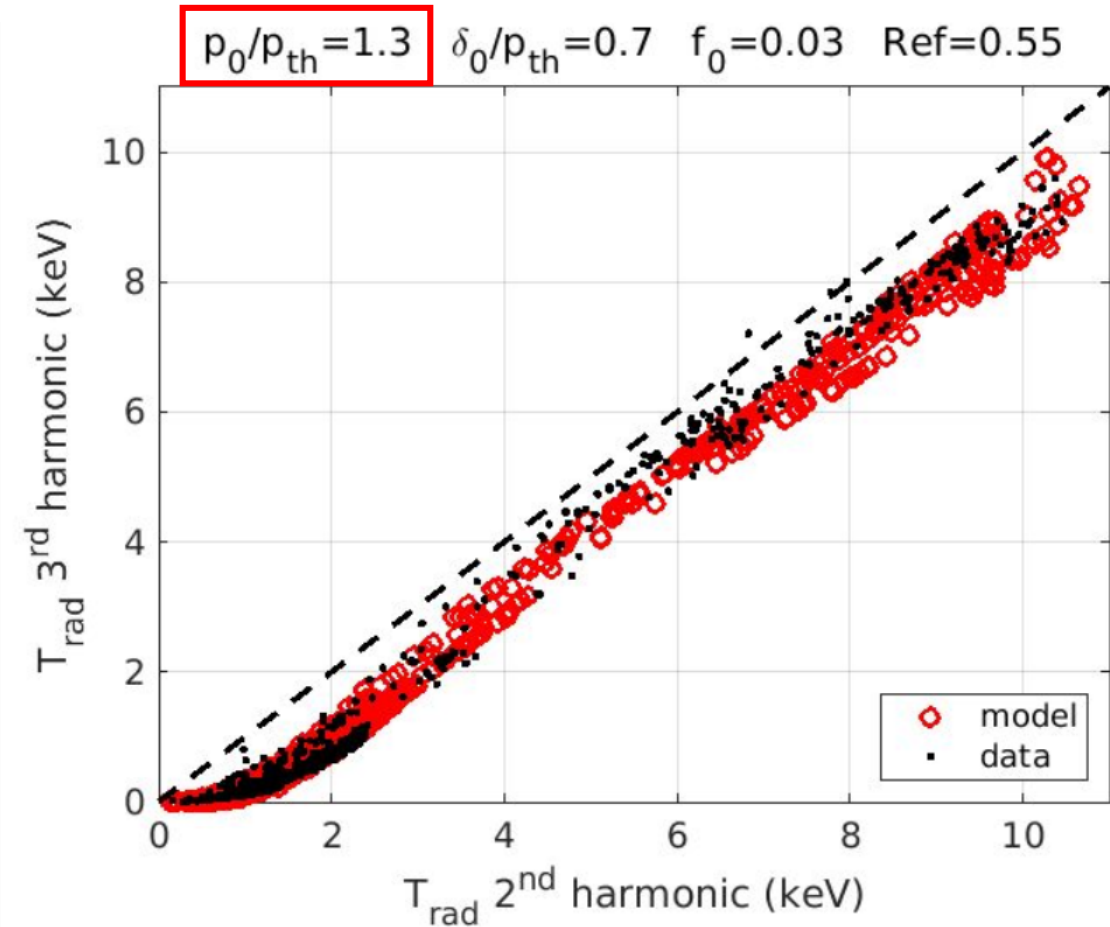
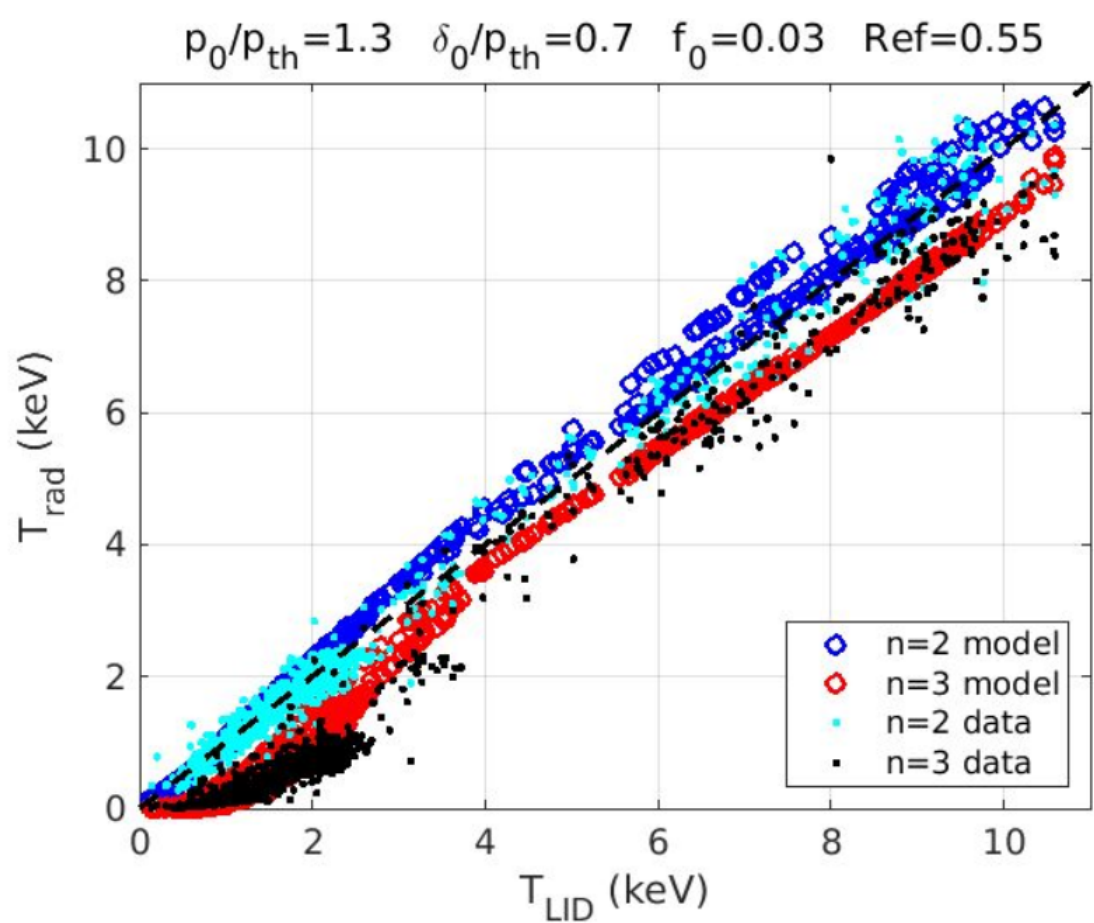
99520, 99795, 99796, 99797, 99799, 99805, 99861, 99862, 99863, 99878, 99943, 99944, 99948



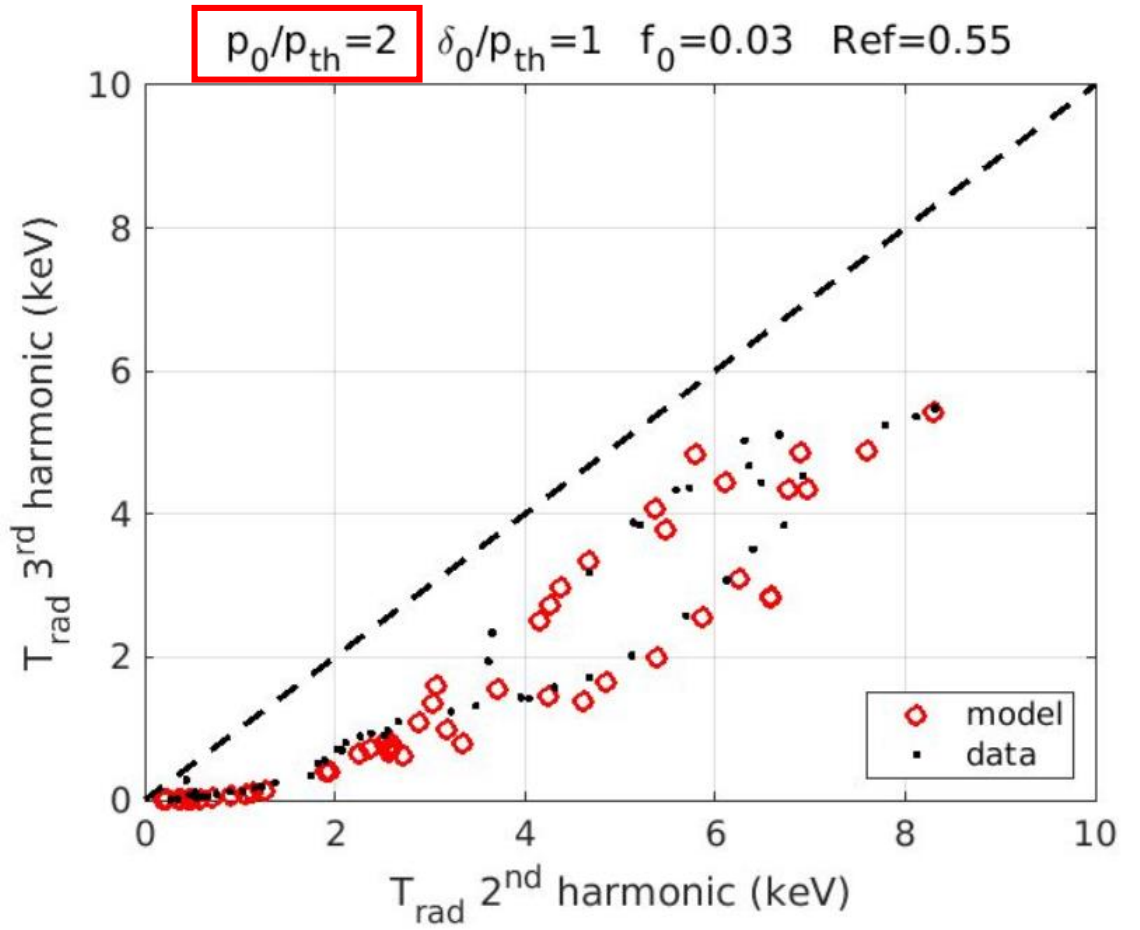
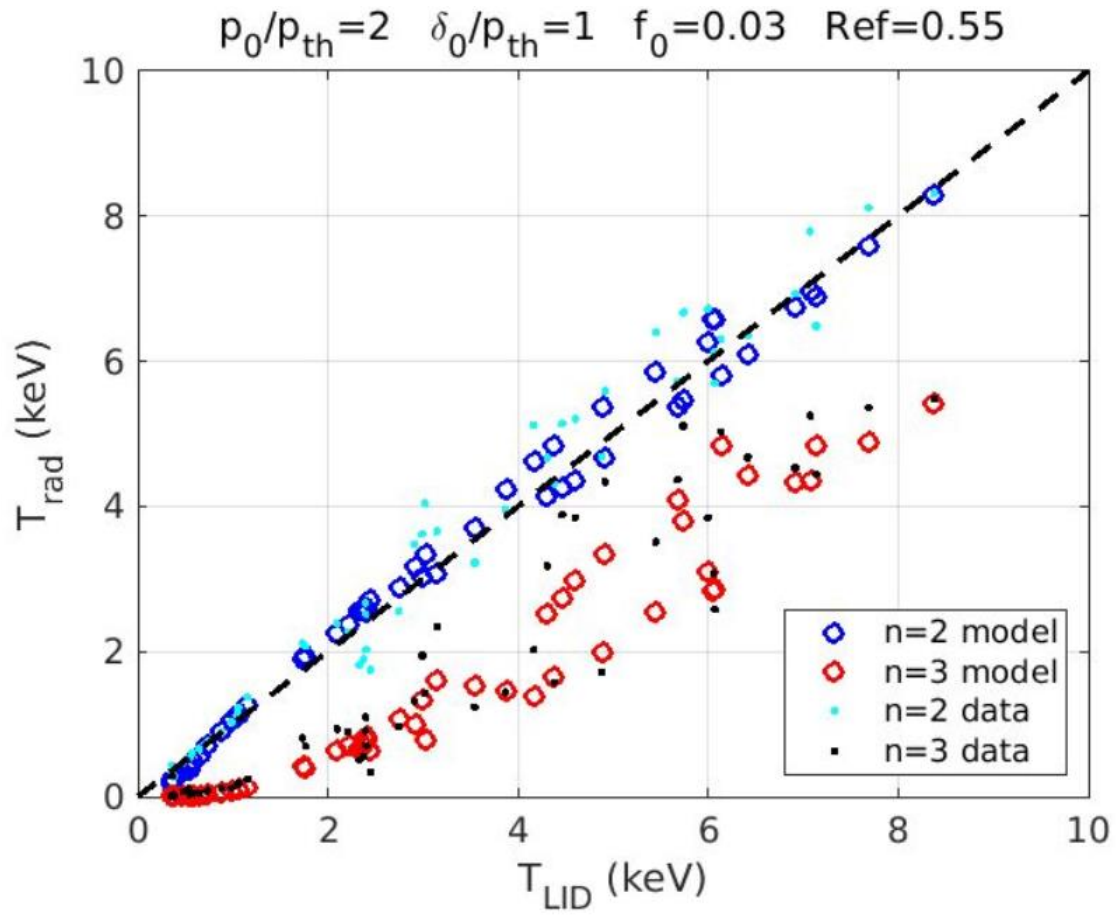
DT hybrid database



99448, 99449, 99450, 99452, 99455, 99541, 99542, 99543, 99544, 99594, 99595, 99596, 99760, 99761, 99866, 99867, 99868, 99869, 99908, 99910, 99912, 99914, 99949, 99950, 99951, 99953



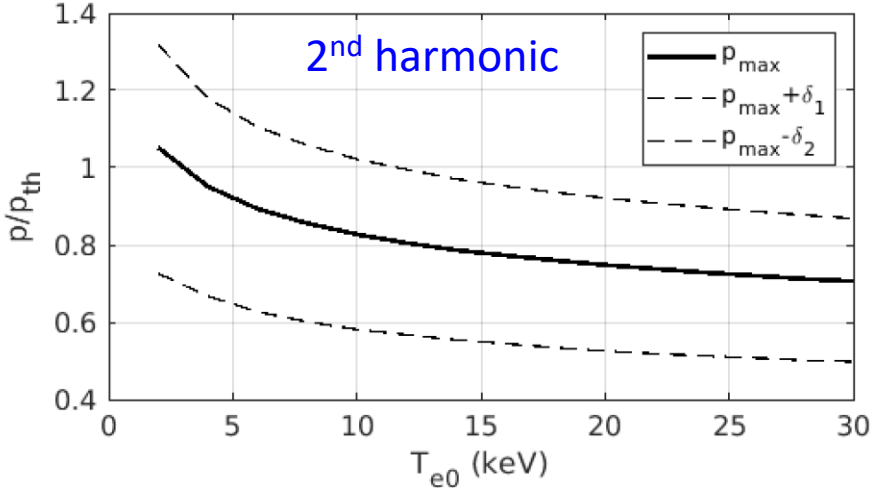
DD M18-03 experiment, pulse 96850



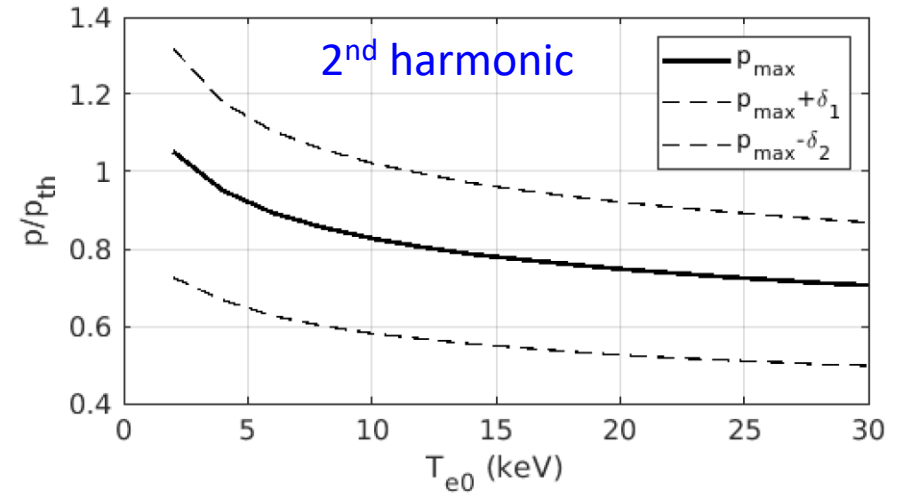
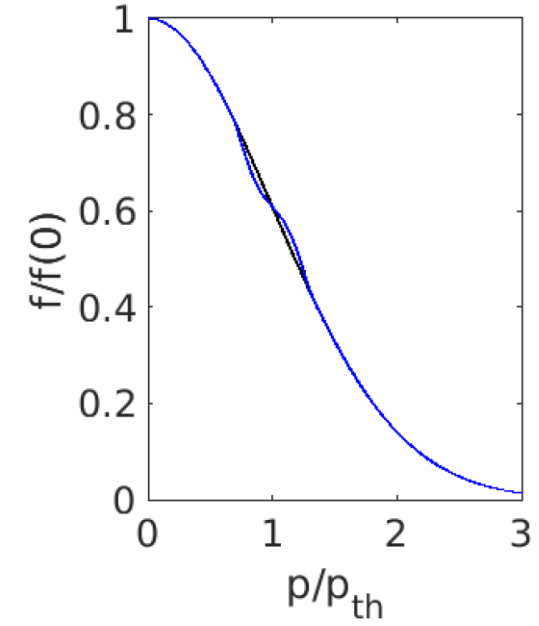
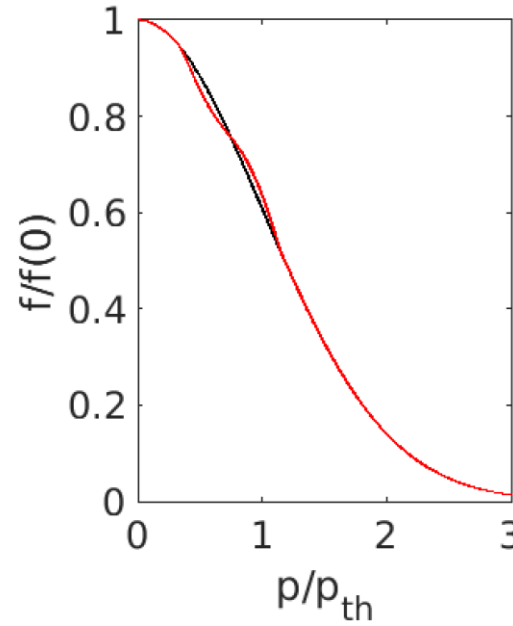
Scenario for specific studies on Energetic Particles.

Low density ($< 5 \cdot 10^{19} \text{ m}^{-3}$). High temperature phase with ICRH only

ITER: two cases with different perturbations



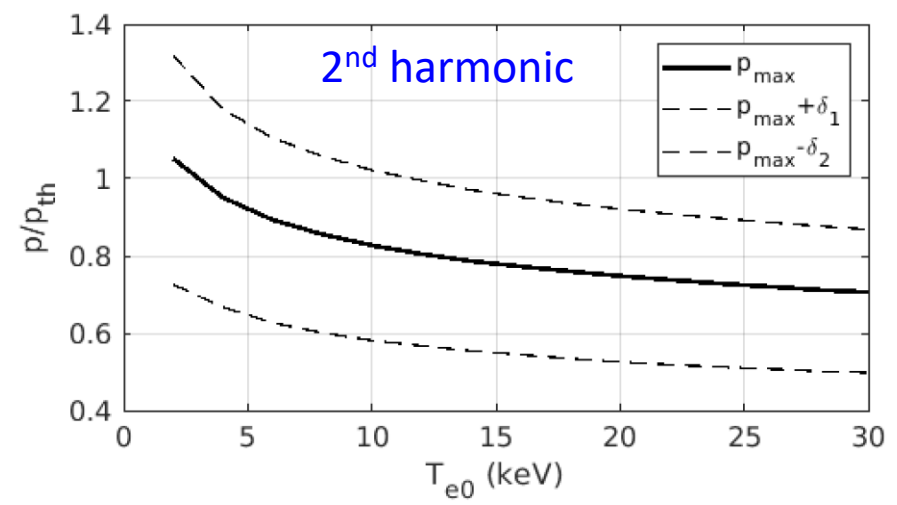
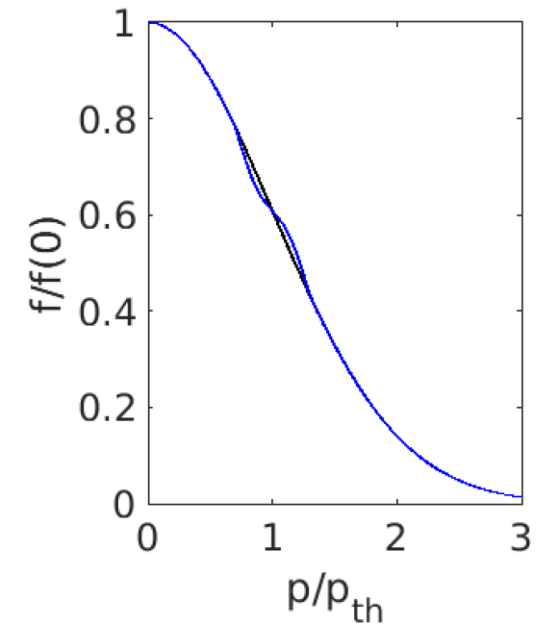
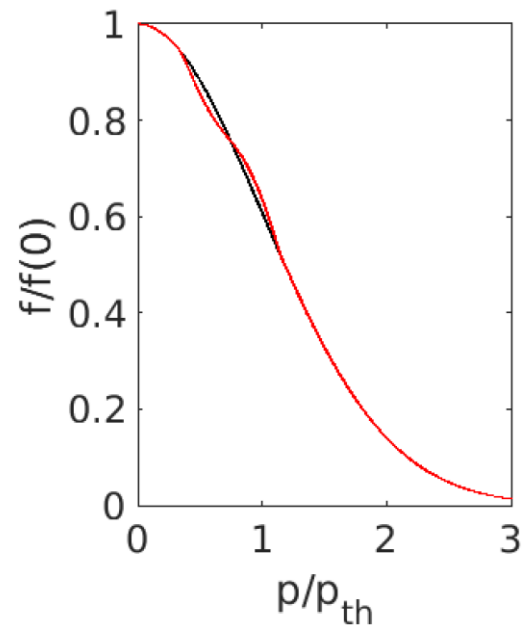
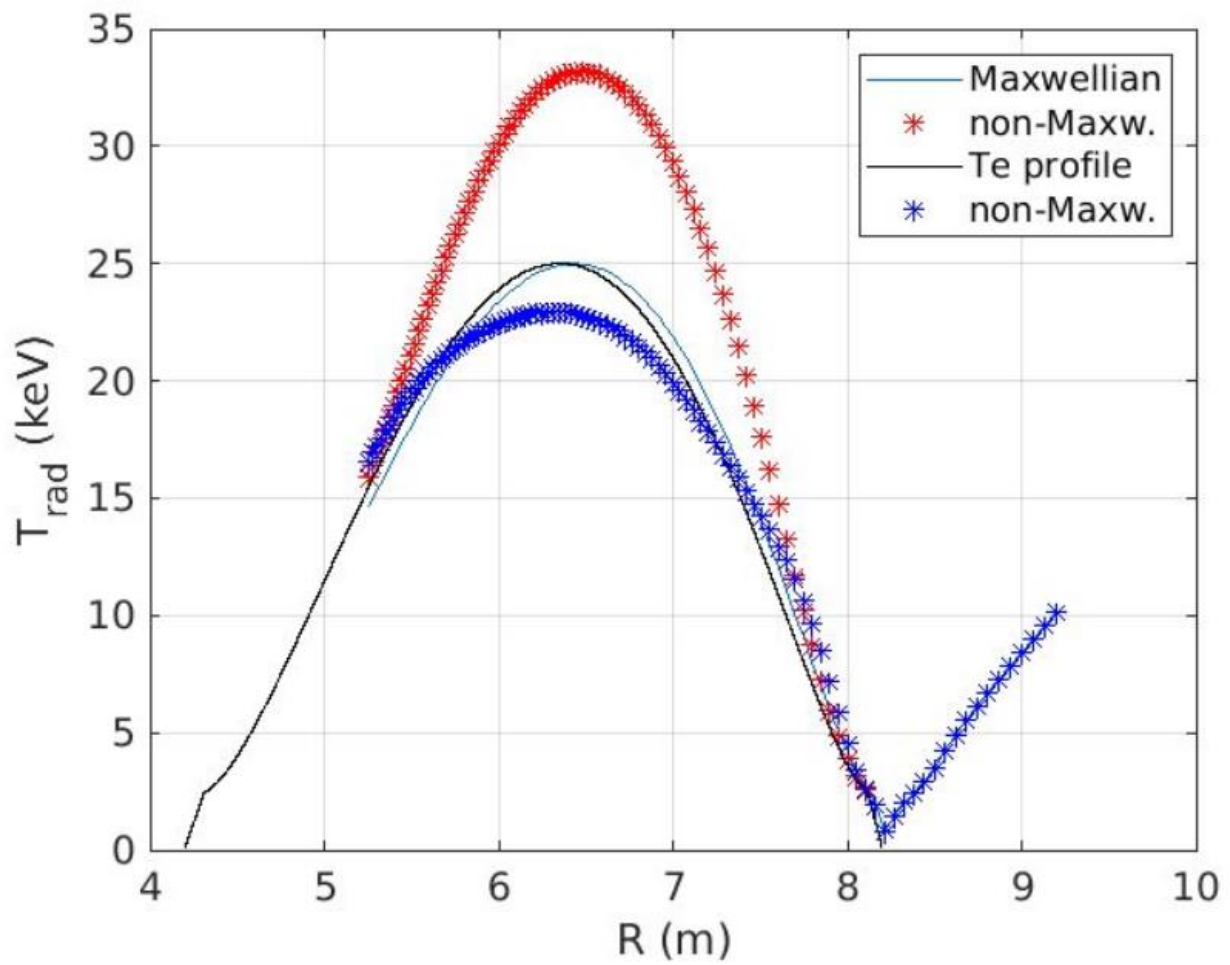
ITER: two cases with different perturbations



ITER: two cases with different perturbations



2nd harmonic





► Using the model of perturbed electron distribution we have shown that :

- a perturbation of < 5% yields the expected effect at $T_e \sim 5-10$ keV ($\Delta T_{\text{rad}} \sim 1$ keV)
- even if present, the perturbation cannot be observed at low temperature
- Thomson spectrum is broad and insensitive to such perturbations

► Consequences :

- In principle, Thomson scattering seems more reliable for T_e measurements at high temperature
- ECE is very effective for constraining the distribution function → tool to explore new physics

► Origin of the effect is under investigation. Two possible causes :

- fast ion collisional relaxation on electrons → needs full kinetic calculation with integro-differential collision operator → **work in progress (R. Dumont)**
- Landau damping of fast ion driven (or other high- β) MHD modes
 - bipolar electron distributions observed in the magnetosheath → *C.H.K. Chen et al., Nat. Commun. 10, 740 (2019)*
 - interpretation as Landau damping of Kinetic Alfvén Waves confirmed by gyrokinetic simulations → *S.A. Horvath et al., Phys. Plasmas 27, 102901 (2020)*
 - gyrokinetic modelling with GENE for JET parameters → **work in progress (S. Mazzi)**



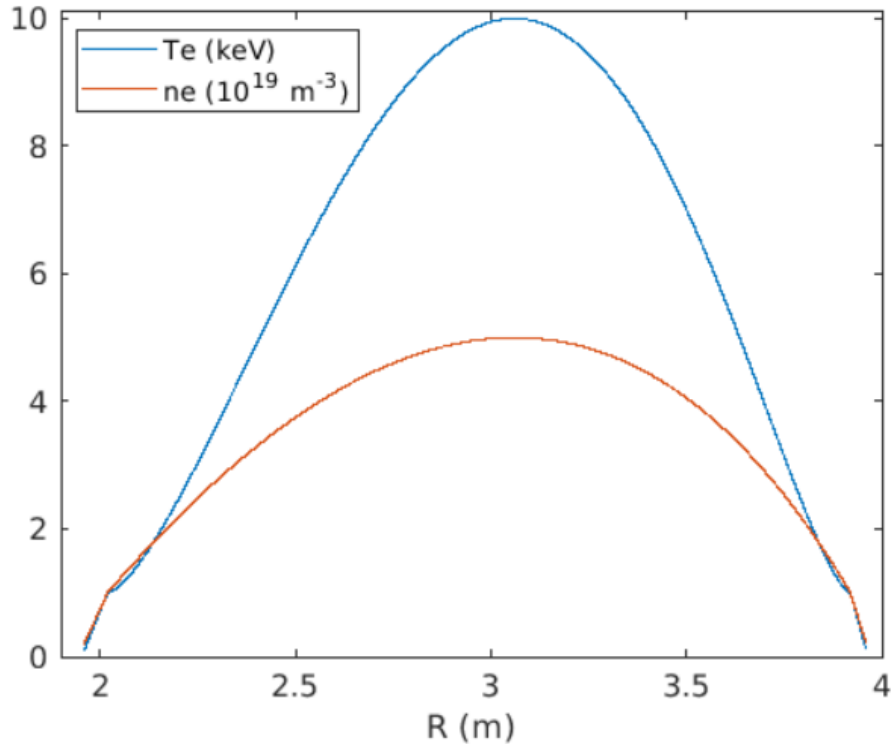
Backup slides

What is seen by 2nd harmonic ECE (in space)

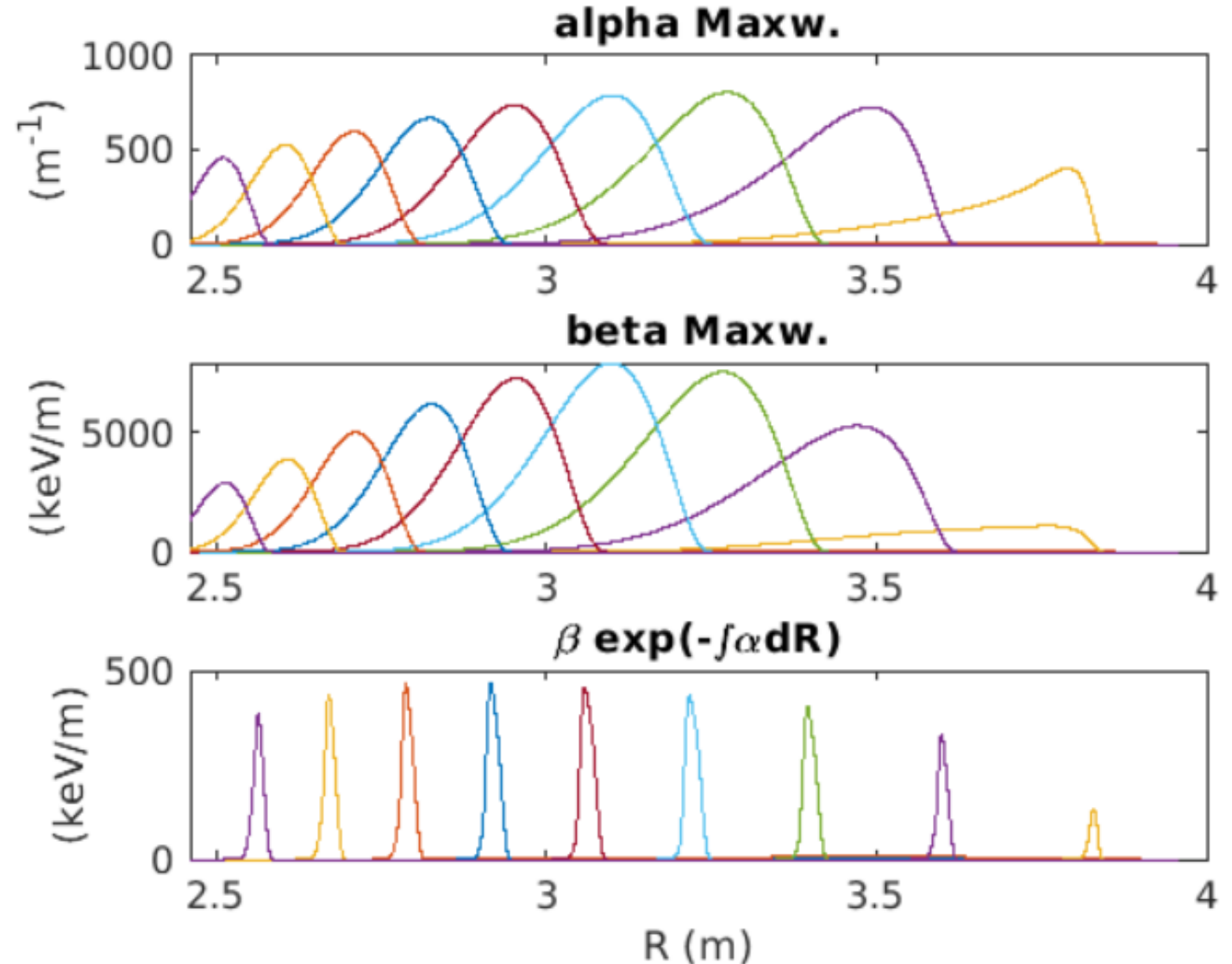


In the following:

analytic density and temperature profiles.
analytic equilibrium with Shafranov shift.



absorption/emission coefficients at different frequencies



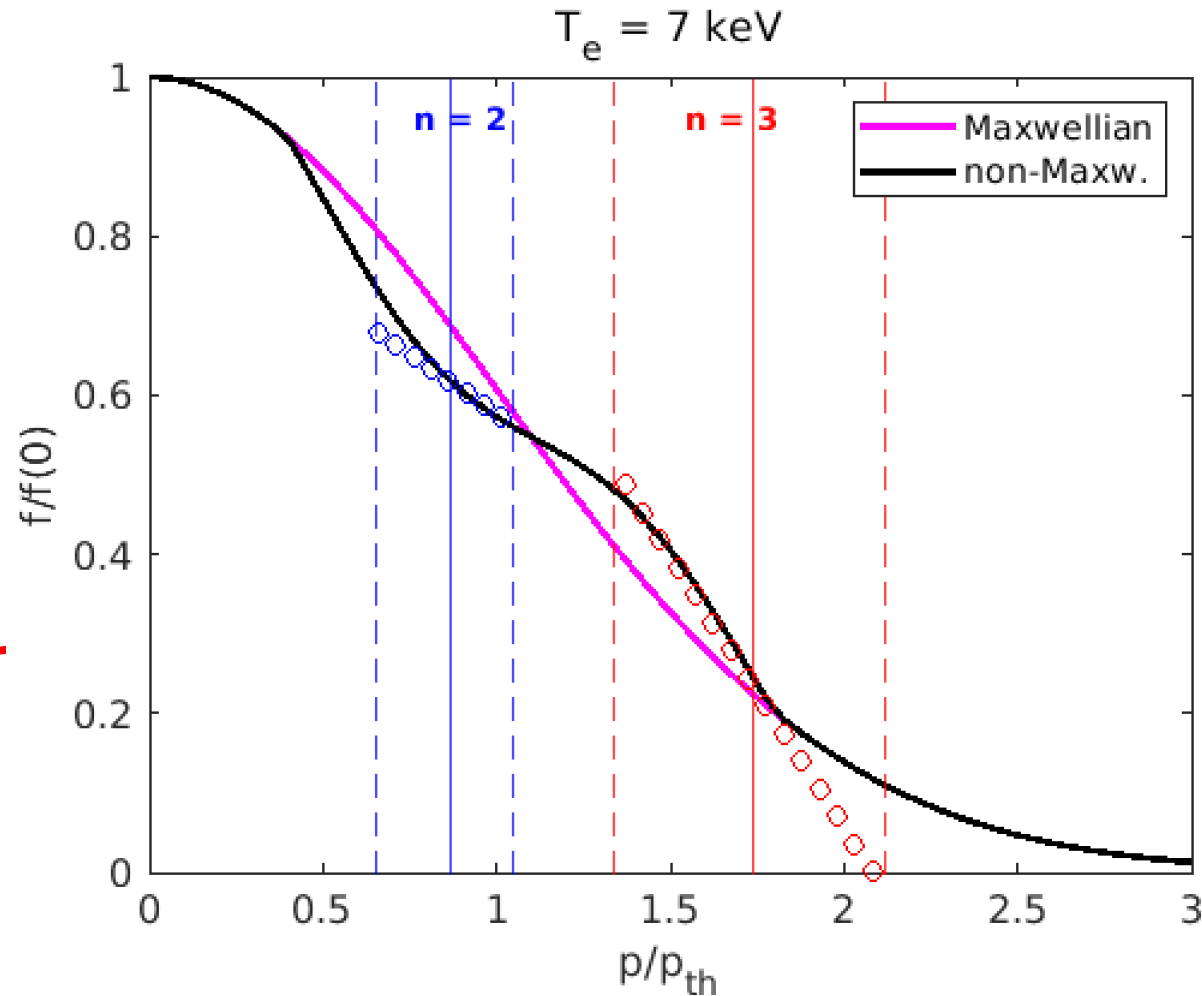
ECE sets constraints on the distribution function



$$f_M = A e^{-E_k/T_e}$$

$$T_e = - \frac{f_M}{df_M/dE_k}$$

Note: here slopes are exaggerated



on average:

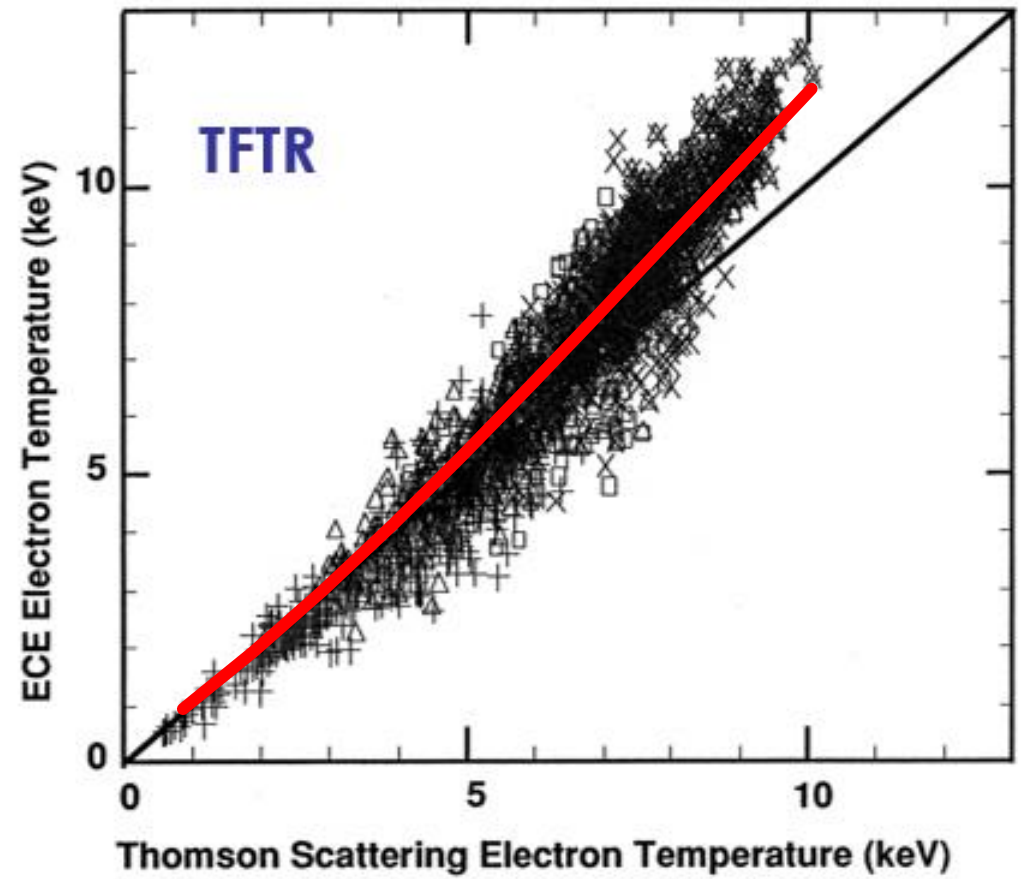
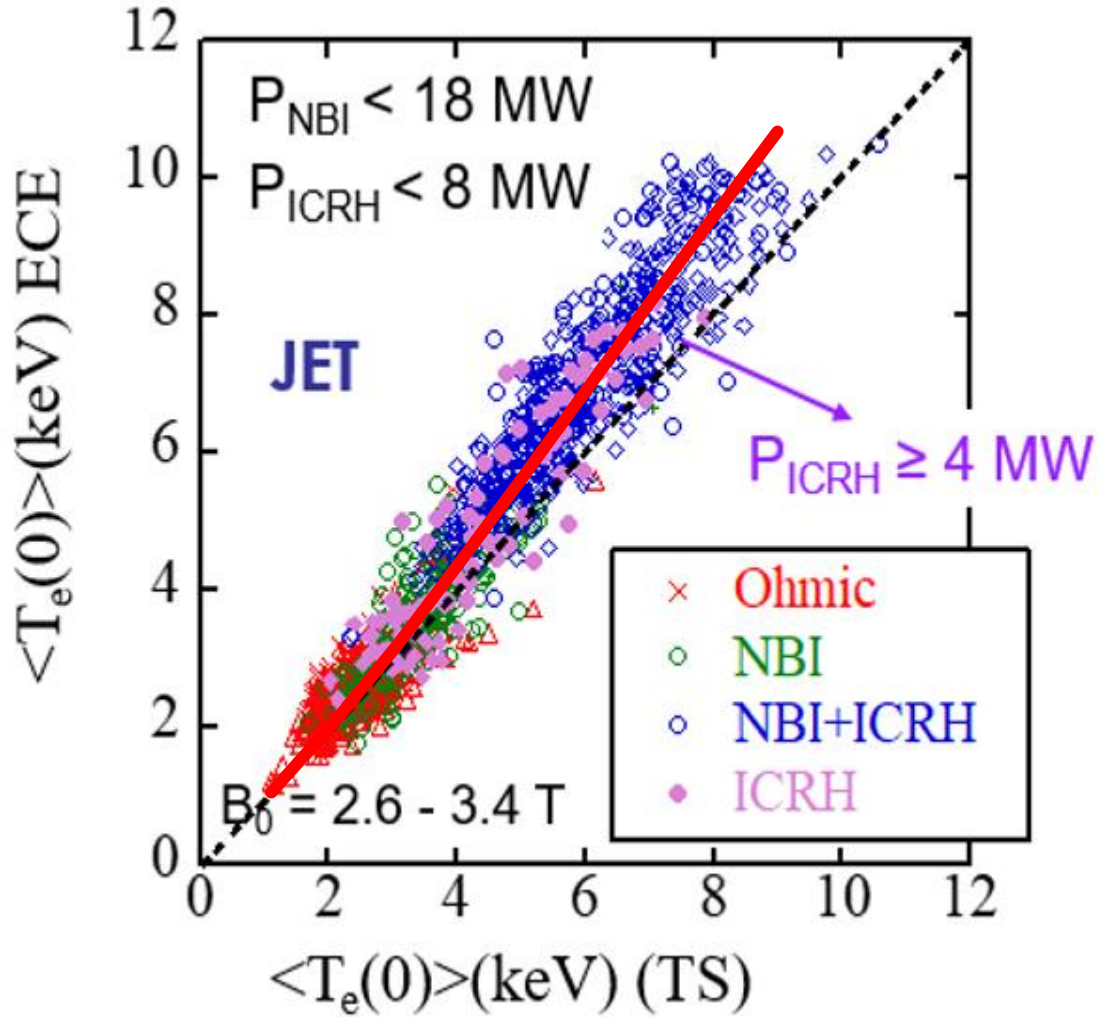
$$n=2 \rightarrow T_{\text{rad}} > T_e$$

$$n=3 \rightarrow T_{\text{rad}} < T_e$$

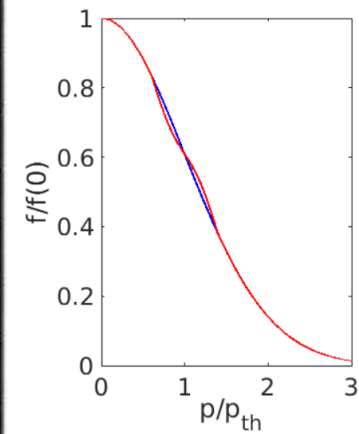
solid lines: maximum of $\beta \exp(-\int \alpha dR)$

dashed lines: width at half-height

Test of model perturbation on old data: one size fits all ?

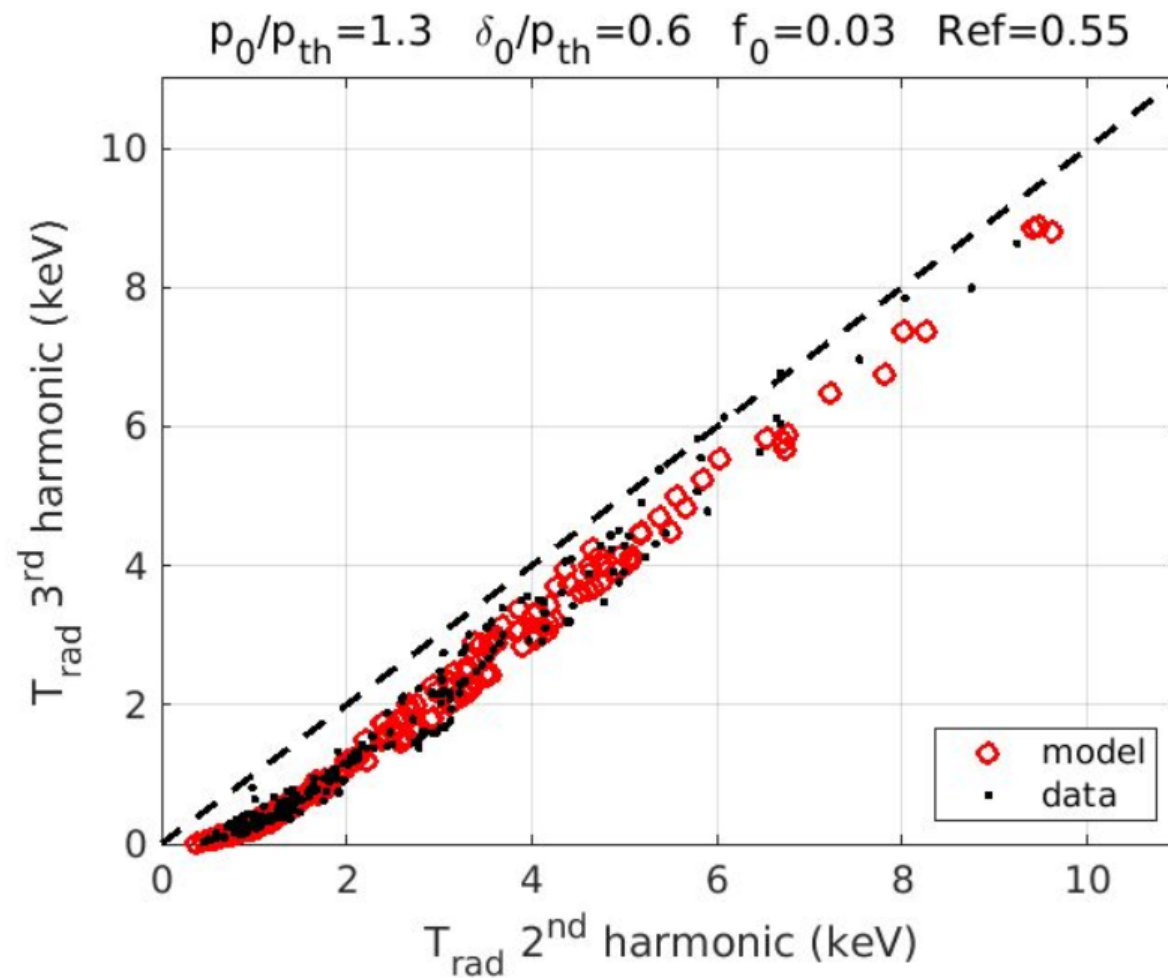
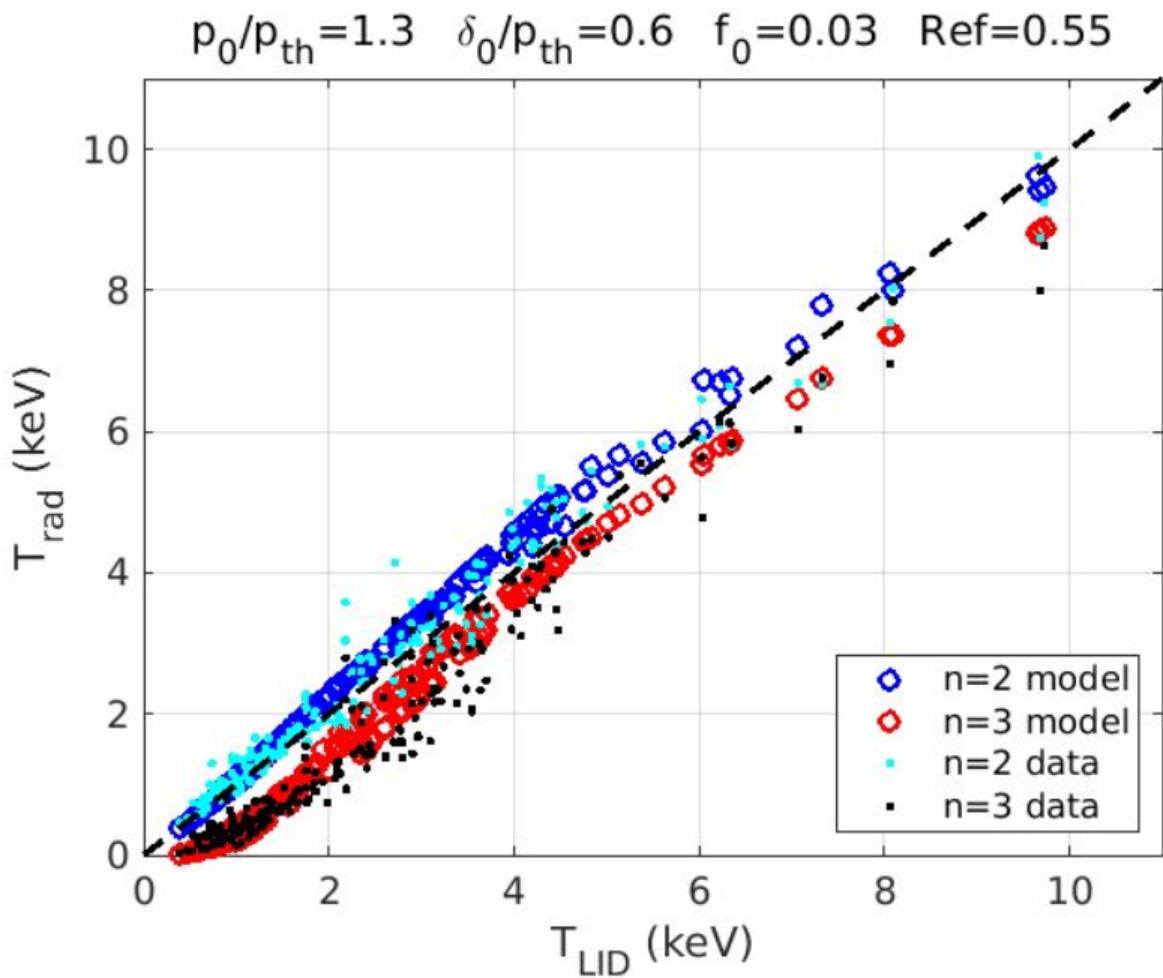


$f_0 = 0.03$
 $p_0/p_{th} = 1$
 $\delta/p_{th} = 0.4$

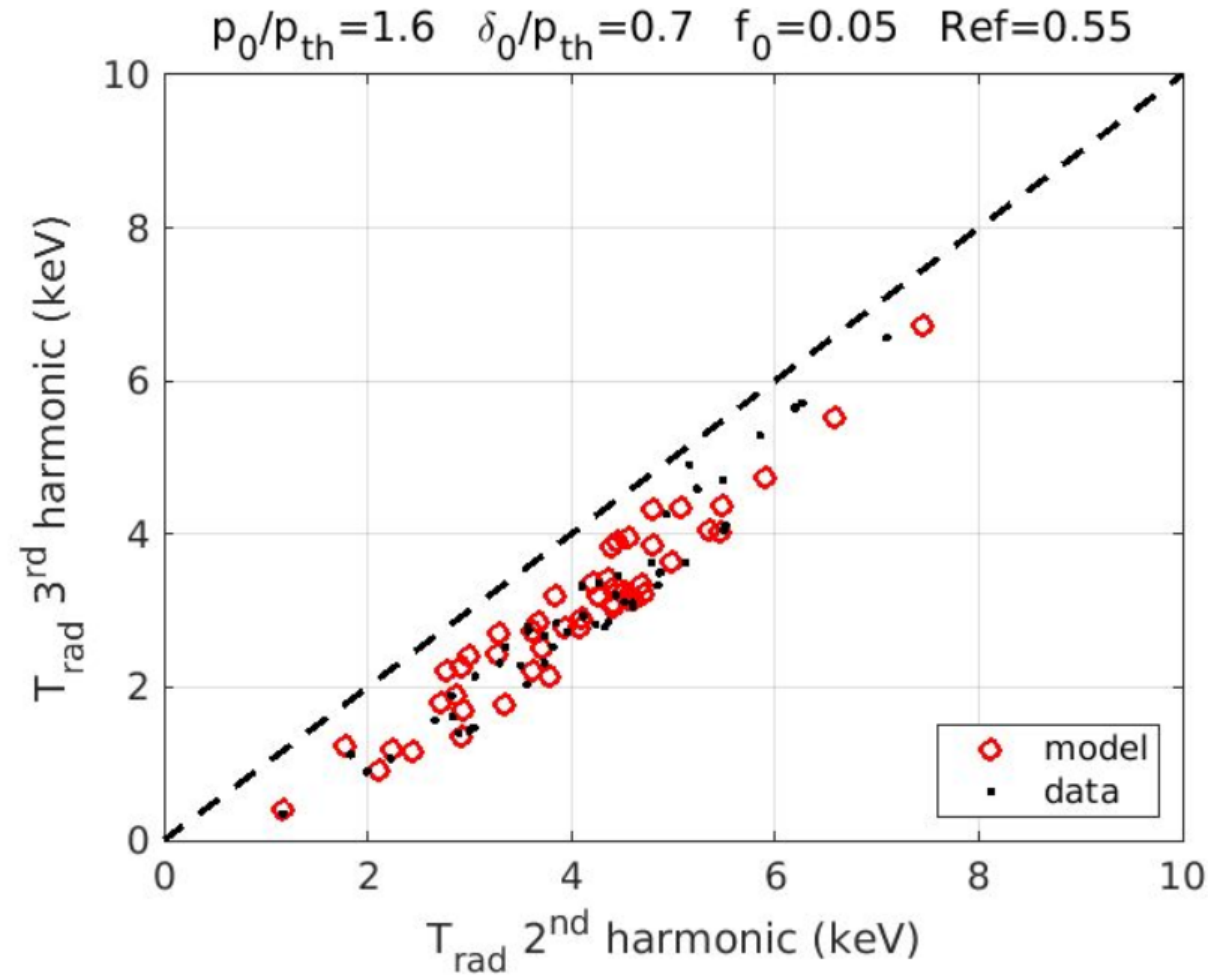
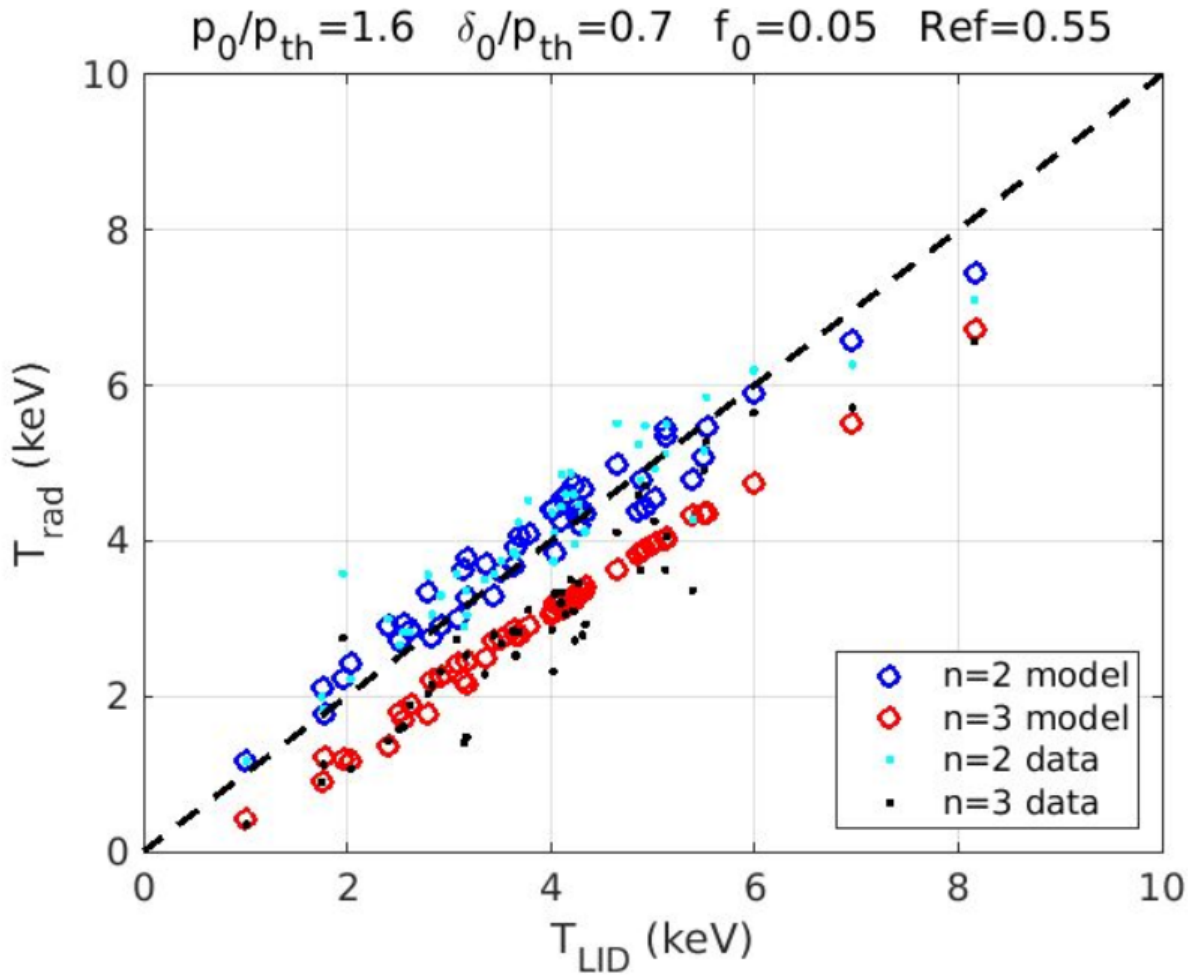


mean values of density and magnetic field used

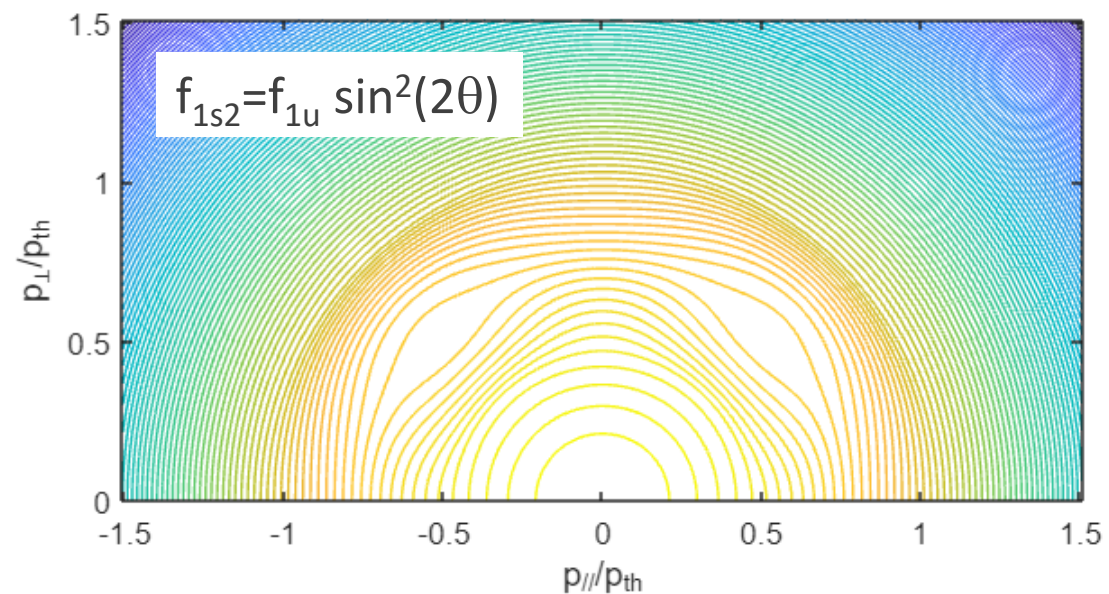
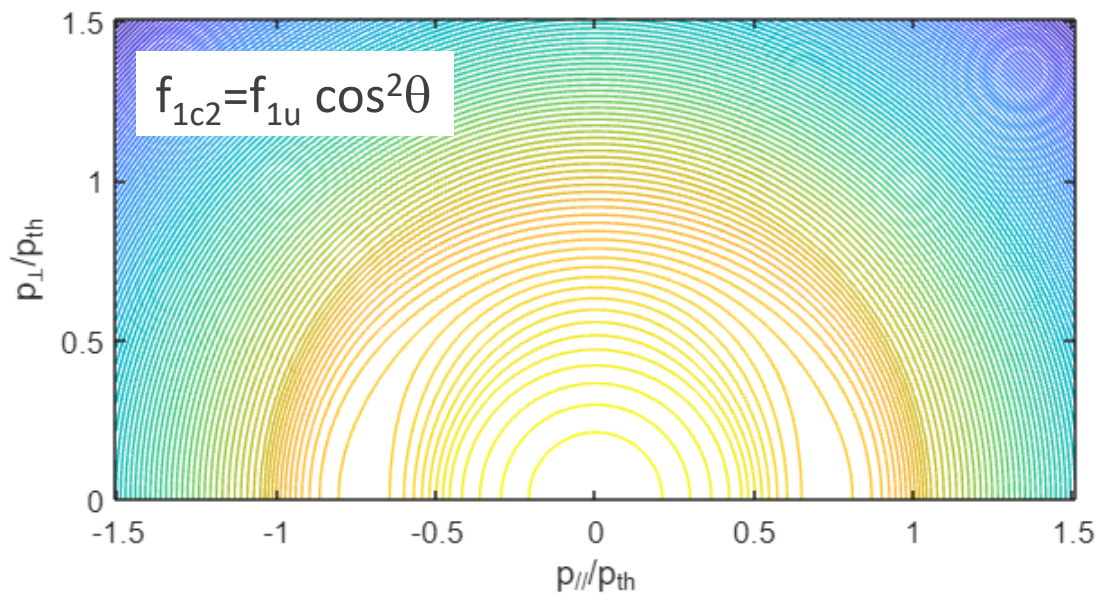
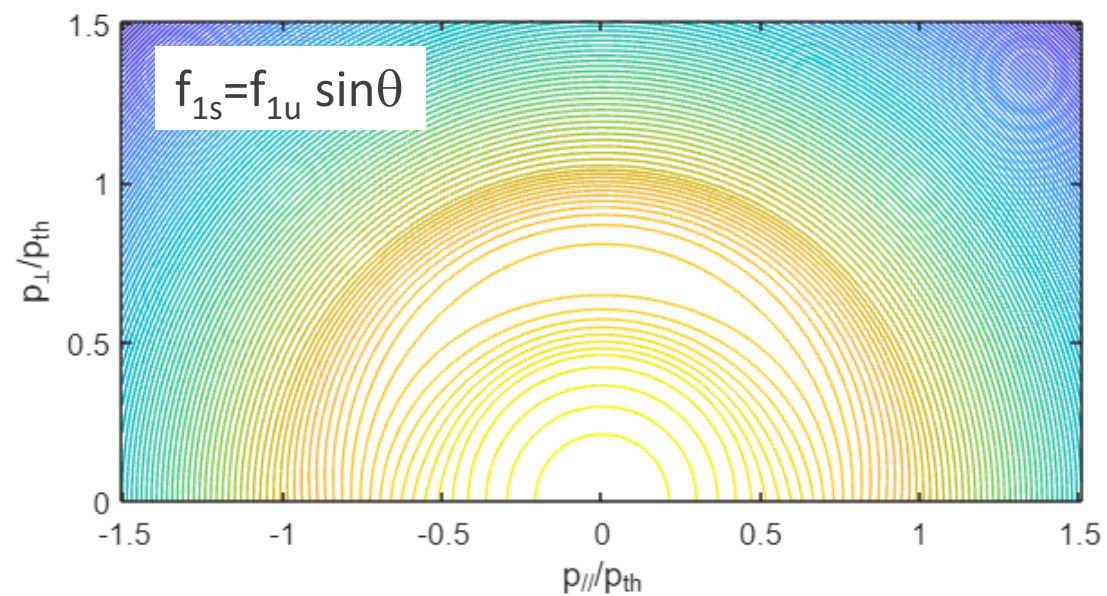
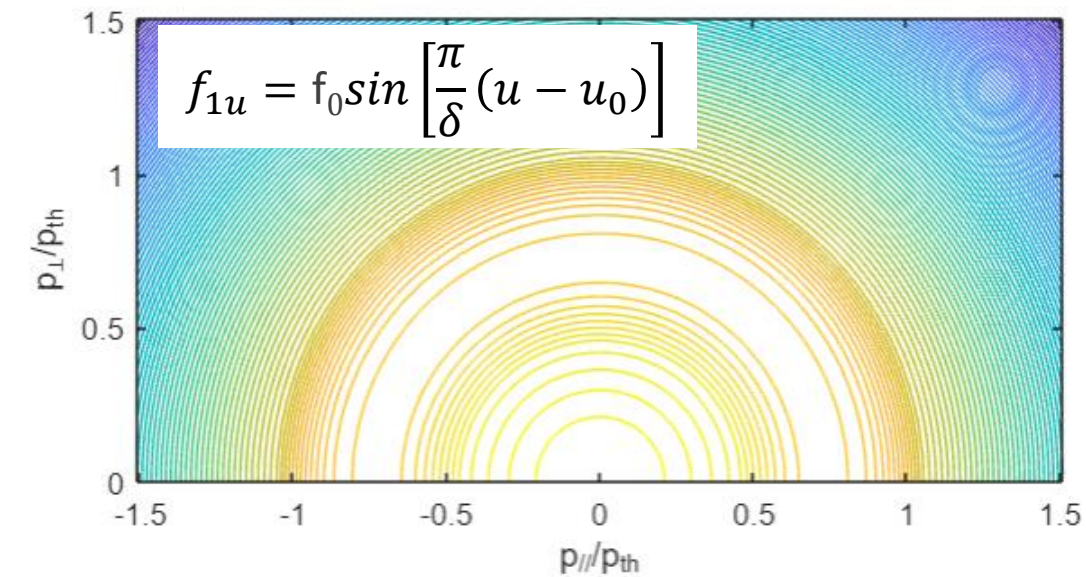
All DT data points with NBI only



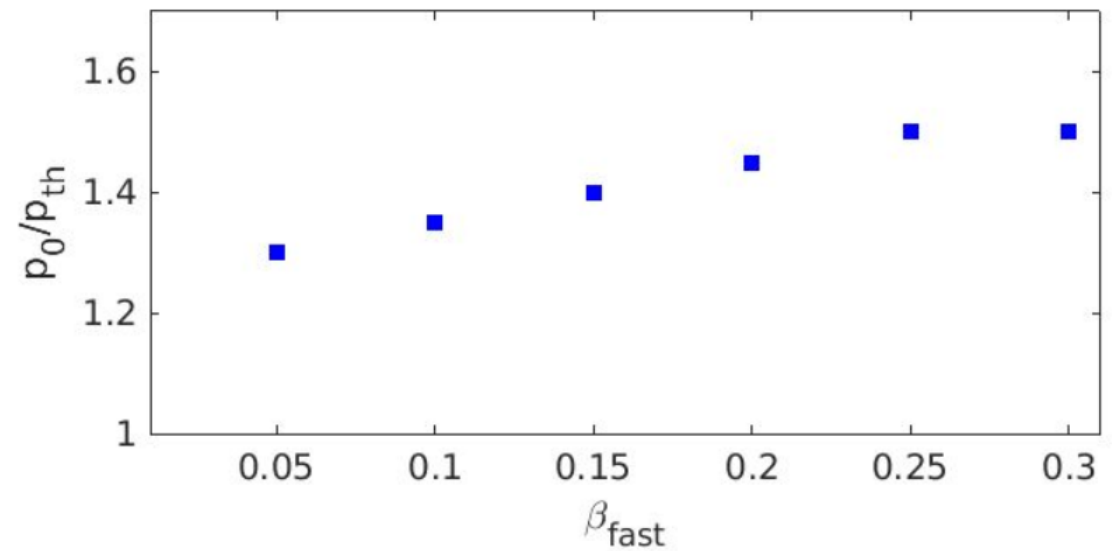
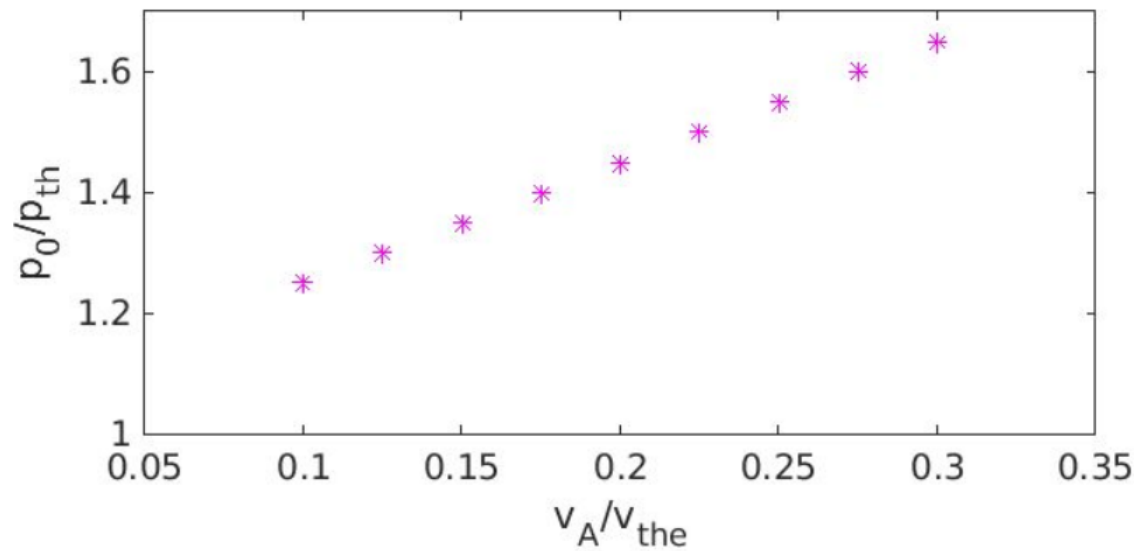
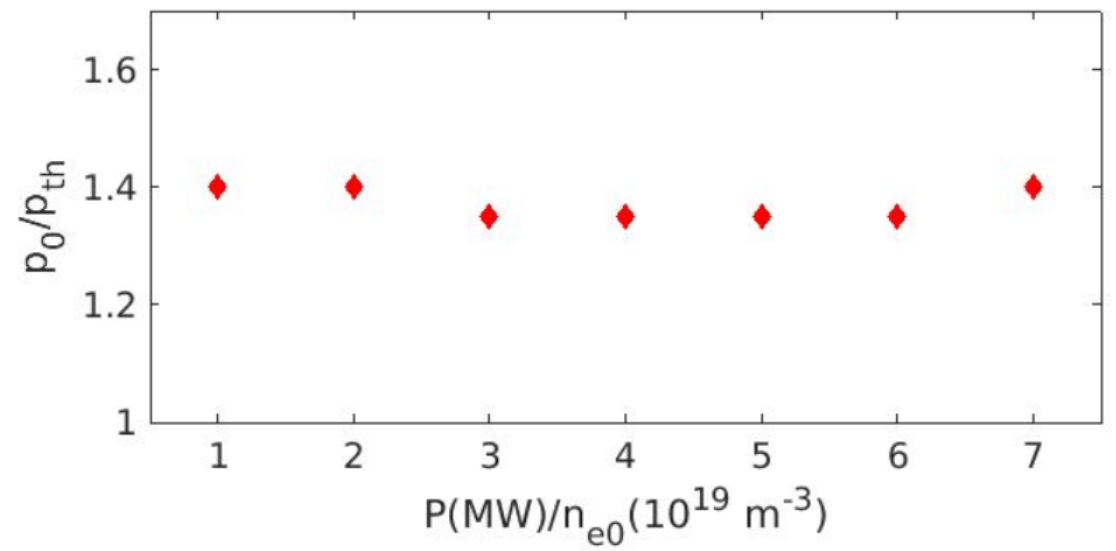
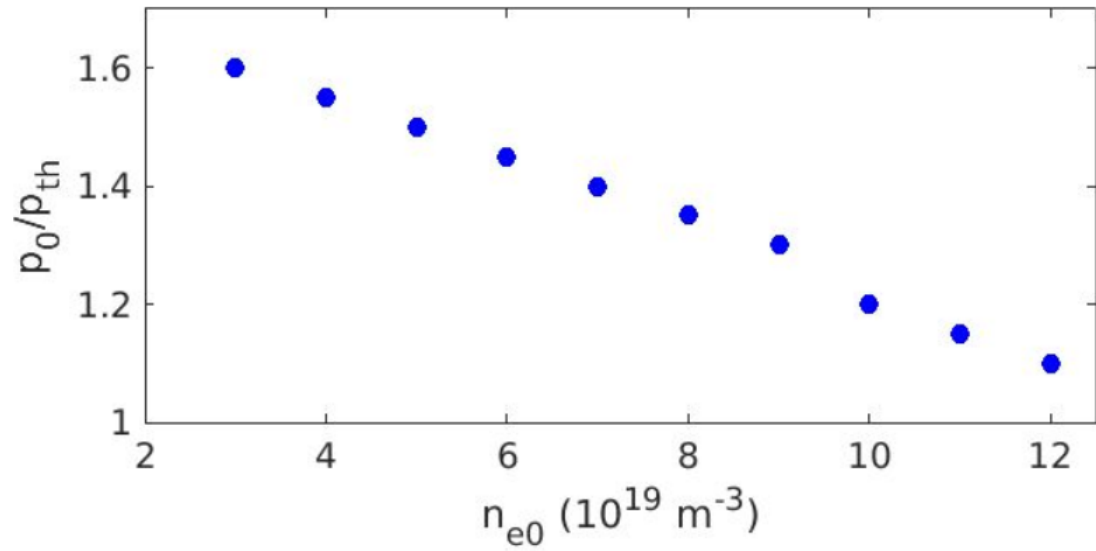
All DT data points with ICRH only



Perturbed electron distributions



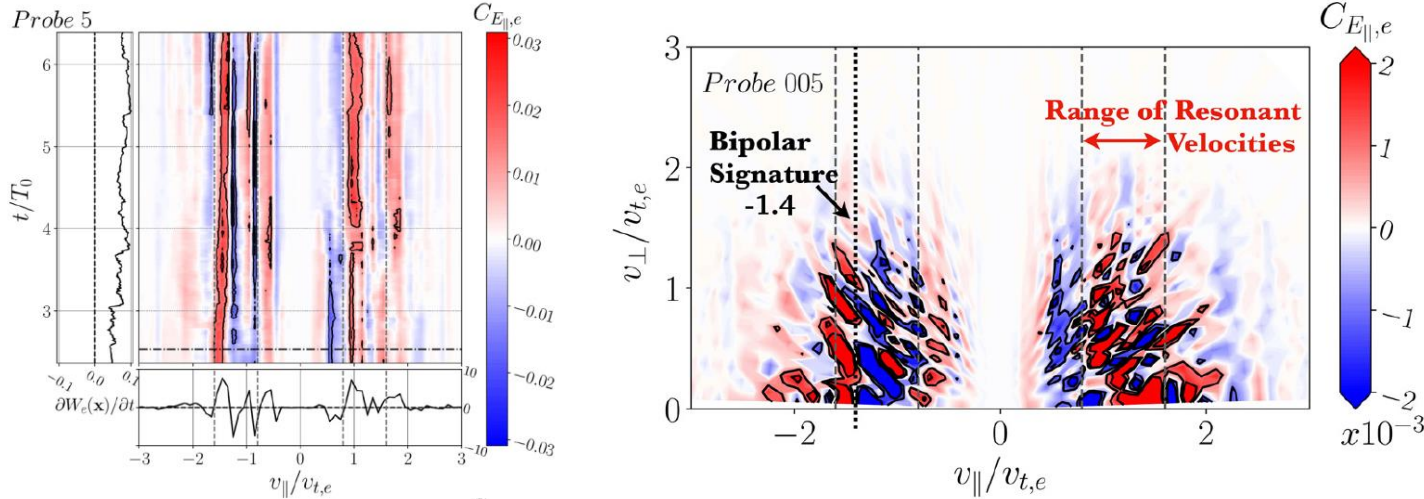
Use of the model for full database analysis: p_0/p_{th} as fit parameter



Landau damping of Kinetic Alfvén Waves in the Magnetosheath



- ▶ [S.A. Horvath et al., Phys. Plasmas 27, 102901 \(2020\)](#)
- ▶ Gyrokinetic simulations of KAW absorption by the electrons in the Earth's magnetosheath
- ▶ **Bipolar signature** of Landau damping of KAW found in computations



[C.H.K. Chen et al., Nat. Commun. 10, 740 \(2019\)](#)

In-situ measurements of energy transfer from turbulence to electrons in the Earth's magnetosheath, from the Magnetospheric Multiscale (MMS) mission

Bipolar signature of Landau damping of KAW found in measurements

