Application of a Quasioptical code PARADE: modeling of ECRH, ECCD and ECE diagnostics in toroidal fusion devices

Kota Yanagihara[1)] and Shin Kubo[2)]

[1)]*Naka Fusion Institute,*
National Institute for Quantum Science and Technology (QST)
[2)]*Chubu University*

QST

PARADE for ECRH, ECCD, and ECE modeling

Various improvements on ECRH, ECCD, and ECE modeling by Quasioptical code PARADE with relativistic effects in JT-60SA and JA-DEMO are presented today.

In order to predict EC applications in reasonable costs with high accuracy,
We applied,

〈1〉 Quasioptical beam model for ECRH (NOT geometrical rays)

〈2〉 Adjoint technique for ECCD calculation

〈3〉 Quasioptical beam model for ECE (NOT geometrical rays)

〈4〉 Fully relativistic dispersion for absorption
(NOT weakly relativistic coefficient in quasi-perp. limit)

〈5〉 Weakly relativistic dispersion for propagation
(NOT cold dispersion)

ECRH&CD&E code w/ multi-rays and relativity has been already proposed, but quasioptical one is not reported before.

Topic 1 <ECRH>

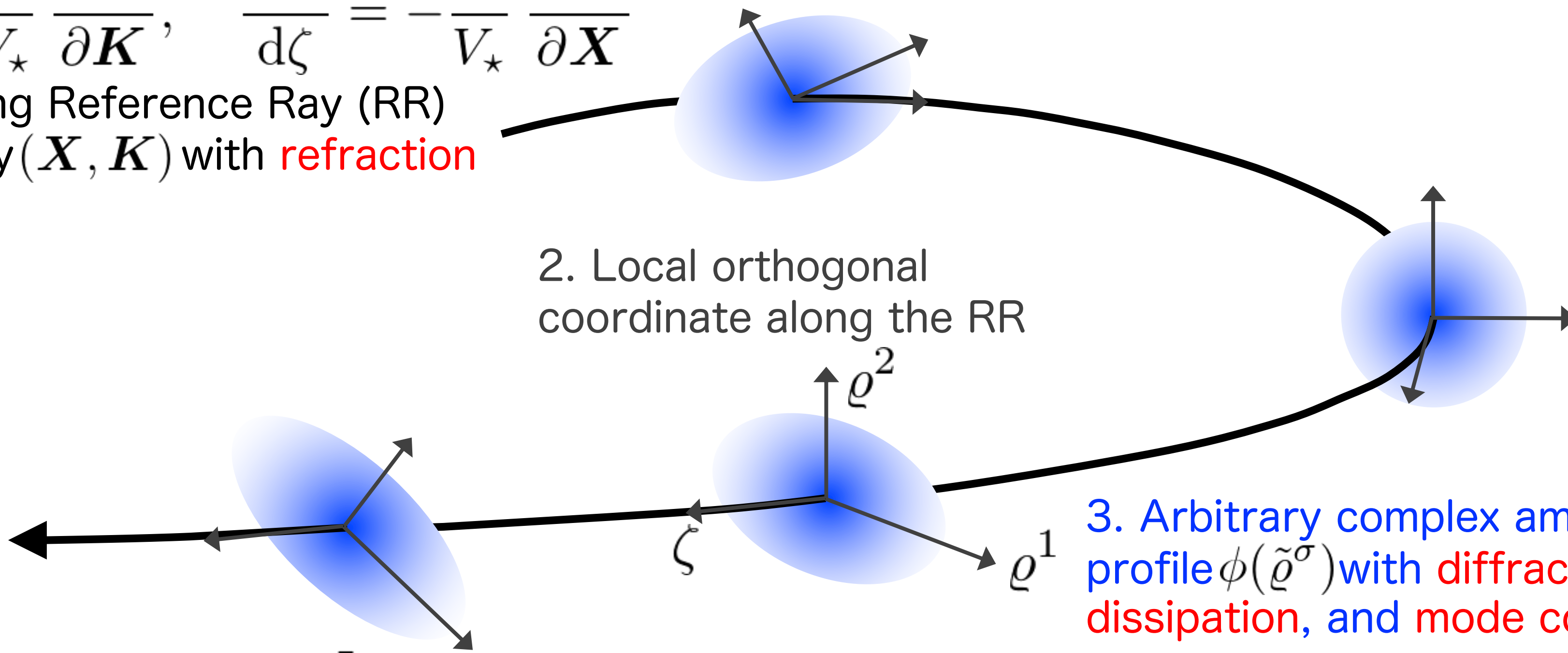
Power deposition w/ Quasioptical beam model

Quasioptical ray tracing code PARADE [1-4]

Schematic image of calculations implemented in PARADE.

$$\frac{\mathrm{d}\boldsymbol{X}}{\mathrm{d}\zeta} = \frac{1}{V_\star}\frac{\partial H_\star}{\partial \boldsymbol{K}}, \qquad \frac{\mathrm{d}\boldsymbol{K}}{\mathrm{d}\zeta} = -\frac{1}{V_\star}\frac{\partial H_\star}{\partial \boldsymbol{X}}$$

1. Bending Reference Ray (RR) trajectory $(\boldsymbol{X}, \boldsymbol{K})$ with refraction



3. Arbitrary complex amplitude profile $\phi(\tilde{\varrho}^\sigma)$ with diffraction, dissipation, and mode conversion

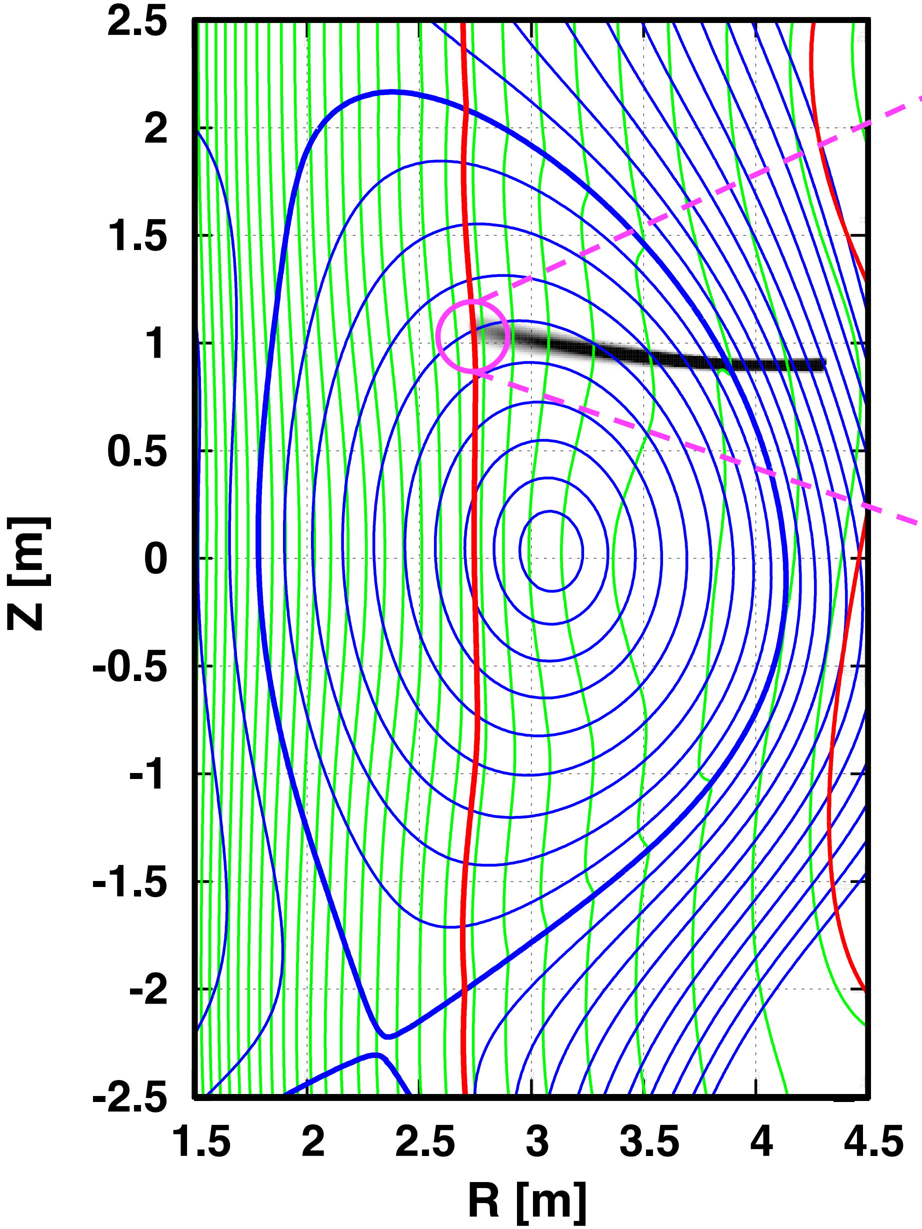
$$\frac{\partial \phi}{\partial \zeta} = \frac{1}{V_\star}\left[-i(\tilde{L}_{\star\sigma\bar{\sigma}}\tilde{\varrho}^\sigma\tilde{\varrho}^{\bar{\sigma}} - U_\star) + \frac{i}{2}\tilde{\Phi}_\star^{\sigma\bar{\sigma}}\partial^2_{\sigma\bar{\sigma}} + \Gamma - \tilde{\vartheta}^\sigma_{\star\bar{\sigma}}\tilde{\varrho}^{\bar{\sigma}}\partial_\sigma - \frac{\tilde{\vartheta}^\sigma_{\star\sigma}}{2}\right]\phi$$

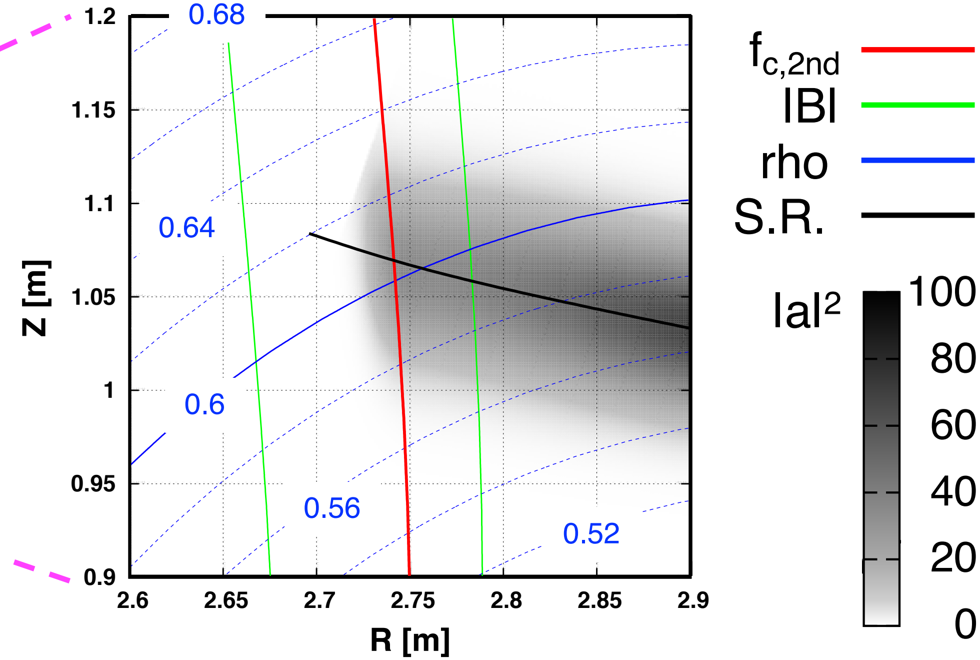
PARADE traces wave beam with its arbitrary quasioptical envelope in inhomogeneous anisotropic media.

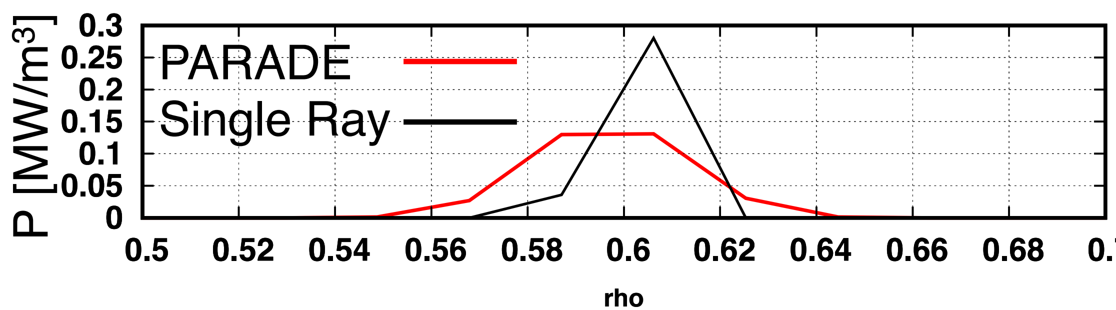
[1] I. Y. Dodin *et al.*, Phys. Plasmas (2019)
[2] K. Yanagihara *et al.*, Phys. Plasmas (2019a)
[3] K. Yanagihara *et al.*, Phys. Plasmas (2019b)
[4] K. Yanagihara *et al.*, Phys. Plasmas (2021)

Quasioptical width broadens power deposition

Prediction for JT-60SA scenario 2. 138 GHz X mode (θ_t, θ_p) = (0,0).



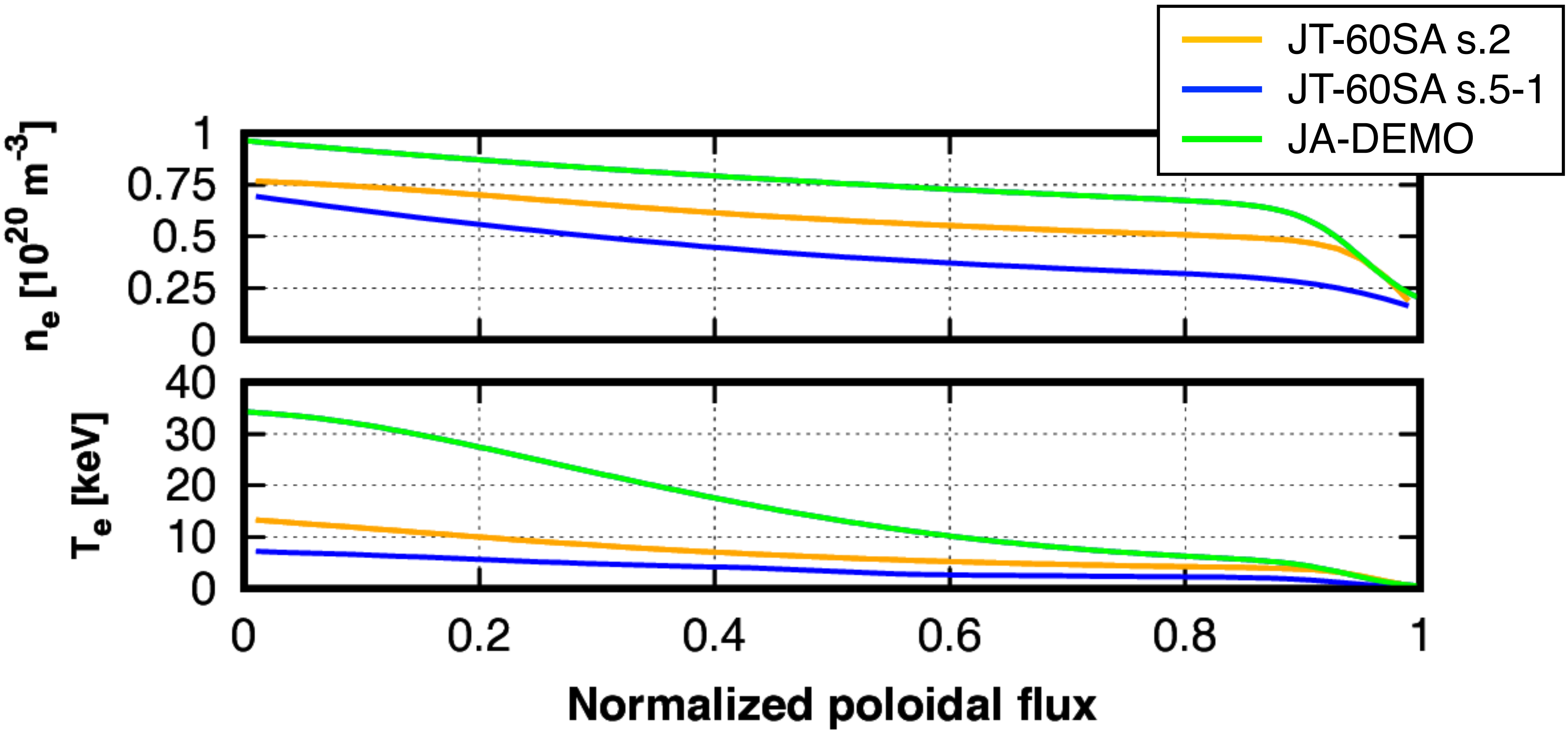




Power deposition profile is broadened.

Electron density and temperature profile

Typical operation scenario 2 & 5-1 of JT-60SA, and JA-DEMO.



These profiles are used for PARADE simulations in this presentation.

Topic 2 <ECCD>

Current drive module based on Adjoint technique

Driven current predictions for tokamak applications

Adjoint technique [5,6] for reasonable simulation

[5] M. Taguchi, Plasma Phys. Controlled Fusion (1989)
[6] N. B. Marushchenko *et al.*, Phys. Plasmas (2011)

$$\langle J_{||} \rangle = \frac{e v_{\rm th}}{\nu_{\rm e0}} \frac{\langle b \rangle}{\langle b^2 \rangle} \langle \int du^3 \frac{\partial \chi}{\partial u} \mathbf{\Gamma}_{\rm rf} \rangle$$: current model same with [6]

$\langle \cdots \rangle$: flux surface average $\qquad u = v\gamma$

$\nu_{\rm e0}$: collisional frequency $\qquad b = |B|/|B_{\rm max}|$

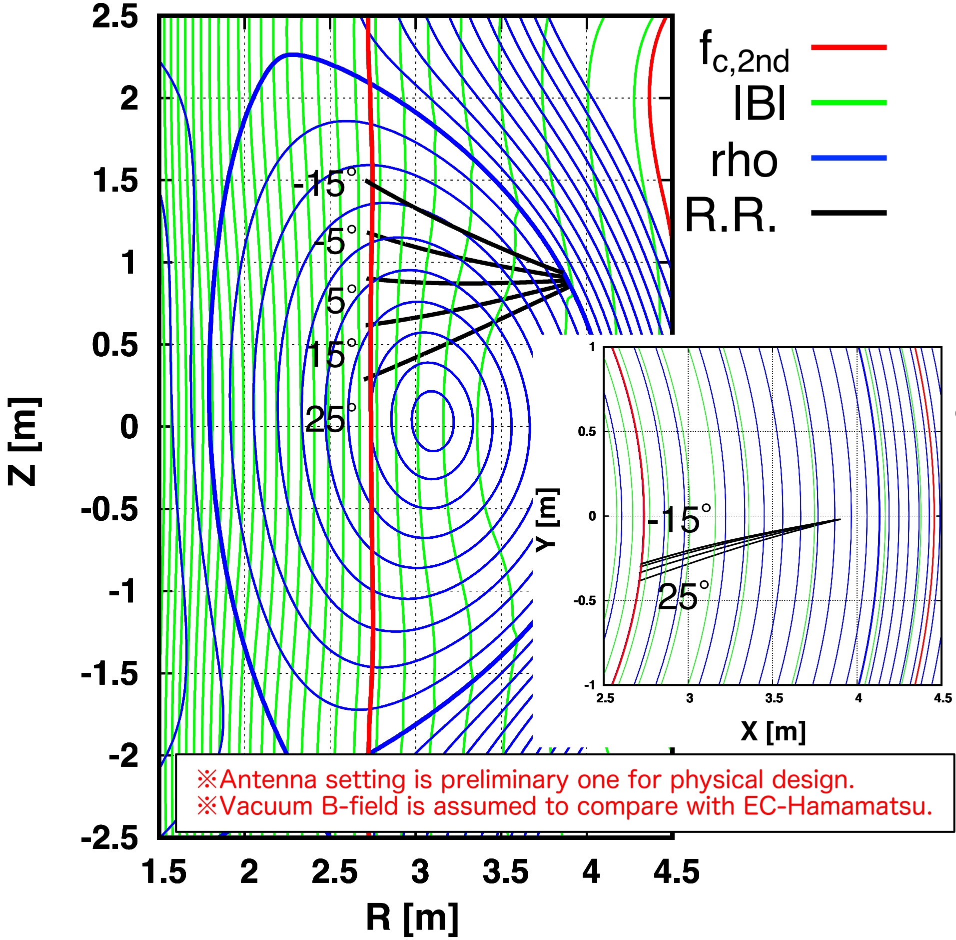
χ : Spitzer function
-> Corresponds to distribution function in weak E-field.
Can be determined immediately, since not to depend on waves.

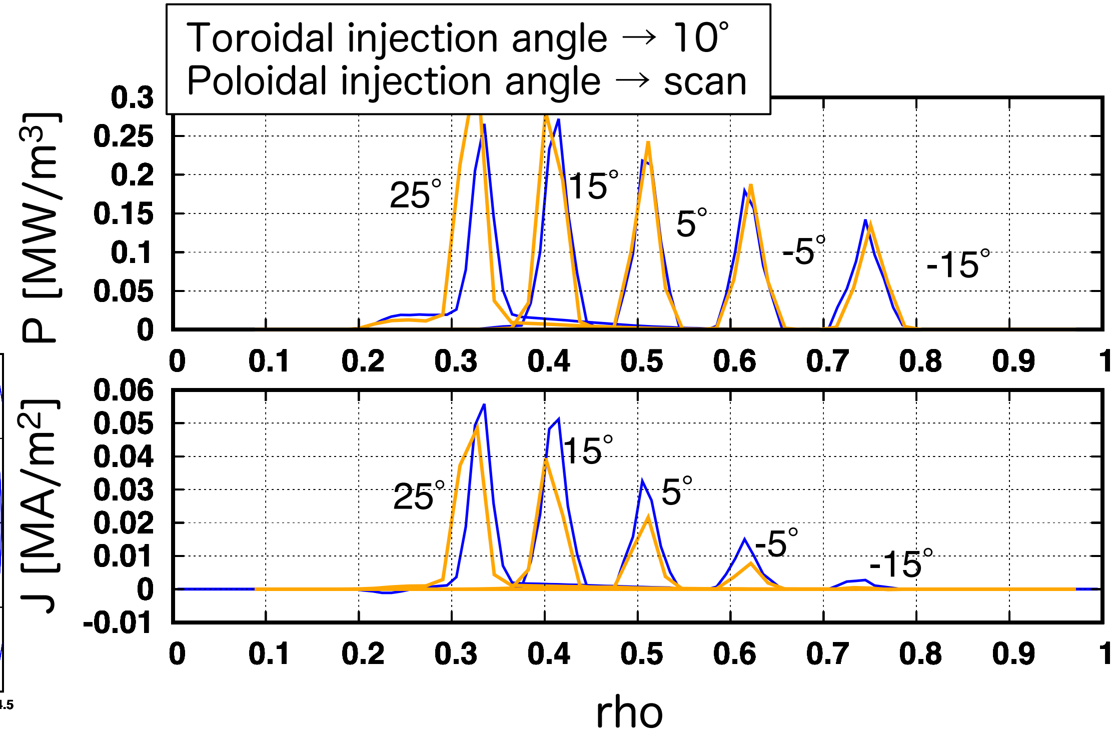
$\mathbf{\Gamma}_{\rm rf} = -\mathrm{D}_{\rm rf} \cdot \nabla f_m$: Quasi-linear diffusion term
-> spatial profile is Improved by PARADE

Widely used due to reasonable computational cost. Namely, evaluate driven current, using Spitzer function instead of a distribution function.
Model equations are same with [6], but spatial profiles of wave vector and amplitude are improved by quasioptical analysis of PARADE.
(u-space 3D integral is reduced to 1D problem as shown in appendix.)

Comparison w/ Multi-ray model used in JT-60SA

Power depo. & current profile in JT-60SA scinario2. 138 GHz X mode.





PARADE : Orange lines

-> Quasioptical-beam model / $D_{Cold,H}$ / $D_{FR,A}$

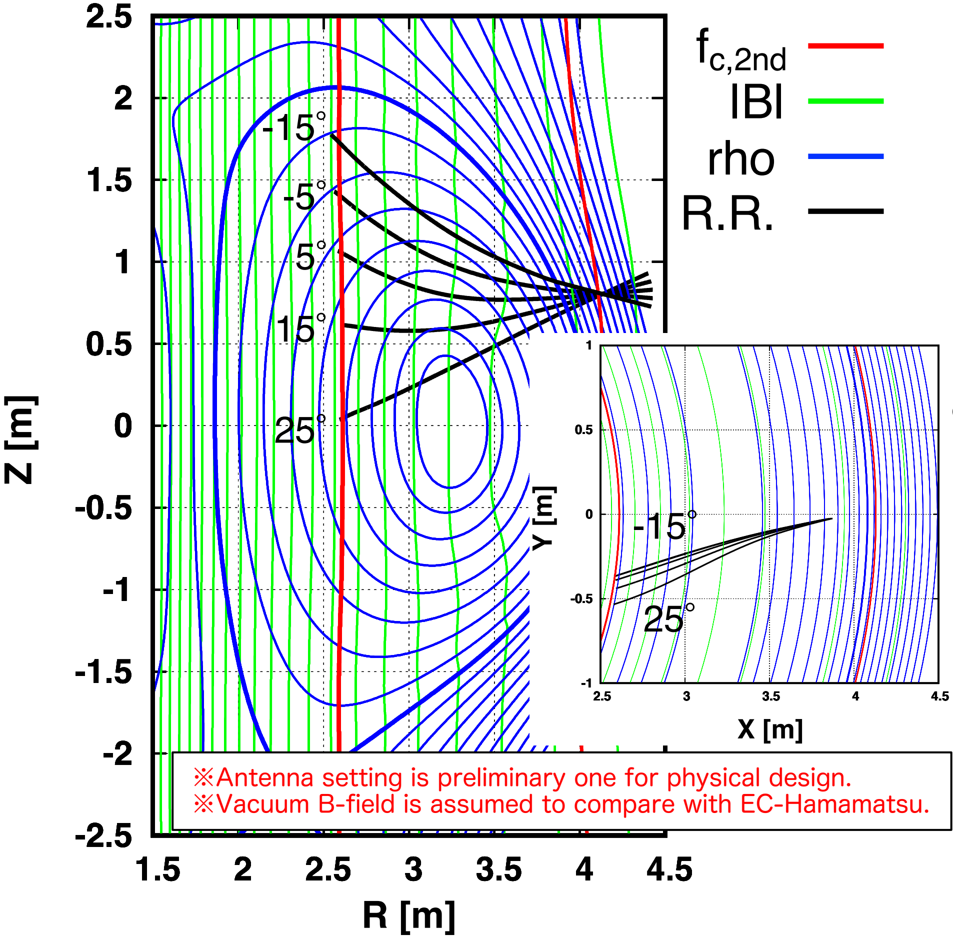
Multi-ray (EC-Hamamatsu) : Blue lines

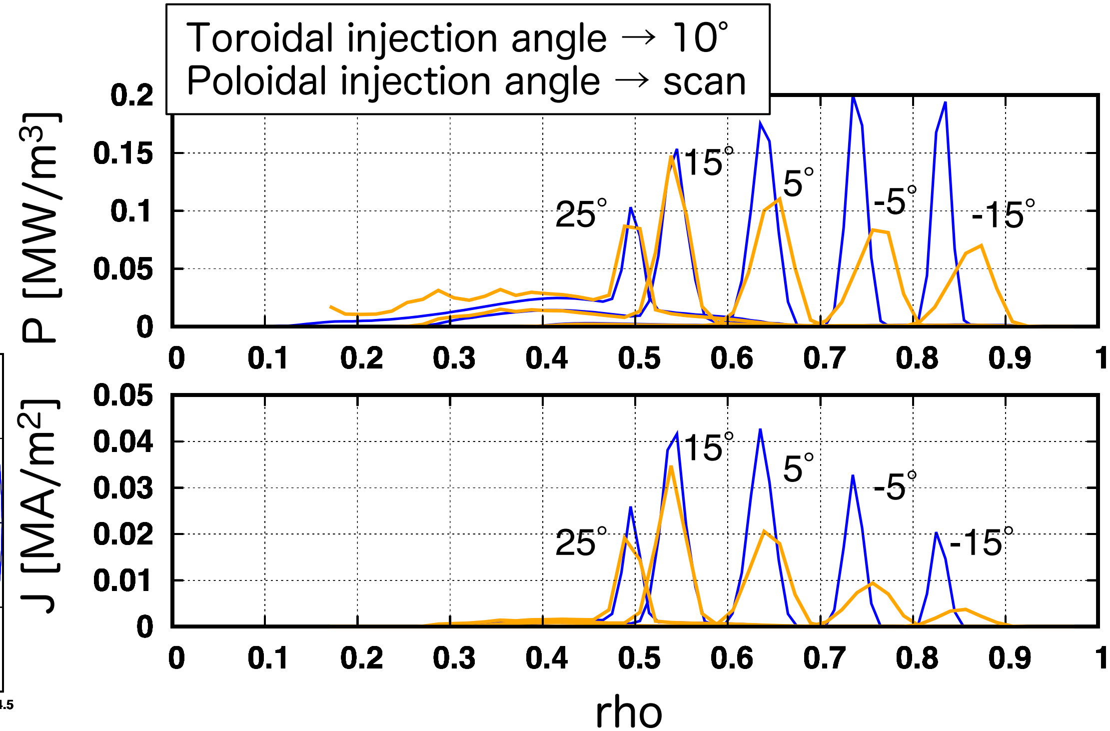
-> Multi-ray model / $D_{cold,H}$ / $D_{WR,A}$

De-focusing beam w/ long path -> Multi-rays are asymptotic to quasioptical beam.
10-15 keV and deep toroidal injection angle -> difference bet. $D_{WR,A}$ and $D_{FR,A}$ is small.
In such a proper situation, reasonable results are obtained.

Comparison w/ Multi-ray model used in JT-60SA

Power depo. & current profile in JT-60SA scinario5-1. 110 GHz X mode.





PARADE : Orange lines
-> Quasioptical-beam model / $D_{Cold,H}$ / $D_{FR,A}$

Multi-ray (EC-Hamamatsu) : Blue lines
-> Multi-ray model / $D_{cold,H}$ / $D_{WR,A}$

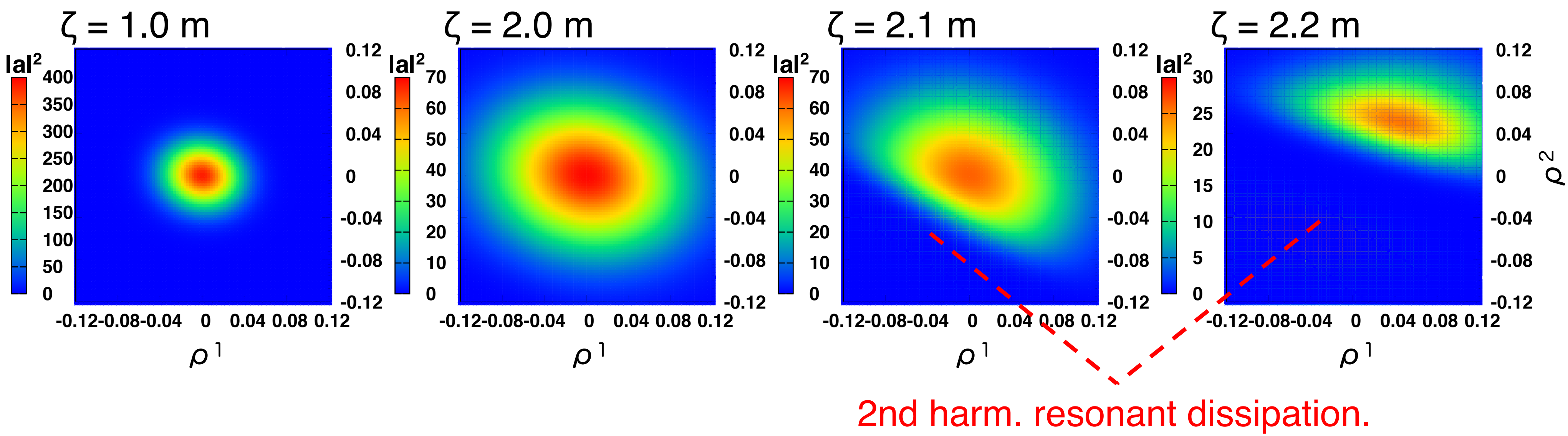
Dissipation-Diffraction interaction, in other words, Absorption-Propagation interaction (see [4,7, next slide]) makes broadening of power deposition & driven current profile. Conventional ray tracing and most quasioptical codes cannot account this effect.

[4] K. Yanagihara *et al.*, Phys. Plasmas (2021)
[7] K. Yanagihara *et al.*, Nucl. Fusion (2021)

Dissipation-Diffraction Interaction

Intensity profile on beam cross sections along the R.R. in JT-60SA scinario5-1. 110 GHz X mode (θ_t, θ_p) = (10°,-15°).

NOTE THAT, Reference Ray (RR) is on (ρ^1, ρ^2) = (0.0 ,0.0).



Beam power is dissipated non-uniformly due to inhomogeneous structure of 2nd harm. resonance. Then, beam profile is shifted from the RR position, (ρ^1, ρ^2) = (0.0,0.0), and broadened due to diffraction.

Topic 3 <ECE>

Electron cyclotron emission module with quasioptical model

EC emission model (1)

Radiation transfer equation, conventionally used on a geometrical optics, is modified to suit for quasioptical system.

Let us start from the conservation low of the wave action flux (or Poynting flux)

$$\frac{\mathrm{d}}{\mathrm{d}\zeta}\int \mathrm{d}^2\varrho\,\phi^+(\zeta,\varrho^1,\varrho^2)\phi(\zeta,\varrho^1,\varrho^2) = \frac{2}{V_\star}\int \mathrm{d}^2\varrho\,\phi^+(\zeta,\varrho^1,\varrho^2)\Gamma(\zeta,\varrho^1,\varrho^2)\phi(\zeta,\varrho^1,\varrho^2)$$

where $\phi^+\phi = V_\star(\zeta)a^+a$

It can be rewritten to simple form as,

$$\frac{\mathrm{d}P(\zeta)}{\mathrm{d}\zeta} = \frac{2}{V_\star}\widehat{W}[\Gamma](\zeta)\,P(\zeta)$$

where $P(\zeta) = \int \mathrm{d}^2\varrho\,\phi^+(\zeta,\varrho^1,\varrho^2)\phi(\zeta,\varrho^1,\varrho^2)$

$$\widehat{W}[f](\zeta) = \frac{\int \mathrm{d}^2\varrho\,\phi^+(\zeta,\varrho^1,\varrho^2)f(\zeta,\varrho^1,\varrho^2)\phi(\zeta,\varrho^1,\varrho^2)}{\int \mathrm{d}^2\varrho\,\phi^+(\zeta,\varrho^1,\varrho^2)\phi(\zeta,\varrho^1,\varrho^2)},$$

EC emission model (2)

Radiation transfer equation, conventionally used on a geometrical optics, is modified to suit for quasioptical system.

Introduce the source term $\mathfrak{B}$ to consider the radiation as,

$$\frac{\mathrm{d}P(\zeta)}{\mathrm{d}\zeta} = \mathfrak{B}(\zeta) - \mathfrak{A}(\zeta)\,P(\zeta)$$

where $\mathfrak{A}(\zeta) = -\frac{2}{V_\star}\widehat{W}[\Gamma](\zeta)$

$$\mathfrak{B}(\zeta) = \widehat{W}[\beta](\zeta) = \frac{\int \mathrm{d}^2\varrho\,\phi^+(\zeta,\varrho^1,\varrho^2)\beta(\zeta,\varrho^1,\varrho^2)\phi(\zeta,\varrho^1,\varrho^2)}{\int \mathrm{d}^2\varrho\,\phi^+(\zeta,\varrho^1,\varrho^2)\phi(\zeta,\varrho^1,\varrho^2)}$$

This representation is quite similar with a conventional model,

$$N_r^2\frac{\mathrm{d}}{\mathrm{d}\zeta}\left(\frac{I(\zeta)}{N_r^2}\right) = \beta(\zeta) - \alpha(\zeta)\,I(\zeta)$$

Hence, we can follow a derivation of it, and obtain the modified equation that,

$$P(\zeta) = \int \mathrm{d}\zeta\,\mathfrak{B}(\zeta)\mathrm{e}^{-\tau(\zeta)} \qquad \text{where } \tau(\zeta) = \int_0^\zeta \mathrm{d}\zeta'\,\mathfrak{A}(\zeta')$$

$$T_{\mathrm{rad}} = \frac{8\pi^3c^2}{\omega^2}P$$

Topic 4 <D_A>

Fully relativistic dispersion for Anti-Hermitian part : absorption

Numerical integrated D_A for any (=fully relativistic) f_0

Summarized from classic hot dispersion tensor [8,9] as,

[8] K. Yanagihara, Nucl. Fusion (2022)
[9] M. Bornatici *et al.*, Phys. Plasmas (1994)

$$D_A = \frac{\varepsilon_a}{\varepsilon_0} = -\pi X \sum_{\mathrm{n}=-\infty}^{\infty} \int \frac{\mathrm{d}u^3}{\gamma} \delta(R_\mathrm{n}) \boldsymbol{V}_\mathrm{n} \boldsymbol{V}_\mathrm{n}^* \boldsymbol{G} \cdot \nabla f_0$$

$$\boldsymbol{V}_\mathrm{n} = u_\perp \begin{pmatrix} \frac{1}{2}[J_{\mathrm{n}-1}(b) + J_{\mathrm{n}+1}(b)] \\ \frac{i}{2}[J_{\mathrm{n}-1}(b) - J_{\mathrm{n}+1}(b)] \\ \frac{u_\parallel}{u_\perp} J_\mathrm{n}(b) \end{pmatrix} \qquad R_\mathrm{n} = \mathrm{n}Y + N_\parallel \frac{u_\parallel}{c} - \gamma$$

$$\boldsymbol{G} \cdot \nabla = \frac{1}{u_\perp} \left(\gamma - N_\parallel \frac{u_\parallel}{c}, N_\parallel \frac{u_\perp}{c} \right) \cdot \begin{pmatrix} \partial_\perp \\ \partial_\parallel \end{pmatrix} \qquad b = \frac{N_\perp}{Y} \frac{u_\perp}{c}$$

$$\int \frac{\mathrm{d}u^3}{\gamma} = 2\pi \int u_\perp \mathrm{d}u_\perp \int \mathrm{d}u_\parallel \qquad X = \frac{\omega_p^2}{\omega^2} \qquad Y = \frac{\omega_c}{\omega}$$

Instead of using Z or F functions, directly adopt a relativistic-Maxwell distribution function into this model and numerically integrate. (※Any distributions are available.) The problem is the calculation cost of 2D integral in velocity space. However, we succeeded to reduce it to 1D problem along the resonance curve.

1-dimentional integral along a resonance curve

Schematic image in velocity space

Resonance Curve (RC) satisfies

$$R_\mathrm{n} = \mathrm{n}Y + N_{\|}\frac{u_{\|}}{c} - \gamma = 0$$

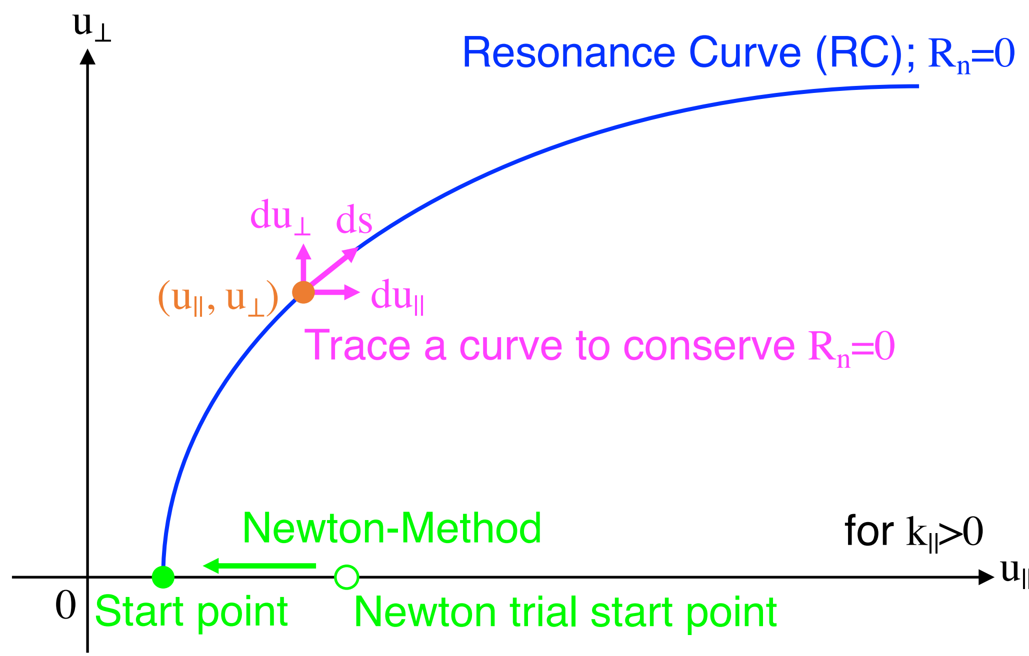
2D integral w/ $\delta(R_n{=}0)$ can be reduced 1D integral along the RC as,

$$\int \mathrm{d}u_\perp \int \mathrm{d}u_{\|}\delta(R_\mathrm{n}) = \int \mathrm{d}s$$

We can trace the RC by using [8],

$$\frac{\mathrm{d}u_{\|}}{\mathrm{d}s} = -\frac{\partial R_\mathrm{n}}{\partial u_\perp} = \frac{u_\perp}{\gamma c}$$

$$\frac{\mathrm{d}u_\perp}{\mathrm{d}s} = \frac{\partial R_\mathrm{n}}{\partial u_{\|}} = \frac{k_{\|}}{\omega} - \frac{u_{\|}}{\gamma c}$$



Start point can be found on $u_{||}$ axis by using Newton-Method.

Newton trial start point can be chosen easily from non-relativistic resonance condition,

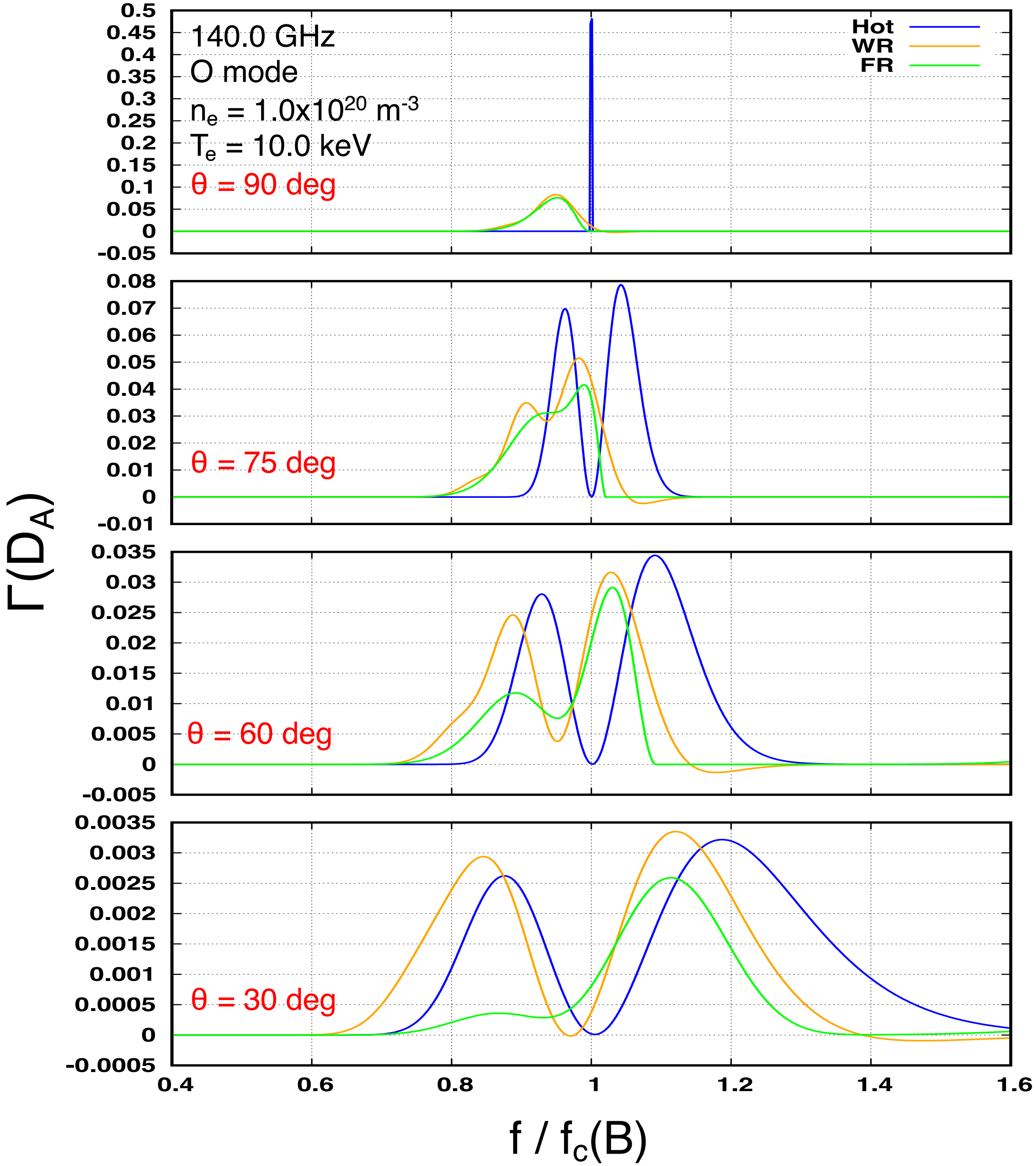
$$u_{\|} = (\omega - \mathrm{n}\omega_c)/k_{\|}$$

-> Substantial reduction of calculation cost is succeeded by introducing RC trace.

[8] K. Yanagihara, Nucl. Fusion (2022)

Injection-angle dependence of each dissipation model

Dissipation coefficients Γ w/ $D_{Hot,A}$, $D_{WR,A}$, and $D_{FR,A}$, as functions of f/f_c



All Γ show Doppler broadening, which is significant in parallel-injection.

Γ_{Hot} gets too much narrow and large peak in perp.-injection, while Γ_{WR} & Γ_{FR} are in good agreement.

Γ_{Hot} and Γ_{WR} seems to be over-estimated, for down-shift side of Doppler broadening, especially for parallel-injection.

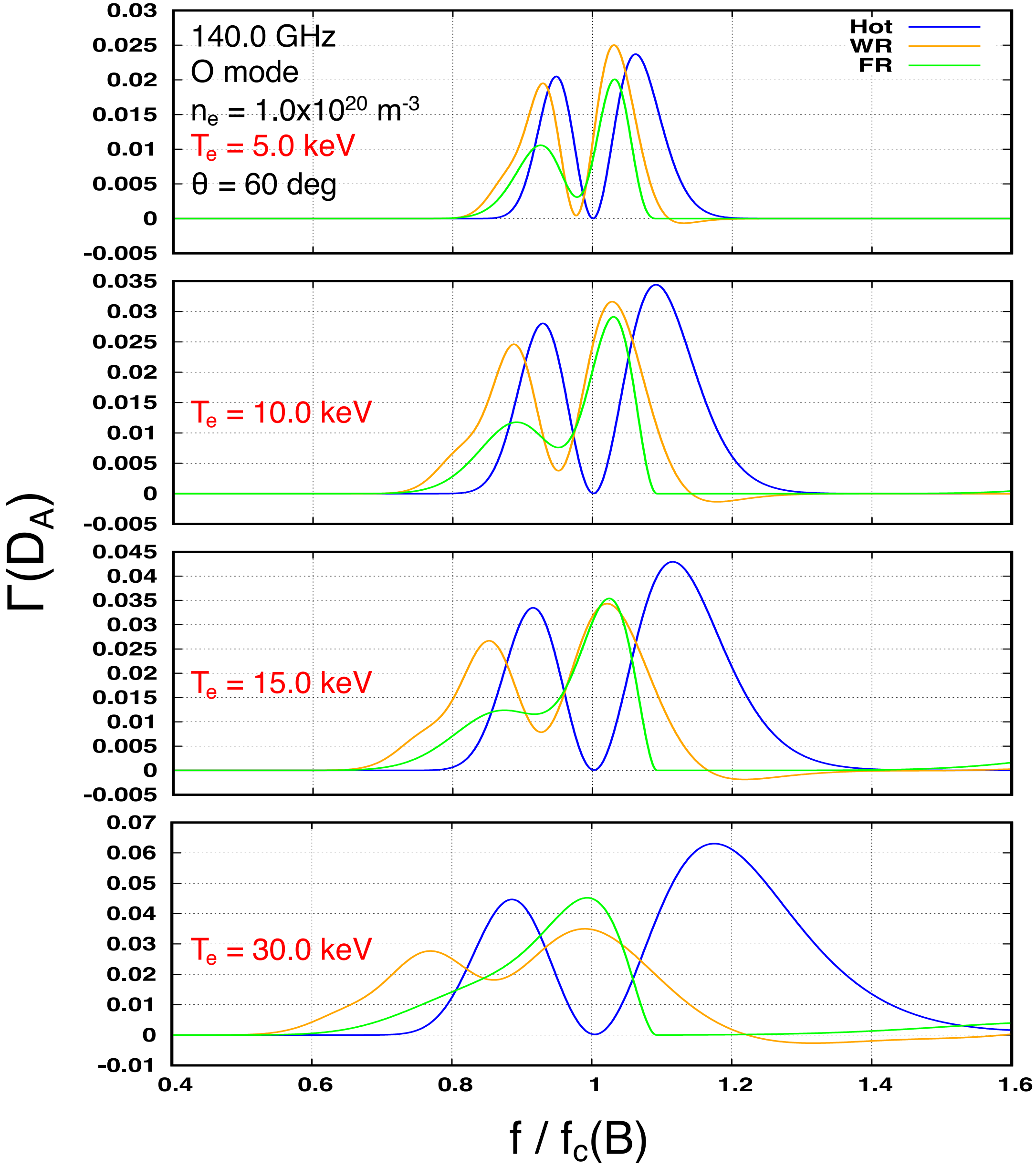
→$D_{FR,A}$ is essential for dissipation modeling w/ oblique injection-angle.

Also...
Γ_{WR} has negative values in all angles, which need numerical cutoff in practice.

[8] K. Yanagihara, Nucl. Fusion (2022)

Temperature dependence of each dissipation model

Dissipation coefficients Γ w/ $D_{Hot,A}$, $D_{WR,A}$, and $D_{FR,A}$, as functions of f/f_c



Γ_{WR} & Γ_{FR} show relativistic down-shift, which become significant in high temp..

Γ_{Hot} and Γ_{WR} seems to be over-estimated, for down-shift side of Doppler broadening, especially for high temp..

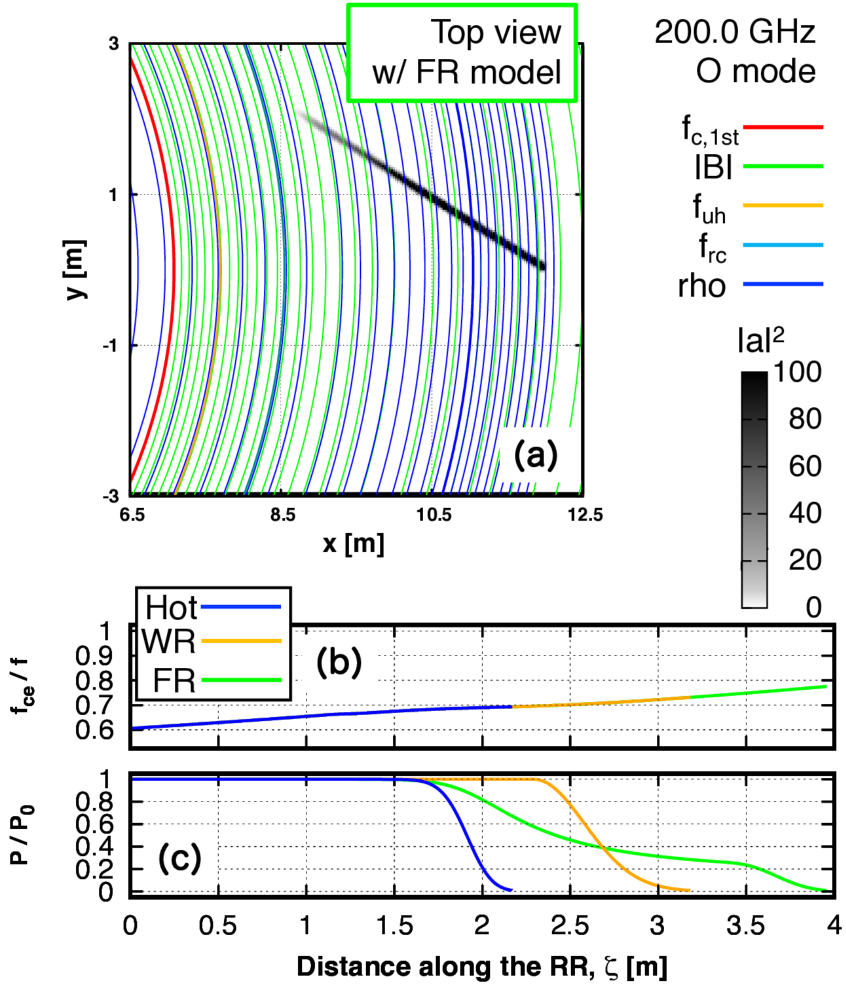
→$D_{FR,A}$ is essential for dissipation modeling in high temp. condition.

Also...
Γ_{WR} has negative values in all angles, which need numerical cutoff in practice.

[8] K. Yanagihara, Nucl. Fusion (2022)

Power absorption w/ fully relativistic D_A in JA-DEMO

Power absorptions simulated by PARADE with Hot, WR, and FR model



Hot : 1st reso.
WR : 1st reso.
FR(1.8-3.5 [m]): 2nd reso.
FR(3.5- [m]): 1st reso.

FR revealed a contamination of second-order resonance, which was not found by Hot and WR.

Outside launch is difficult for localized heating.

As a solution, we propose top launch.

Example for JA-DEMO, perpendicularly-injected from out-side position.

WR and FR model give different result in obliq. injection.

-> Full-Relativity should be considered in high T_e condition.

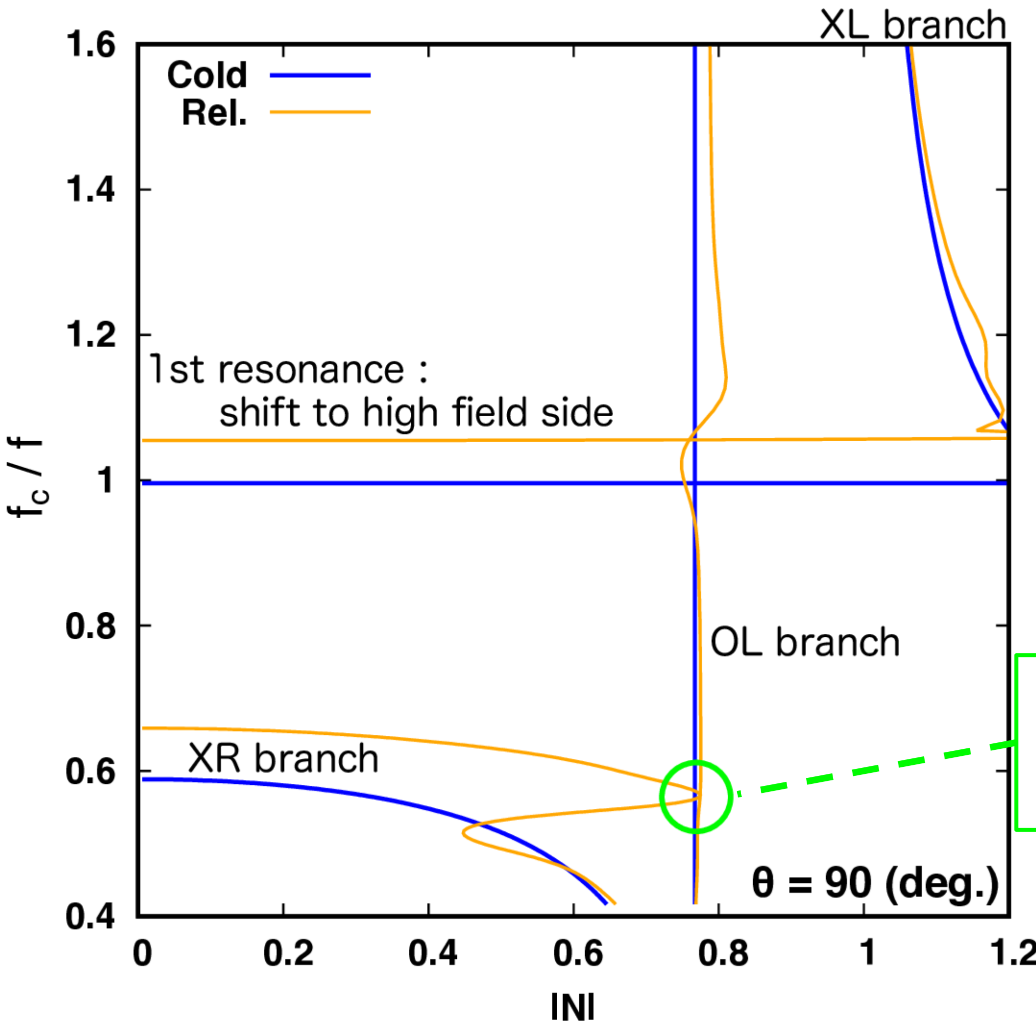
[8] K. Yanagihara, Nucl. Fusion (2022)

Topic 5 <D_H>

Weakly relativistic dispersion for Hermitian part : propagation

Weakly relativistic dispersion for Hermitian part

D_H governs wave propagations.



$|D_H|$ = 0 for Cold and WR [10].

[10] F. Volpe, Phys. Plasmas (2007)

Freq = 140.0 GHz
n_e = 1.0×10^{20} m^{-3}
T_e = 15.0 keV

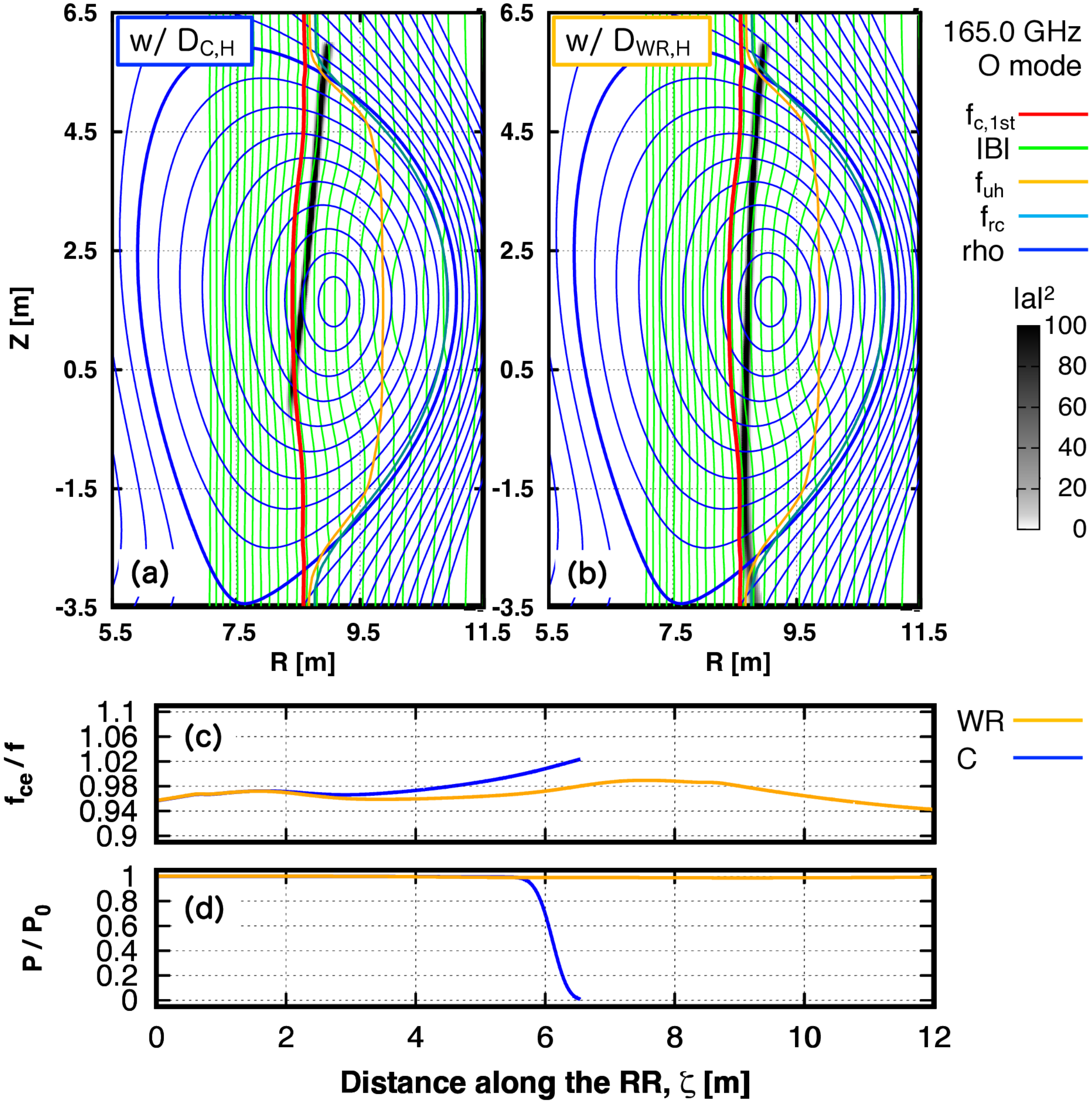
OX mode conversion can occur, but let us leave this interesting problem for future work.

OL & XL : WR branches deviate from Cold branches near the 1st resonance.
XR : WR branch deviates from Cold branch near the 2nd resonance.
→ $D_{WR,H}$ makes different propagation path from conventional $D_{C,H}$ near resonances.

[8] K. Yanagihara, Nucl. Fusion (2022)

Weak relativity makes additional beam bending

WR D_H makes extra-bending of beam path near the resonance.



Simulations using PARADE with Cold and WR D_H.

Propagation path is corrected by installing WR.

In this example, WR trajectory does not reach resonance and does not absorbed at all, while Cold trajectory completely absorbed.

If ones believe Cold D_H on JA-DEMO…
Mis-identification of heating position, damage the facing wall with power of 100 MW/m^2, and reflected stray can also damage the diagnostics.

[8] K. Yanagihara, Nucl. Fusion (2022)

Summary

- We report today Application results of Quasioptical code PARADE to ECRH, ECCD, and ECE modeling with relativistic effects in JT-60SA and JA-DEMO plasmas.
- In order to predict ECRH, ECCD, and ECE in reasonable costs with high accuracy, we introduced 5 topics as follows.
- 1. Quasioptical beam model gives broadening of power deposition profile.
- 2. Current drive module based on Adjoint technique to suit for PARADE gives reasonable result.
- 3. EC emission module with quasioptical model gives qualitatively consistent result but need more detailed validations.
- 4. Fully relativistic dispersion with 1D integral for D_A gives different absorption rate, especially in high T_e condition and large $N_{//}$ condition.
- 5. Weakly relativistic dispersion for D_H gives variation of a beam propagation path near the resonance.
- Topics 1-3 will be reported in a new paper with the same (or similar) title as this presentation. Topics 4,5 have been reported in [8].

[8] K. Yanagihara, Nucl. Fusion (2022)